CSCE 2211 Spring 2024 Applied Data Structures
Assignment #3

Dr. Amr Goneid ***Date: Mon Mar 4, Due: Mon Mar 11, 2024***

Solve the following problems and report the solutions in soft form on the current document using Microsoft Office Word or a MathType editor.

1. (15 Points)

For the following code segments, find the worst case number of operations ***T(n)*** and the complexity of the segment in terms of the Big-O

Code Segment	***Operations to count***	***T(n)***	***Big-O***
i = 1 ; ***while (a[i-1] <= a[i] && i < n) i++ ;***	*Comparisons*		
k = 1; ***for (i = 1; i <= n/2; i++)*** ***for (j = 1; j <= n; j += 2) k *= 2 ;***	*Multiplications*		
k = 0; ***for (i = 0; i < n-1; i++)*** ***for (j = i+1; j < n; j++) k *= 3 ;***	*Multiplications*		
for (i = 0; i < n; i++) ***for (j = 0; j < n; j++)*** ***if (a[i,j] != a[j,i]) a[j,i] = a[i,j];***	*Array Element Accesses* *[ ]*		
double Foo (double a[ ], double x, int n) ***{ double y =a[0]; double p = 1;*** ***for (int i = 1; i <= n; i++) {*** ***p = p * x; y = y + a[i] * p; }*** ***return y;*** ***}***	*Double arithmetic operations*		

2. (10 points)

Find the Big-O for the following number of operations:

No. of Operations	***O(. . .)***
$T(n) = 10n + 2n\log n + 5\sqrt{n}\log n$	
$T(n) = 2^n + n^2 + n!$	
$T(n) = \frac{n^2 - 1}{n+1} + 20\log n^3$	
$T(n) = 1 + 2 + 4 + 8 + ... + 2^{n-1}$	

3. (15 points)

A 2-D array ***A[0..n-1][0..n-1]*** represents a binary image of *n* by *n* pixels. An element of ***A*** is either "0" representing a black pixel or "1" representing a white pixel. The following algorithm receives the image ***A*** and produces another binary image ***B*** of the same size

```
ALGORITHM Enigma (A , B, n)
{
   for (int i = 0; i < n; i++)
     for (int j = 0; j < n; j++)
           B[i][j] = (A[j][i] + 1) % 2;
}
```

- What does this algorithm do?
- Find the number of arithmetic operations ***T(n)*** done for a given (***n***).
- What is the Big-O complexity of the algorithm in terms of (***n***).

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4. (15 points)

Consider the following two algorithms:

```
ALGORITHM Fun (a[ ], int s, int e)
{
  int m = s ;
  for  j =  s+1 to e
           if (a[j] < a[m])  m = j ;
  return m;
 }

ALGORITHM Process (a[0..n-1 ] , int n )
{
   for i = 0 to n-2 {
           m = Fun (a , i , n-1) ;
           t = a[i];  a[i] = a[m];  a[m] = t; }
}
```

- What are the objectives of these two functions?
- Find the number of array element comparisons T(n) done by a call of ***Fun (a , s, e),*** and by a call of ***Process(a,n)*** as a function of (n).
- What are the corresponding Big-O complexities in terms of (n)?

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5. (15 points)

For an array of size (n > 1), suppose algorithm (A) takes $n\ (\log n)^2$ microseconds to process the array and algorithm (B) takes $16\ n^{1/2} \log n$ microseconds to do the same job:

- For what values of (n) does program (A) take less time than (B)?
- For each of these algorithms, what will be the time spent to process an array of size $n = 2^{10}$?

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6. (15 points)

Consider a randomly ordered array ***X[0..n-1]*** of floating point numbers and the following algorithm

```
ALGORITHM BFCP (X, n)
{
  y = ∞;
  for i = 1 to n − 1 do
    for j = i + 1 to n do
          z = |X[i] – X[j]|
          if (z < y) { y = z; K = i; L = j;}
  return y , K, L;
 }
```

- What does this algorithm do?
- Find the number of comparisons ***T(n)*** done for a given (***n***).
- What is the Big-O complexity of the algorithm in terms of (***n***).

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7. (15 points)

Consider the following two functions:

```
int Fun1 (int n)
{
      if (n == 1) return 0;    else return (1 + Fun1 (n/2));
}
```

```
int Fun2 (int n)
{
      int L = 0;
      while (n > 1) { n = n/2;   L++;  }
      return L;
}
```

- Trace the two functions for n = 1 , 2 , 4 , 8. To do this, build a table containing (n), the returned value from each function for that (n), and the total number of integer arithmetic operations T(n) needed to achieve the result.

n	Fun1(n)	T1(n)	Fun2(n)	T2(n)
1				
2				
4				
8				

- What is the complexity (Big-O) of each function?
- What are the objectives of these two functions and in what way do they differ?
