CSCE 2211 Exercises
Exercises (6): Disjoint Sets

1. Consider the union-find disjoint set structure. Prove that in case of the simple union, the worst-case find cost is ***O(n),*** where ***(n)*** is the size of the sets.

2. Consider 8 disjoint sets with parents 0, 1, 2,….., 7. Show their tree structure after each of the following simple union operations (in the given order):
   union (4,5) then **union (6,7)** then **union (4,6)**

3. For the sets you obtained in question 2, show their tree structure after the operation **union(3,4)** in case of:
   a) Simple Union
   b) Union by size
   What are the maximum Tree heights in the above two cases?

4. For the two set structures you obtained in question 3, compute the average cost per node for the **find** operation.

5. Implement a disjoint set class that uses **Union-by Height**.

6. Consider 16 disjoint sets with parents 1, 2,….., 16. Show the result of the following sequence of instructions:
   union(1,2), union(3,4), union(3,5), union(1,7), union(3,6), union(8,9), union(1,8), union(3,10), union (3,11), union(3,12), union(3,13), union(14,15), union(16,0), union(14,16), union (1,3), union(1, 14) when the unions are:

   a. performed arbitrarily (simple union)
   b. performed by size.
   c. performed by height.

7. Suppose we want to add an extra operation, **remove(x)**, which removes x from its current set and places it in its own. Show how to modify the union/find algorithm.