Joint Resource Management with Distributed Uplink Power Control in Full-Duplex OFDMA Networks

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Abstract—Resource allocation problems in full-duplex orthogonal frequency division multiple access (FD-OFDMA) networks are challenging due to their combinatorial, non-convex nature. In this paper, user pairing, subcarrier and power allocation in a single cell FD-OFDMA network are considered. A joint optimization problem is formulated to maximize the network’s sum rate while satisfying downlink (DL) and uplink (UL) transmission power constraints. Due to the sheer complexity of the proposed formulation, mainly due to its combinatorial nature, an efficient, iterative two-step solution algorithm for the joint problem is proposed. In the first step, based on defining the DL user equipment (UE) signal to noise ratio (SNR) threshold, which is the least SNR that can be detected by the DL-UE, an algorithm is proposed for user pairing and subcarrier assignment. In the second step, the power allocation problem for the assigned users’ pairs is formulated and solved using the Alternating Direction Method of Multipliers (ADMM) in the high signal-to-interference-noise ratio regime. Finally, numerical results are presented to validate the performance of the proposed algorithms. We show that the performance of our proposed computationally-efficient two-step algorithm is very close to the sum rate upper bound derived from solving the dual problem.

Keywords: ADMM, Full Duplex, OFDMA, Power Allocation, Resource Allocation, SNR Threshold, Subcarrier Allocation.

I. INTRODUCTION

Modern wireless communication networks are continuously required to offer a significant increase in the network capacity in order to be able to support the enormous growth in the number of wireless communication users. Accordingly, efficient resource allocation algorithms have become a crucial need. However, most of the existing communication networks waste the available resources by utilizing half-duplex (HD) communication. Theoretically speaking, enabling the network nodes to simultaneously transmit and receive data at the same time slot and same channel, i.e. utilizing full-duplex (FD) communication, can double the aggregate network throughput.

A. Literature Review

Although FD communication was considered unfeasible, because of the high self-interference (SI) from the node transmission on the node reception, the recent evolution in SI cancellation techniques [2]–[5] reinvigorates the attention to FD communication and nominates the FD communication as a technique that is able to supply the needed high rates [6]. In [7], it was shown that both user diversity gain and FD communication gain can be achieved, and the performance of FD communication highly depends on the strength of the residual SI. Additionally, studying the recent development and future directions of resource allocation in different FD systems attracts recent research work [8]–[10] to explore the new network resources in different domains, including power, space, frequency, and device dimensions. It is found that FD can outperform HD in both interference-unaware and interference-aware scenarios [11]. Additionally, in [12], the authors derived necessary conditions for the FD mode achieves a better energy efficiency (EE)-spectral efficiency (SE) tradeoff than the HD mode. Moreover, it was proved that it is possible for the industry to design the optimal EE-oriented resource allocation strategy while guaranteeing a given required SE. Accordingly, to fully utilize the available FD resources while taking into consideration the new challenges that will arise from deploying FD, efficient and novel resource allocation schemes are strongly needed [13], [14].

In addition, deploying FD in multiple access networks like orthogonal division multiple access (OFDMA) recently gains a lot of attention. However, different from HD-OFDMA networks that require subcarrier (SC) allocation, in FD-OFDMA each subcarrier serves simultaneous transmissions in the uplink (UL) and the downlink (DL) modes. Therefore, efficient pairing between users transmitting in the UL and users receiving in the DL into independent transceivers is required to decrease the co-channel interference (CCI) introduced from the UL transmission on the DL reception 1. This additional optimization requirement increases the complexity of the resource allocation in FD-OFDMA networks due to the combinatorial nature of the subcarrier assignment and users’ pairing. In [15], a joint subcarrier scheduling and power allocation problem to maximize the sum rate under both perfect and imperfect SI cancellation scenarios is proposed.

1Throughout the paper, users who are transmitting in the UL are denoted by UL users and users who are receiving in the DL are denoted by DL users.
For the perfect cancellation scenario, subcarrier scheduling and power allocation are optimized by applying the Lagrange duality method. For the imperfect cancellation, an iterative algorithm based on the projected gradient method is proposed. In [16], the joint problem of subchannel assignment and power allocation in FD-OFDMA network considering the inter-node interference is investigated with both full and limited channel state information (CSI) knowledge. In the case of limited CSI, a low-complexity inter-node interference estimation is presented. In [17], the joint optimization problem of transmission mode selection, subcarrier assignment, relay selection, subcarrier pairing as well as power allocation is investigated for OFDMA networks. The binary assignment problem is transformed into a maximum weighted bipartite matching problem which can be solved by the classical Hungarian method.

The aforementioned work did not consider the user pairing optimization in maximizing the FD-OFDMA sum rate. However, user pairing is considered in [18], [19], in which the joint optimization problem of subcarrier assignment, UL-DL user pairing, and power allocation is solved by the dual method in which it is decomposed into a primal problem and a dual problem. The concave-convex procedure is used to transform the primal problem into a tractable form through sequential convex approximations while the sub-gradient method is utilized to solve the dual problem. In [20], the effects of different system parameters on the FD-OFDMA network performance are studied. Moreover, a joint resource allocation problem which aims at maximizing the network sum rate by considering mode selection, user pairing, subcarrier allocation and power control is proposed and solved by relaxing the subcarrier assignment variables to the continuous domain. In [21], the FD-OFDMA allocation problem is discussed and solved using the matching theory. In [22], a joint algorithm that aims at maximizing the utility sum of users while fully exploiting the capacity benefit of FD communication is proposed. The key idea of the proposed algorithm is the assignment of a transmission mode, users and transmit power levels jointly for a frequency resource block, which is a group of contiguous subcarriers, based on the awareness of residual SI. Moreover, since maximizing the network’s sum rate will affect the fairness among the network’s users, some work discussed the fairness problem in FD-OFDMA networks. Fairness can be achieved through multiple approaches like maximizing the max-min fairness rate, by imposing the minimum rate constraint for each UL and DL user, or guaranteeing at least on subcarrier for each UL and DL user. Maximizing the system max-min fairness rate in an FD multi-user OFDMA system is addressed in [23], where the uplink/downlink transmission direction assignment, user paring, and power allocation problem are jointly optimized to maximize the system max-min fairness rate. To solve the joint NP hard problem, the authors proposed efficient methods based on simple relaxation and greedy rounding techniques. In [24], the authors proposed a queue-aware, fair scheduling and power allocation problem for FD-OFDMA networks. The proposed problem aims at maximizing the user equipments (UE) signal-to-interference noise ratio (SINR) values, while at the same time enforcing fairness among the UEs. Solving such a problem requires information on the UE radio conditions, their queue statuses, as well as an innate definition of fairness. Accordingly, the authors define a UE pair priority and formulate the problem with the objective of maximizing the sum of these priorities. It is shown that the proposed approach improves fairness among the user equipment at no cost in the system’s performance.

B. Contributions

To maximize the network DL and UL sum rate while satisfying transmission power constraints, we consider the joint users’ pairing, subcarrier allocation, and power allocation problem. Our main contributions in this work are as follows:

1) We propose a computationally efficient, polynomial-complexity joint user pairing and subcarrier allocation algorithm which is based on defining the least detectable received SNR by the UE, i.e., the signal-to-noise (SNR) threshold [25], [26]. The existence of the SNR threshold is a practical specification of the DL-UE that will help in defining a set of candidate DL, UL, and subcarriers that have a very low CCI level. It is shown that the complexity of the proposed algorithm is lower than the algorithm presented in [21].

2) Different from the water filling power allocation [27] presented in [21], we analyze and solve the power allocation problem for the proposed FD-OFDMA network. The power allocation problem’s approximation for the high signal-to-interference noise ratio (SINR) is proved to be convex, and hence, it is formulated and solved distributively using the Alternating Direction Method of Multipliers (ADMM) [28], [29]. Additionally, the proposed power allocation problem is different from the power allocation problem in the FD bidirectional channel presented in [31] as the power allocation in the case of FD-OFDMA is done over all subcarriers which results in different objective function and constraints.

3) From numerical results, it is shown that the convergence of the proposed power allocation algorithm is faster than the centralized interior-point based algorithm. Additionally, we show that our proposed algorithm can achieve a performance that is very close to the optimal solution. Additionally, these results validate the effectiveness of the proposed low-complexity algorithm in achieving a good performance that is close to the upper bound obtained by the iterative approximation approach considered in [19]; however, it should be stated that proposed sum rate maximization may not achieve fairness among the network’s users, however, fairness is out of our work scope and will be

The optimal power allocation in FD-OFDMA networks is different from the traditional water-filling in HD-OFDMA [30] as in HD-OFDMA, both the UL and DL transmissions are independent, in which water-filling is proved to be optimal. The main idea of water-filling is to allocate more power for the channel with better signal-to-interference noise ratio (SINR) either in the UL or the DL transmission. However, in the case of FD, the presence of residual SI from the UL transmission on the DL transmission, and the presence of the CCI from the UL transmission on the DL transmission will make both UL and DL transmissions dependent. Hence, the optimal power allocation will be different.
considered in our future work. As mentioned before, fairness in FD-OFDMA networks is discussed in [23], [24].

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the joint user pairing, subcarrier, and power allocation problem, along with the proposed solution algorithm are presented. In Section IV, numerical analysis is presented to validate the performance of the proposed solution algorithm. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

In this paper, we consider a single cell, time division duplex (TDD) network with a FD-AP operating in OFDMA with S subcarriers. In order to realize FD feasibility, the AP is equipped with a special FD radio which provides linear cancellation, non-linear cancellation, and analog cancellation to cover up for the self-interference, nonlinear harmonic components, quantization and transmitter noise [2]–[4]. All subcarriers are assumed to be perfectly orthogonal, i.e., there is no inter-subcarrier interference. There are N HD single antenna users, and therefore, in a given time slot, the AP connects with N/2 UL users and N/2 DL users. In every time slot, the AP assigns a given subcarrier s to simultaneously serve the n\textsuperscript{th} UL user transmission along with the m\textsuperscript{th} DL user transmission. In that case, the received UL signal at the AP will be affected by the SI from the AP DL transmission. Additionally, the received signal from the AP at the m\textsuperscript{th} DL user will suffer from the CCI from the UL transmission that shares the same subcarrier. Therefore, in order to improve the network spectral efficiency, it is needed to decrease the interference on both the UL and DL transmission. The SI on the UL transmission is controlled by the FD radio implemented at the AP [2]–[4]. On the other hand, the CCI on the DL transmission can be controlled by proper pairing between the DL and UL users that share the same subcarriers. In other words, the DL-UL pair that is chosen to share a given subcarrier should guarantee a small CCI. The system model with user pairing and subcarrier allocation is shown in Fig. 1; as shown, pairing is most probable between UL and DL users which are far away from each other to limit the interference from the UL transmission on the DL transmission.

Additionally, in order to maximize the network sum rate, it is needed to optimize the power allocation among different subcarriers for both the DL and UL transmissions. Based on the above assumptions, if we consider that the m\textsuperscript{th} DL user transmission is paired with the n\textsuperscript{th} UL user transmission on the s\textsuperscript{th} subcarrier, then the received SINR for the m\textsuperscript{th} DL user is given by

\[
\Gamma_{m|DL_s} = \frac{P_s D_{m-AP}^{-\alpha} |h_{m-AP}^s|^2}{\sigma^2 + CCI_{m,n}},
\]

where, \(P_s\) is the s\textsuperscript{th} subcarrier DL AP transmission power, \(D_{m-AP}^{-\alpha}\) denotes the large scale propagation fading between the m\textsuperscript{th} user and the AP with distance \(D_{m-AP}\) and path loss exponent \(\alpha\), \(h_{m-AP}^s\) is the channel coefficient between the m\textsuperscript{th} user and the AP transmission antenna on the s\textsuperscript{th} subcarrier \(^3\), where all the channel coefficients are assumed to be an i.i.d. zero mean complex Gaussian random variables with unit variance, i.e., Rayleigh fading, \(\sigma^2\) is the additive white Gaussian noise (AWGN) variance, and \(CCI_{m,n}^s\) denotes the co-channel interference on the m\textsuperscript{th} DL user from the n\textsuperscript{th} UL user transmission; the value of \(CCI_{m,n}^s\) is given by

\[
CCI_{m,n}^s = P_{ns} D_{m-n}^{-\alpha} |h_{m-n}^s|^2,
\]

where \(P_{ns}\) is the n\textsuperscript{th} UL transmission power in a given subcarrier s. As clear from (2), the value of the CCI is decreased by choosing a DL-UL pair with large mutual distance \(D_{m-n}\). Furthermore, the received SINR from the n\textsuperscript{th} UL user at the AP receiving antenna is given by

\[
\Gamma_{n|UL_s} = \frac{P_s D_{n-AP}^{-\alpha} |h_{n-AP}^s|^2}{\sigma^2 + P_s/C},
\]

where \(P_s/C\) represents the residual SI (RSI) after using a FD radio with a cancellation parameter \(C > 1\), which is available at the AP [2], [32]. Accordingly, the value of the RSI is controlled by the implemented FD radio. From the received DL SINR and the received UL SINR, calculated in (1) and (3), respectively, the network DL and UL sum rates per unit time and unit bandwidth (bits/sec/Hz) are given, respectively, by

\[
R_{T|DL} = \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} \sum_{s=1}^{S} a(m,n,s) \log_2(1 + \Gamma_{m|DL_s}),
\]

\[
R_{T|UL} = \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} \sum_{s=1}^{S} a(m,n,s) \log_2(1 + \Gamma_{n|UL_s}),
\]

where the first summation sums over all DL users, the second summation sums over all UL users, and the last summation

\(^3\)Throughout this paper, \(h_{m-n}^s\) denotes the channel coefficient between the transmitter y and the receiver x on the s subcarrier.
sums over all subcarriers. The coefficient \( a(m, n, s) \) is a binary variable that indicates the user-pairing and the subcarrier assignment. \( a(m, n, s) = 1 \) means that the \( m \)th DL user is paired with the \( n \)th UL user and the \( (m-n) \) pair is served by the \( s \)th subcarrier; otherwise, the value of \( a(m, n, s) \) will be equal to zero. Finally, the network sum rate per unit time and unit bandwidth \((\text{bits/sec}/\text{Hz})\) is the sum of the DL and UL rates calculated in (4), and is given by

\[
R_T = R_{T\mid DL} + R_{T\mid UL}. \tag{5}
\]

III. JOINT USER PAIRING, SUBCARRIER AND POWER ALLOCATION IN FULL-DUPLEX OFDMA NETWORKS

In this section, we optimize the user pairing, power allocation, and subcarrier allocation to maximize the UL and DL network sum throughput given in (5), while satisfying the transmission power constraints imposed on the AP and the UEs. When formulating the joint allocation problem, it should be noted that each of the DL and UL users is allowed to receive and send data, respectively, on multiple subcarriers. However, each subcarrier is allowed to be assigned only once to a single transceiver pair. Therefore, the joint resource allocation problem is given by

\[
\max_{A,P_s,P_{ns}} R_T
\]

\[
\text{s.t.} \begin{align*}
\sum_{s=1}^{S} P_s &\leq P_{\text{max}}, \\
\sum_{s=1}^{S} P_{ns} &\leq P_{ul\mid \text{max}}, \forall n \in \{1\cdots N/2\}, \\
\sum_{s=1}^{S} \sum_{n=1}^{N/2} a(n,m,s) &= 1 \forall s^* \in \{1\cdots S\},
\end{align*} \tag{P1}
\]

where \( A \) is a vector that contains all the \( a(m, n, s) \) variables for all combinations of UL users, DL users, and subcarriers. \( P_s = [P_1, P_2, \ldots, P_S]^T \) is a vector that includes the DL transmission powers on each subcarrier and \( P_{ns} = [P_{n1}, P_{n2}, \ldots, P_{N/2n1}, P_{n2}, \ldots, P_{N/2nS}]^T \) is a vector that includes all the UL users transmission powers on different SCs. The first constraint in (P1) guarantees that the AP total transmission power will not exceed the maximum allowable transmission power \( P_{\text{max}} \). Similarly, the second constraint is to limit the UL transmission power to its maximum value denoted by \( P_{ul\mid \text{max}} \). Finally, the last constraint guarantees that each subcarrier is assigned to a single pair. It must be noticed that the formulation proposed in (P1) is a hard problem due to its combinatorial nature as a result of the presence of the binary variables \( a(m, n, s) \). Therefore, obtaining the optimal solution using brute-force exhaustive search will be very challenging especially for a large number of users and subcarriers. Accordingly, a suboptimal solution algorithm is proposed. First, we propose a solution algorithm for joint user pairing and subcarrier allocation to find the values of \( a(m, n, s) \) variables. Second, we derive a solution for the power allocation problem. Finally, we explain the iterative algorithm for solving the joint resource allocation problem in (P1).

A. Joint User Pairing and Subcarrier Allocation Solution Algorithm

In the beginning, to pair the DL and UL users into independent transceivers, we must reassure that the main purpose of the pairing process is to decrease the CCI. Therefore, it is not required to estimate the channel between each DL-UL pair; it is sufficient to estimate the SNR or the received powers between each DL-UL pair and choose the ones with the least interference. Therefore, during the UL channels’ estimation, the DL users can overhear the UL users’ pilot transmission to the AP and report the UL interference levels estimation results to the AP. However, if the received SNR from specific UL transmissions on some particular subcarriers is below

4Due to the centralized nature of the cellular network, in which the AP fully control the users procedures, the AP is capable of adjusting the channel estimation procedures with the existing network’s users.

5The UL channels can be estimated by having the UL users periodically inserting reference pilot signals in the transmitted data. The transmission patterns of these reference signals are adjusted such that in a given time-frequency resource, a single UL user will be sending its reference signal. For more information on channel estimation techniques for OFDM networks, please refer to [33], [34].
the DL-UE SNR threshold, the DL user will not report any received powers from these UL transmissions. Accordingly, it is favorable to pair the DL user with these UL transmissions. Obviously, by following this pairing procedure, the CCI can be significantly reduced, and hence, the sum rate is expected to increase. The proposed pairing and subcarrier assignment algorithm is described in Algorithm 1. At the beginning, after having all the required UL channels, DL channels and UL-DL SNRs available at the AP, the AP constructs an initial candidates’ set \( \Phi_i \) by including all \((m, n, s)\) combinations that were not reported to the AP in the training phase. In other words, in the training phase, the \(n^{th}\) UL transmission was not detected by the \(m^{th}\) DL user on the \(s^{th}\) subcarrier because the received SNR at the DL user is smaller than the SNR threshold detectable at the DL UE denoted by \(SNR_{th}\). Accordingly, \( \Phi_i \) is defined as,

\[
\Phi_i = \{(m, n, s), m \in \{1 \cdots N/2\}, n \in \{1 \cdots N/2\}, s \in \{1 \cdots S\} | CCI_{m-n}/\sigma^2 < SNR_{th}\}, \tag{6}
\]

where \( CCI_{m-n}/\sigma^2 \) is the received SNR from the \(n^{th}\) UL transmission at the \(m^{th}\) DL user on the \(s^{th}\) subcarrier; the value of \( CCI_{m-n} \) is given in (2). Since each subcarrier is allowed to be assigned once to a single DL-UL pair, therefore, the next step is to check for conflicts between different DL-UL pairs on different SCs. If there is no conflict on a given SC, the algorithm will pick the DL-UL pair that achieves the highest sum rate on that SC, and includes the picked pair along with paired with an UL user in a certain subcarrier, the following conditions must be satisfied:

1) the received SNR from the UL transmission is below the DL user SNR threshold,
2) when a conflict occurs between multiple pairs, the pair with highest sum rate is chosen.

It should be noticed that the complexity of joint user pairing and subcarrier allocation described in Algorithm 1 is in the order of \(O(SN^2)\). When comparing the exhaustive search solution complexity with the complexity of the proposed algorithm, it can be readily seen that the exhaustive search complexity grows exponentially with the number of users and subcarriers, i.e. \(O(2^SN)\). Additionally, the transmitter-subchannel-receiver threesided matching algorithm, proposed in [21], is in the order of \(O(N^3S^3)\), while as mentioned above, the proposed algorithm complexity is polynomial in the number of users and subcarriers, in other words, it has lower complexity than the exhaustive search and the algorithm proposed in [21].

For a better understanding of the joint user pairing and subcarrier allocation, we provide an illustrative example. Consider a system with 4 DL users, 4 UL users, and 4 subcarriers. As mentioned before, in the UL channels estimation, the DL users will overhear the UL pilot transmission, and report the received SNR to the AP. Accordingly, after the training phase, the AP will have a report for the received power between each DL and UL user pair, as shown in Table I, where \(\{a, b, c, d\}\) is the set of DL users, and \(\{r, t, w, v\}\) is the set of UL users. The check marks indicate the received SNR at the DL user from a given UL on a given SC is below the SNR threshold. The first step is to define the set \( \Phi_f \), in which the AP determines the candidate pairs whose received SNRs between the UL and DL users are less than \(SNR_{th}\). Accordingly, \( \Phi_f \) is given by

\[
\Phi_f = \{(d,v,1), (a,t,2), (c,r,3), (c,t,3), (d,t,3), (b,w,4), (d,v,4), (c,r,4)\}. \tag{7}
\]

From the candidate pairing chances obtained in \( \Phi_f \), it can be noticed that there are no conflicting pairs on both the first and the second subcarriers. However, for subcarrier 3 and subcarrier 4, the choice among conflicting pairs will be based on maximizing the sum rate. Accordingly, after calculating the sum rates for \( (c,r,3), (c,t,3), (d,t,3) \) and \( (b,w,4), (d,v,4), (c,r,4) \) and choosing the ones with highest sum rates, the final pairing set will be given by

\[
\Phi_f = \{(d,v,1), (a,t,2), (c,r,3), (b,w,4)\}. \tag{8}
\]

The next step is to allocate power to the transceiver pairs on the given subcarriers specified by \( \Phi_f \).

B. Power Allocation in FD OFDMA

The next step after user pairing and subcarrier allocation is to optimize the power allocation among the existing pairs. The solution of the power allocation problem in FD OFDMA will be different from the traditional water-filling in HD networks due to the correlation between the UL and DL transmissions.

### TABLE I: Reporting received power between different UL-DL pairs

<table>
<thead>
<tr>
<th>Subcarrier₁</th>
<th>Subcarrier₂</th>
<th>Subcarrier₃</th>
<th>Subcarrier₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>d</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

We should note that, during the pilot transmission, it is assumed that the power is uniformly distributed among the subcarriers. Pilot transmission power adjustment is out of our work scope.
Algorithm 2: Power Allocation Algorithm for OFDMA FD network

Data: all CSI information, $P_{\text{max}}$, $P_{\text{ul}|\text{max}}$, $\alpha$, $P_s^0$, $P_{ns}^0$, $Z_s^0$ and $Z_{ns}^0$, $\lambda$

Result: Find $P_s$ and $P_{ns}$ maximizing $R_T$ while keeping total transmission power constraints $P_{\text{max}}$ for the AP and $P_{\text{ul}|\text{max}}$ for the UL transmissions.

Initially:

<table>
<thead>
<tr>
<th>k</th>
<th>$P_s^{k+1}$</th>
<th>$P_{ns}^{k+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>prox$_{\mathcal{A}(P_s)}(P_s^k - Z_s^k - U_s^k)$</td>
<td>prox$<em>{\mathcal{A}(P</em>{ns})}(P_{ns}^k - Z_{ns}^k - U_{ns}^k)$</td>
</tr>
<tr>
<td>2</td>
<td>$P_s^{k+1} = P_s^k + U_s^k$</td>
<td>$P_{ns}^{k+1} = P_{ns}^k + U_{ns}^k$</td>
</tr>
<tr>
<td>3</td>
<td>$Z_s^{k+1} = Z_s^k + U_s^k$</td>
<td>$Z_{ns}^{k+1} = Z_{ns}^k + U_{ns}^k$</td>
</tr>
<tr>
<td>4</td>
<td>$U_s^{k+1} = U_s^k + P_s^{k+1} - Z_s^{k+1}$</td>
<td>$U_{ns}^{k+1} = U_{ns}^k + P_{ns}^{k+1} - Z_{ns}^{k+1}$</td>
</tr>
<tr>
<td>5</td>
<td>if $P_s = Z_s \forall s \in {0, ..., S}$ and $P_{ns} = Z_{ns} \forall s \in {0, ..., S}$ and $n \in {1, ..., N/2}$ then</td>
<td>else $k = k + 1$</td>
</tr>
<tr>
<td>6</td>
<td>$P_s = P_s^k$</td>
<td>return to step 1</td>
</tr>
<tr>
<td>7</td>
<td>$P_{ns} = P_{ns}^k \forall s \in {0, ..., S}$ and $n \in {1, ..., N/2}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>end</td>
<td></td>
</tr>
</tbody>
</table>

The power allocation problem for the proposed model is derived from the initial problem formulated in (P1) after considering only the active subcarriers which are assigned to the formed users’ pairs in the user pairing and subcarrier allocation step. For instance, in the illustrative example, after obtaining the final pairing set is given in (8), the sum rate $R_T^{\text{opt}}$ is given by

$$R_T^{\text{opt}} = \log_2(1 + \Gamma_{d|DL_1}) + \log_2(1 + \Gamma_{d|UL_1}) + \log_2(1 + \Gamma_{u|DL_2}) + \log_2(1 + \Gamma_{u|UL_2}) + \log_2(1 + \Gamma_{c|DL_3}) + \log_2(1 + \Gamma_{c|UL_3}) + \log_2(1 + \Gamma_{b|DL_4}) + \log_2(1 + \Gamma_{b|UL_4}).$$

Therefore, the power allocation problem is given by

$$\max_{P_s, P_{ns}} R_T$$

\begin{align}
\text{s.t. } & \sum_{s=1}^{S} P_s \leq P_{\text{max}}, \\
& \sum_{s=1}^{S} P_{ns} \leq P_{\text{ul}|\text{max}}, \forall n \in \{1 \cdots N/2\}.
\end{align}

In solving the power allocation problem, we will consider the high SINR case because the main objective of the user pairing and subcarrier allocation step is to guarantee the least possible interference as well as the highest possible rate on each subcarrier. Therefore, after the pairing and subcarrier assignment step, it is reasonable to consider the high SINR case. In that case, the proposed power allocation problem can be readily proved to be approximately convex, which can be solved using ADMM. First, we start by proving the convexity of the power allocation problem at high SINR approximation. Afterwards, we present the steps of the ADMM solution algorithm.

Proposition 1. In high SINR case, the power allocation problem (P2) is approximately a convex optimization problem. It is rewritten as

$$\min_{P_s, P_{ns}} F(P_s) + G(P_{ns})$$

\begin{align}
\text{s.t. } & \sum_{s=1}^{S} P_s \leq P_{\text{max}}, \\
& \sum_{s=1}^{S} P_{ns} \leq P_{\text{ul}|\text{max}}, \forall n \in \{1 \cdots N/2\},
\end{align}

where

$$F(P_s) = -\sum_{s=1}^{S} \log_2 \left( \frac{P_s D_{m\rightarrow Ap}^m h_s^{m\rightarrow Ap}^2}{\sigma^2 + P_s/C} \right),$$

$$G(P_{ns}) = -\sum_{s=1}^{S} \log_2 \left( \frac{P_{ns} D_{n\rightarrow Ap}^m h_s^{n\rightarrow Ap\rightarrow n}^2}{\sigma^2 + P_{ns} D_{n\rightarrow Ap}^m h_s^{n\rightarrow Ap\rightarrow n}^2} \right).$$

It should be noted that the objective function and the constraints functions, in (P3), are fully separable in the DL transmission powers $P_s$ and the UL transmission powers $P_{ns}$. Since the constraints are all affine constraints, then the convexity of the approximate high SINR problem (P3) can be proved by proving the convexity of $F(P_s)$ with respect to $P_s$ and the convexity of $G(P_{ns})$ with respect to $P_{ns}$. The proof of Proposition 1 is presented in Appendix A.

There are many algorithms that are used to solve these types of convex problems. In this paper, we are going to adopt the ADMM algorithm, which is a method for solving generic convex constrained problems and only uses the proximal operator of the objective function and projection onto the constraint set [28], [29], [37]. The convergence of the ADMM algorithm is discussed in [29]. Additionally, it is shown that ADMM achieves a linear convergence rate [38], [39]. The main advantage of using ADMM is its ability to solve separable optimization problems in a distributed fashion. The first step towards solving (P3) using ADMM is to rewrite the constrained optimization problem as a sum of the objective function and an indicator function of the convex set of the constraints. The solution of the power allocation problem in (P3) can be described as follows.

Proposition 2. The ADMM solution algorithm for the problem
Algorithm 3: Joint User Pairing, Subcarrier and Power Allocation Algorithm for OFDMA FD network

**Data:** all CSI information, $P_{\text{max}}$, $P_{u_{\text{max}}}$, $\alpha$, $P_s$, $P_{\text{ns}}$, $Z_s$, and $Z_{\text{ns}}$, $\lambda$, $\beta$, $SNR_{th}$

**Result:** Find $a(m,n,s)$ in $[1\cdots N/2]$, $n \in [1\cdots N/2]$, and $s \in [1\cdots S]$, $P_s$ and $P_{\text{ns}}$ maximizing $R_T$, while keeping total transmission power constraints $P_{\text{max}}$ for the AP and $P_{u_{\text{max}}}$ for the UL transmissions.

**Intially:** Assume DL and UL power are distributed equally among active users and subcarriers.

- $\Phi_1 = \emptyset$, $\Phi_f = \emptyset$,
- iteration $t = 1$.

**Step 1:** Joint User Pairing and Subcarrier Allocation

- a. Form the initial candidate pair set $\Phi_1$ given in (6),
- b. Apply Steps described in Algorithm 1 to find $A(t)$.

**Step 2:** Power Allocation

- a. From $\Phi_f(t)$, formulate power allocation problem in P3.
- b. Run the power allocation scheme described in Algorithm 2 to find $P_s(t)$ and $P_{\text{ns}}(t)$.

**Step 3:** Check convergence

- a. if $A(t) = A(t-1) \& R_T(t) = R_T(t-1)$ then
- $A = A(t)$,
- $P_s = P_s(t)$
- $P_{\text{ns}} = P_{\text{ns}}(t)$
- else $t = t+1$
- Return to Step 1 with the calculated transmission powers from Step 2 to form $\Phi_1(t+1)$

(P3) is given by

$$
\begin{align*}
    P_{s}^{k} & := \text{prox}_{\lambda f(P_s)}(P_{s}^{k} - Z_{s}^{k} - U_{s}^{k}), \\
    Z_{s}^{k+1} & := \Pi_{C_{DL}}(P_{s}^{k} + U_{s}^{k}), \\
    P_{\text{ns}}^{k+1} & := \text{prox}_{\lambda g(P_{\text{ns}})}(P_{\text{ns}}^{k} - Z_{\text{ns}}^{k} - U_{s}^{k}), \\
    Z_{\text{ns}}^{k+1} & := \Pi_{C_{UL}}(P_{\text{ns}}^{k} + U_{\text{ns}}^{k}), \\
    U_{s}^{k+1} & := U_{s}^{k} + P_{s}^{k+1} - Z_{s}^{k+1}, \\
    U_{\text{ns}}^{k+1} & := U_{\text{ns}}^{k} + P_{\text{ns}}^{k+1} - Z_{\text{ns}}^{k+1},
\end{align*}
$$

(11)

where

$$
\begin{align*}
    f(P_s) & = -\log_2 \left( \frac{P_s D_m^{\alpha} |h_m^{s} - A|^2}{\sigma^2 + P_s / C} \right), \\
    g(P_{\text{ns}}) & = -\log_2 \left( \frac{P_{\text{ns}} D_{n-\text{Ap}}^{\alpha} |h_{n-\text{Ap}}^{s}|^2}{\sigma^2 + P_{\text{ns}} D_{n-\text{Ap}}^{\alpha} |h_{n-\text{Ap}}^{s}|^2} \right).
\end{align*}
$$

(12)

The variables $Z_s$ and $Z_{\text{ns}} \forall s \in [1\cdots S]$ and $n \in [1\cdots N/2]$ are equivalent to the variables $P_s$ and $P_{\text{ns}} \forall s \in [1\cdots S]$ and $n \in [1\cdots N/2]$, respectively. $\text{prox}_{\lambda f(P_s)}(\nu)$ and $\text{prox}_{\lambda g(P_{\text{ns}})}(\nu)$ are the proximal operators on $f(P_s)$ and $g(P_{\text{ns}})$, respectively, with a scaling factor of $\lambda$. Finally, $\Pi_{C_{DL}}(\nu)$ and $\Pi_{C_{UL}}(\nu)$ are the Euclidean projections onto the constraints set $C_{DL}$ and $C_{UL}$, which are defined, respectively, by

$$
\begin{align*}
    C_{DL} & = \{z_1, z_2, \ldots, z_S \mid \sum_{s=1}^{S} P_s \leq P_{\text{max}}, \forall n \in [1\cdots N/2]\}, \\
    C_{UL} & = \{z_1, z_2, \ldots, z_{SN/2} \mid \sum_{n=1}^{N/2} P_{\text{ns}} \leq P_{u_{\text{max}}} \forall n \in [1\cdots N/2]\}.
\end{align*}
$$

(13)

**Proof:** Since the optimization problem given in (P3) is fully separable in the DL and UL transmission powers, then the first step is to write (P3) as two distinct convex optimization problems. The first problem is in terms of the DL transmission powers (P3.1) and the second problem is in terms of the UL transmission powers (P3.2). The first optimization problem (P3.1) is given by

$$
\begin{align*}
    \min_{P_s} \sum_{s=1}^{S} f(P_s) & \quad \text{s.t.} \sum_{s=1}^{S} P_s \leq P_{\text{max}},
\end{align*}
$$

(14)

where $f(P_s)$ is defined in (12). Afterwards, (P3.1) is reformulated into the canonical form defined in [28], as follows

$$
\begin{align*}
    \min_{P_s} \sum_{s=1}^{S} f(P_s) + \Psi(Z_s),
\end{align*}
$$

(15)

where $\Psi(Z_s) = I_{C_{DL}}(z_1, z_2, \ldots, z_S)^8, I_{C_{DL}}$ is an indicator function which is defined as

$$
I_{C_{DL}}(x) = \begin{cases} 0, & x \in C_{DL}, \\ +\infty, & x \notin C_{DL}, \end{cases}
$$

where the set $C_{DL}$ is defined in (13). Similarly, the second problem is in terms of the UL transmission powers (P3.2), and is given by

$$
\begin{align*}
    \min_{P_{\text{ns}}} \sum_{s=1}^{S} g(P_{\text{ns}}) & \quad \text{s.t.} \sum_{s=1}^{S} P_{\text{ns}} \leq P_{u_{\text{max}}} \forall n \in [1\cdots N/2],
\end{align*}
$$

(16)

where $g(P_{\text{ns}})$ is defined in (12). Afterwards, (P3.2) is reformulated into the canonical form as follows

$$
\begin{align*}
    \min_{P_{\text{ns}}} \sum_{s=1}^{S} g(P_{\text{ns}}) + \Theta(Z_{\text{ns}}),
\end{align*}
$$

(17)

where $\Theta(Z_{\text{ns}}) = I_{C_{UL}}(z_1, z_2, \ldots, z_{SN/2}), I_{C_{UL}}$ is an indicator function which is defined as

\footnote{In the canonical form definition, precisely speaking, in $\Psi(Z_s)$ definition, a change of variables from $P_s$ to $z_s$ is required $\forall s \in \{0, 1, \cdots, S\}$, where $Z_s = [z_1, z_2, \cdots z_S]^T$ [28], [29].}
Algorithm 1. Using the user pairing and subcarrier allocation results, the power allocation problem is formulated in (P2). Assuming the high SINR case, the problem’s approximation in the case of the high SINR case (P3) is formulated and solved using Algorithm 2. Finally, if the users’ pairing, subcarrier allocation, and sum capacity remain unchanged, the algorithm will stop and the final solution for the joint problem is obtained. Otherwise, the algorithm will rerun the joint user pairing and subcarrier allocation described in Algorithm 1 with the new power allocation and then the power allocation scheme described in Algorithm 2.

IV. Numerical Analysis

In this section, first, we evaluate the FD power allocation performance. Second, we study the effects of different system parameters on the power allocation problem as well as the joint allocation problem. We are considering a square grid with the access point in its center. All the users are uniformly distributed inside the grid. It is assumed that the SNR threshold $\text{SNR}_\text{th} = 20\, \text{dB}$, the SI cancellation parameter $C = 70\, \text{dB}$, the maximum DL transmission power $P_{\text{max}} = 2\, \text{W}$, and the maximum UL transmission power for each UL user $P_{UL|\text{max}} = 1\, \text{mW}$. The proposed system is simulated using Monte Carlo simulations on MATLAB. Unless stated otherwise, the simulation parameters are given in Table II.

<table>
<thead>
<tr>
<th>$N$</th>
<th>16 users</th>
<th>$S$</th>
<th>16 SCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SNR_{\text{th}}$</td>
<td>$-20, \text{dB}$</td>
<td>$\alpha$</td>
<td>2.7</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>$2, \text{W}$</td>
<td>$P_{UL</td>
<td>\text{max}}$</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>$-110, \text{dBm}$</td>
<td>$C$ [3]</td>
<td>$10^7$</td>
</tr>
</tbody>
</table>

The complete solution algorithm for the proposed allocation problem is given in Algorithm 2. In the beginning, it is assumed that all UL channel state information (CSI), DL CSI, and mutual SINR information between UL and DL users are available at the AP. Next, the parameters $\alpha$, $P_{\text{max}}$, and $P_{UL|\text{max}}$ are determined. Afterwards, initial vectors $P_s^0$, $P_{ns}^0$, $Z_s^0$, and $Z_{ns}^0$ are determined. Then, in steps 1 through 6, the algorithm continues on updating the values of $P_s$, $Z_s$, $P_{ns}$, and $Z_{ns}$. It must be noticed that the dual variables $U_s$ and $U_{ns}$ are updated to measure the deviation of $P_s$ from $Z_s$ and $P_{ns}$ from $Z_{ns}$, respectively. The algorithm will stop iterating when $P_s$ converges to $Z_s$ and $P_{ns}$ converges to $Z_{ns}$. It should be emphasized that steps 3 and 4, which are used to update $P_{ns}$ and $Z_{ns}$, are performed in parallel at each UL user. However, the remaining steps of Algorithm 2, which include updating $P_s$, $Z_s$, $U_s$, and $U_{ns}$ and checking the algorithm convergence, are performed at the AP.

Finally, the complete solution algorithm for solving the joint user pairing and subcarrier allocation, formulated in (P1), is given in Algorithm 3. Initially, to solve the user-pairing and subcarrier allocation, we set $\theta_t = 0$, and the number of iteration $t = 1$. The first step, after assuming equal power allocation over the UL and DL transmissions, is to apply the joint user pairing and subcarrier algorithm explained in.

Finally, the problems in (P3.1.1) and (P3.2.2) can be solved using the algorithm given in (11), where the proximal operator and the Euclidean projection are given, respectively, by

$$\text{prox}_{\lambda f}(\nu) = \arg\min_x f(x) + (1/(2\lambda))\|x - \nu\|^2_2,$$

$$\Pi_C(\nu) = \arg\min_{x \in C} \|x - \nu\|_2.$$

$$I_{C_{UL}}(x) = \begin{cases} 0, & x \in C_{UL}, \\ +\infty, & x \notin C_{UL}. \end{cases}$$

(15)
A. Power Allocation Performance Evaluation

In this section, the behavior of the power allocation problem proposed in Section III-B is evaluated. First, we validate our solution algorithm by comparing its performance with the interior point algorithm [40], implemented in Matlab Optimization Toolbox. Second, we compare the required run time needed by both algorithms to solve the power allocation problem in (P3). Finally, we study the effect of the SI cancellation parameter (C) and the mutual distance between users on the network sum throughput.

1) Validating Proposed ADMM Algorithm for Power Allocation Problem: Fig. 2 shows the variation of the sum throughput $R_T$ with the SI cancellation parameter $C$, for different number of users and subcarriers. Additionally, it compares the proposed ADMM solution and the interior-point algorithm. It is expected that increasing $C$ will cause an increase in the sum throughput as a result of decreasing the RSI level. Furthermore, increasing the number of subcarriers is expected to offer more transmission channels, and as a consequence, the sum throughput will increase as well. This behavior can be validated from the results shown in Fig. 2, as the enhancement of the sum capacity is noticed by increasing the value of $C$ from $70dB$ to $110dB$. Furthermore, increasing the number of subcarriers causes an increase in the network sum rate. Additionally, it is clear that the proposed distributed ADMM solution matches the interior point solution for different network conditions. Furthermore, Table III shows the difference between the required run time for both the proposed ADMM algorithm and the interior-point algorithm for different numbers of users and subcarriers. Even though both ADMM and interior-point algorithm have linear convergence rates [38]–[40], the per-iteration complexity of the interior-point algorithm is expected to be much higher than that of ADMM because in the case of the interior-point algorithm, the solution of the power allocation problem in (P3) is less than that required by the interior-point algorithm implemented in Matlab Optimization Toolbox. Additionally, it is observed that the difference between the required run time between the two algorithms increases with the number of available subcarriers. For instance, the ADMM reduced the required run time by about 21% for $S = 16$ and $N = 24$, and this reduction in run time increased to about 77% for $S = 64$ and $N = 24$. Based on the results shown in Fig. 2 and Table III, it is quite clear that the proposed ADMM algorithm requires less run time than that taken by the interior-point algorithm, to solve the power allocation problem in (P3) with almost similar performance.

Fig. 3 shows the variation of the sum throughput $R_T$ with the grid size, for different numbers of subcarriers. First of all, since the users are uniformly distributed along the considered square grid, then as the grid size becomes bigger, the average distance between the AP and the users becomes larger. Hence, both the UL and DL received powers decrease. Therefore, it is expected that the larger the grid size the smaller the achieved capacity. The decrease in $R_T$ with respect to the grid size can be verified from the results shown in Fig. 3. Additionally, the results in Fig. 3 show the proposed ADMM solution and the interior-point solution result in similar performance.

\[^9\]The comparison between the ADMM and interior-point algorithms is performed under the same optimality tolerance with the same CPU specifications.
TABLE III: Comparison between run time for the proposed ADMM algorithm and the Interior-Point Algorithm

<table>
<thead>
<tr>
<th>Subcarriers</th>
<th>Users</th>
<th>ADMM</th>
<th>Interior-Point Algorithm</th>
<th>Time Difference percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>24</td>
<td>3.48s</td>
<td>4.45s</td>
<td>21%</td>
</tr>
<tr>
<td>32</td>
<td>24</td>
<td>4.35s</td>
<td>17.72s</td>
<td>75%</td>
</tr>
<tr>
<td>64</td>
<td>24</td>
<td>8.78s</td>
<td>38.89s</td>
<td>77%</td>
</tr>
</tbody>
</table>

B. Joint User Pairing, Subcarrier Allocation and Power Allocation Performance Evaluation

In this subsection, we study and evaluate the performance of the joint allocation problem. First, the proposed solution algorithm performance is validated by comparing it with the optimal solution which is obtained through exhaustive search that enumerates all possible user pairings and subcarrier allocations, and chooses the combination that results in the maximum sum rate. The total number of combinations for exhaustive search equals \((N/2)^{2S}\). It is quite clear that, for large networks, obtaining the optimal solution is very complicated as a result of the large number of combinatorial possibilities of users’ pairings, and subcarrier allocation. Afterwards, we explore how varying different network parameters can affect the achieved sum capacity.

1) Validating Proposed User Pairing and Subcarrier Allocation Algorithm: Fig. 4 compares the performance of the proposed joint allocation algorithm with the exhaustive search solution for a 3-SC, 6-user network, with a varying grid size from 200m to 600m. In that case, the exhaustive search solution runs over 729 different combinations of DL users, UL users, and SC allocations. From the results in Fig. 4, it can be seen that the difference between the proposed algorithm and the exhaustive search solution is almost 1%, which indicates that our proposed approach is very close to being optimal. Also, when the value of \(P_{\text{max}}\) is increased from 2\(W\) to 6\(W\), the proposed algorithm sum rate increased and achieved a performance which is very close to the optimal. Additionally, when calculating the run time for both algorithms, it is found that the proposed algorithm takes much less time to find the solution, as compared to the exhaustive search based approach\(^{11}\). Additionally, the proposed algorithm and the exhaustive search solution are compared when varying the self-interference cancellation parameter in Fig. 5. We can observe that the proposed algorithm approaches the optimal solution for different values of the self-interference cancellation parameter. Which again emphasizes the merits of our proposed algorithm as compared to the "optimal" exhaustive search solution in terms of performance and complexity.

For further validation, the two solution methods are compared for a 4-SC, 4-user network which requires the enumeration of 256 different combinations of DL users, UL users, and SC allocations. In Fig. 6, the two solution methods are compared while changing the grid size, and similar to the results shown in Fig. 4, the difference between the optimal capacity and the capacity achieved by the proposed algorithm is very small. Additionally, in Fig. 7, the upper-bound obtained by solving the dual problem in [19], is simulated for comparison. The three methods are compared for different values of SI cancellation parameter. From the results in Fig. 7, we can observe that the performance of the optimal solution is very close to the upper bound obtained in [19], which means that the duality gap is very small, which matches the results obtained in [41] where it is proved that solving the dual problem of the non-convex multi-carrier spectrum sharing problems gives a solution with a negligible duality gap. Furthermore, it is observed that the proposed algorithm can achieve a performance close to the upper bound, found through solving the dual problem and that is very close to the optimal, exhaustive search based solution.

The next step is to validate the proposed algorithm performance in larger networks. In large networks, the implementation of the exhaustive search solution is impractical. Therefore, we will compare the proposed algorithm performance with the dual upper bound solution presented in [19]. In Fig. 8, the proposed algorithm and the upper bound solution are compared for a network with 16 SC and 16 users and a network with 48 SCs, which is the same as the number of SCS used in certain amendments of the IEEE 802.11 standard of Wi-Fi networks for example, and 20 users. In both cases, we can observe that the gap between the proposed algorithm and the upper bound is very small which validates the effectiveness of the proposed algorithm in achieving a "close to optimal" performance.

Next, we compare the performance of our proposed algorithm against the random pairing FD network and the HD network. In the random pairing FD network, UL users and DL users are randomly paired and allocated to available subcarriers. In the HD network, we assume a square grid where the HD AP is centered in the middle. In the first time slot, the AP serves \(N\) DL users and each subcarrier is allocated to the DL transmission with the highest SNR. Similarly, in the second time slot, the AP serves \(N\) UL user and each subcarrier is allocated to the UL transmission with the highest SNR. In Fig. 9, the three systems are compared while varying the grid size. From the results shown in Fig. 9, it can be noticed that the proposed scheme greatly outperforms both the random pairing and the HD systems. It can be noticed that, for 200\(m\) grid, the proposed algorithm sum rate is 1.5 times that of

\(^{10}\) The implementation of the exhaustive search is done in this work only for comparison purpose.

\(^{11}\) Note that the complexity of the exhaustive search algorithm is exponential in the number of users and the number of subcarriers; therefore, for larger networks, with large number of users and large number, the exhaustive search based approach becomes impractical to implement.
the random pairing sum rate, and 1.91 times that of the HD system sum rate. Furthermore, as the grid size increases to 2Km, the proposed algorithm still guarantees a better rate than that offered by both the random pairing and HD schemes.

Moreover, Fig. 10 shows the variation of the sum rate with the SI cancellation parameter $C$. It is obvious that changing the SI cancellation parameter will not affect the HD system performance. However, increasing $C$ is expected to increase the sum rate for both the proposed and random pairing schemes, as a result of reduced RSI. From the results displayed in Fig. 10, the decrease in the sum rate with SI cancellation coefficient is verified, for both the proposed and random pairing scheme. Additionally, the advantage of the proposed scheme over the random pairing scheme can be justified, as over different values of $C$, the proposed scheme achieves a better rate than random pairing and the HD scheme. Similarly, from the results shown in Fig. 10, it can be noticed that further decrease in $C$ may cause HD to outperform the FD performance. In that case, mode selection will be essential to switch the network to HD operation to maximize the network spectral efficiency. Mode selection is addressed in the work presented in [20] for a FD-OFDMA network with multiple APs.

V. Conclusion

In this paper, a single cell OFDMA network is considered, wherein a joint user pairing, subcarrier, and power allocation algorithm that aims at maximizing the DL and UL sum rate is proposed. A two-step solution algorithm is presented; the first step is the joint user pairing and subcarrier allocation by considering the pairs whose co-channel interference is below the downlink user equipment SNR threshold. The second step is to consider the power allocation in high signal-to-interference-noise ratio. Using the alternating direction method of multipliers the power allocation problem is solved as two separate problems in the uplink and downlink transmissions. Through numerical simulations, we show that the distributed nature of the proposed power allocation solution enables it to converge to the optimal solution faster than the "centralized" interior point based algorithm. We also show that our proposed algorithm achieves a performance that is very close to the sum rate upper bound, obtained from solving the dual problem. This, in turn, proves that our proposed approach achieves a performance that is very close to the optimal, "impractical" exhaustive search based solution.

In the future work, fairness will be considered in the joint optimization problem. The fairness issues while maximizing the networks’ sum rate can be addressed by maximizing the min-max fairness rate, by adding a minimum rate constraint for each user, or by guaranteeing at least one subcarrier for each user in the network.

APPENDIX A

Proof of Proposition 1

In case of high SINR, and after assigning the available subcarrier to the DL-UL user pairs, the sum DL and UL rates in (4) are, respectively, modified to

\[
\hat{R}_{T|DL} = \sum_{s=1}^{S} \log_2(\Gamma_{m|DL_s}),
\]

\[
\hat{R}_{T|UL} = \sum_{s=1}^{S} \log_2(\Gamma_{n|UL_s}).
\] (17)
Accordingly, the sum rate approximation, in high SINR, is given by,

$$\bar{R}_T = \sum_{s=1}^{S} \log_2(\Gamma_m[D]_s) + \log_2(\Gamma_n[U]_s),$$

$$= \sum_{s=1}^{S} \log_2 \left( \frac{P_s D_m^{\alpha h_m - AP} h_m^{\alpha h_m - AP}}{\sigma^2 + P_s/C} \right),$$

$$+ \log \left( \frac{P_n D_m^{\alpha h_m - AP} h_m^{\alpha h_m - AP}}{\sigma^2 + P_n D_m^{\alpha h_m - AP} h_m^{\alpha h_m - AP}} \right),$$

$$= F(P_s) + G(P_{ns}).$$

Accordingly, the power allocation problem is reformulated, as given in (P3). Now the next step, to prove the convexity of the power allocation problem approximation, is to prove the convexity of $F(P_s)$ with respect to $P_s$, and $G(P_{ns})$ with respect to $P_{ns}$. In other words, it is required to prove that both functions have a positive semi-definite Hessian matrix, i.e., $\nabla^2 F(P_s) \geq 0$ and $\nabla^2 G(P_{ns}) \geq 0$. Starting with $F(P_s)$, the first derivative of $F(P_s)$ with respect to $P_s$ is given by

$$\nabla F(P_s) = \left[ \frac{\partial F(P_s)}{\partial P_1}, \ldots, \frac{\partial F(P_s)}{\partial P_S} \right]^T,$$

$$= \left[ \frac{\partial f_1(P_1)}{\partial P_1}, \ldots, \frac{\partial f_s(P_s)}{\partial P_s} \right]^T,$$

where $f_s(P_s) \forall s \in \{1, \ldots, S\}$ is given in (12). The second equality in (19) is due to the fact that $F(P_s)$ is fully separable over $P_s \forall s \in \{1 \ldots S\}$. The value of $\partial f_s / \partial P_s \forall s \in \{1, \ldots, S\}$ is given by

$$\frac{\partial f_s}{\partial P_s} = \frac{-\sigma^2}{P_s \sigma^2 + P_s/C}.$$

The next step is to calculate the value of $\nabla^2 F(P_s)$. Since, $F(P_s)$ is fully separable over $P_s \forall s \in \{1 \ldots S\}$, therefore $\nabla^2 F(P_s) = \text{diag} \left[ \frac{\partial^2 f_1}{\partial s^2}, \ldots, \frac{\partial^2 f_S}{\partial s^2} \right]$ is a diagonal matrix whose diagonal entries are given by

$$\nabla^2 F(P_s)_{(s,s)} = \left( \frac{\sigma^2}{F_s} \times \frac{1/C}{(\sigma^2 + P_s/C)^2} \right) + \left( \frac{1}{\sigma^2 + P_s/C} \times \frac{\sigma^2}{F_s^2} \right),$$

where $\nabla^2 F(P_s)_{(s,s)}$ is the $(s, s)$ element in the Hessian matrix of $F(P_s)$. From (21), it can be noticed that all the diagonal elements $\nabla^2 F(P_s)$ are positive and hence, $\nabla^2 F(P_s)$ is a positive semi-definite matrix. Therefore, $F(P_s)$ is a convex function with respect to $P_s$. Using the same procedures, the convexity of $G(P_{ns})$ with respect to $P_{ns}$ can be proved. Finally, Since both $F(P_s)$ and $G(P_{ns})$ are convex function, and all the constraints in (P3) are linear, therefore, the power allocation problem approximation in high SINR regime is a convex optimization problem.

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**REFERENCES**


Fig. 10: Variation of $R_T$ versus SI cancellation parameter for HD, random allocation FD and proposed scheme. Parameters used to generate this figure: \[ N = 24, S = 16, P_{\text{max}} = 2W, P_{UL,max} = 1mW \]


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