

# Optimization of Wireless Powered Communication Networks with Heterogeneous Nodes

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**Abstract**—This paper studies optimal resource allocation in a wireless powered communication network with two groups of users; one is assumed to have radio frequency (RF) energy harvesting capability and no other energy sources, while the other group has legacy nodes that are assumed not to have RF energy harvesting capability and are equipped with dedicated energy supplies. First, the base-station (BS) with a constant power supply broadcasts an energizing signal over the downlink. Afterwards, all users transmit their data independently on the uplink using time division multiple access (TDMA). We propose two transmission schemes, namely OPIC and OPAC, subject to different energy constraints on the system. Within each scheme, we formulate two optimization problems with different objective functions, namely maximizing the sum throughput and maximizing the minimum throughput, for enhanced fairness. We establish the convexity of all formulated problems which opens room for efficient solution using standard techniques. Our numerical results show the superiority of our realistic system accommodating legacy nodes, along with RF harvesting nodes, compared to the baseline WPCN system with RF energy harvesting nodes only. Moreover, the results reveal new insights and throughput-fairness trade-offs unique to our new problem setting.

**Index Terms**—cellular networks, convex optimization, green communications, RF energy harvesting.

## I. INTRODUCTION

RF energy harvesting has recently emerged as a promising solution to efficiently prolong the limited lifetime of energy-constrained wireless networks. This is due to the fact that RF energy harvesting allows wireless devices to continuously harvest energy from the surrounding radio environment. Since RF signals can carry energy and information at the same time, a dynamic simultaneous wireless information and power transfer scheme called SWIPT is proposed in [1]–[3]. The fundamental trade off between transmitting energy and transmitting information over a point-to-point noisy link is studied in [1]. In [2], the authors introduced dynamic power splitting as a general SWIPT receiver operation and proposed two practical SWIPT receiver architectures, namely, separated and integrated information and energy receivers. In addition,

[3] studied and proposed SWIPT for orthogonal frequency division multiplexing (OFDM) systems.

On the other hand, wireless powered communication networks (WPCNs), a newly emerging type of wireless networks, have recently attracted considerable attention [4]–[6]. In WPCN, users first harvest RF energy on the downlink from wireless energy signals broadcast by a hybrid access point (HAP). Afterwards, users transmit their information signals to the HAP on the uplink using harvested energy in the downlink phase, e.g., using TDMA in [4]. In addition, [5] introduced user cooperation in WPCN as a solution for the doubly near-far phenomenon that results in unfair rate allocation among users as observed in [4]. Moreover, a full-duplex WPCN scheme with energy causality has been proposed in [6], in which all users can continuously harvest wireless power from the HAP till its uplink information transmission slot. In this paper, we study WPCNs with two types of nodes, with and without RF energy harvesting capability in efficient way of utilizing the nodes without RF energy harvesting to enhance the network performance, even beyond the baseline WPCN with all energy harvesting nodes [4]. This constitutes an important step towards more realistic future wireless networks as RF energy harvesting technology gradually penetrates the wireless industry.

Our main contribution in this paper is three-fold. First, we introduce a generalized problem setting for WPCN, in which the network includes two types of nodes, with/without RF energy harvesting capability, of which the problem setting studied in [4] falls as a special case. Second, we develop two optimal resource allocation schemes for the considered WPCNs which are different in the model of how each node without energy harvesting capability exploits its energy supply for uplink information transmissions. Furthermore, we formulate two optimization problems for investigating the maximum sum throughput and the maxmin throughput for each proposed optimal resource allocation scheme. Third, we establish the convexity of the formulated problems and characterize the optimal solution in closed form for one of them and solve the other problems efficiently using known convex problems solvers. Our numerical results demonstrate the fundamental trade off between achieving maximum sum throughput and achieving fair rate allocation among different users and reveal interesting insights about the formulated problems.

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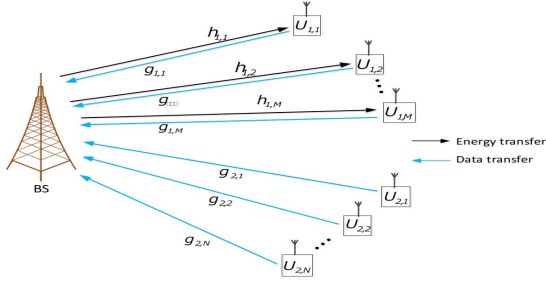


Fig. 1. WPCN with two types of nodes.

The rest of the paper is organized as follows. The system model is presented in Section II. In Section III, the maximum sum throughput and the maxmin throughput optimization problems are formulated and convexity is established for OPIC scheme. Section IV studies the maximum sum throughput and the maxmin throughput optimization problems for OPAC scheme. Numerical results are presented in Section V. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL

We study a WPCN with two types of nodes, as shown in Fig. 1, namely  $M$  RF energy harvesting (Type I) nodes and  $N$  legacy (non-harvesting) nodes (Type II). It is assumed that the BS and all users in both groups are equipped with a single antenna each, operate over the same frequency channel and the radios are half-duplex. Each user of Type I, denoted by  $U_{1,i}$  for  $i = 1, \dots, M$ , is assumed to have no battery or other energy supply and, hence, needs to first harvest energy from the RF energy collected from the BS broadcast on the downlink, primarily to energize Type I nodes. The harvested RF energy can be stored in a rechargeable battery and then used for data transmission on the uplink, e.g., [4]. On the other hand, legacy Type II nodes, denoted by  $U_{2,j}$  for  $j = 1, \dots, N$ , are not equipped with RF energy harvesting circuitry and, hence, are assumed to have dedicated energy supplies for their communication needs on the uplink.

The network operates in a TDMA fashion. For convenience, we assume the block (slot) duration is normalized to 1. At the first  $\tau_0 \in [0, 1]$  fraction of time, the BS broadcasts an energizing signal over the downlink so that each  $U_{1,i}$  could harvest a certain amount of energy and charge its battery. The remaining  $1 - \tau_0$  fraction of time is allocated to uplink data transmissions where  $U_{1,i}$  and  $U_{2,j}$  are assigned certain portion of time denoted by  $\tau_{1,i}$  and  $\tau_{2,j}$ , respectively, for  $i = 1, \dots, M$  and  $j = 1, \dots, N$ . Hence, the slot is split as follows:

$$\tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1. \quad (1)$$

The downlink channel coefficient from the BS to  $U_{1,i}$ , the uplink channel coefficient from  $U_{1,i}$  to the BS and the uplink channel coefficient from  $U_{2,j}$  to the BS are denoted by complex random variables  $h'_{1,i}$ ,  $g'_{1,i}$  and  $g'_{2,j}$ , respectively,

with channel power gains  $h_{1,i} = |h'_{1,i}|^2$ ,  $g_{1,i} = |g'_{1,i}|^2$  and  $g_{2,j} = |g'_{2,j}|^2$ . In addition, it is assumed that all downlink and uplink channels are quasi-static flat fading, i.e., all downlink and uplink channels remain constant over a transmission block, that is a time slot, but can change independently from one block to another. The BS has perfect knowledge of all channel coefficients at the beginning of each block. The transmitted energy signal from the BS to all users, over the downlink, is denoted by  $x_B$  with  $P_B$  average power, i.e.,  $E(|x_B|^2) = P_B$ . Hence, the energy harvested by an arbitrary Type I node,  $U_{1,i}$  in the downlink phase is given by

$$E_{1,i} = \eta_i P_B h_{1,i} \tau_0, \quad (2)$$

where  $\eta_i \in (0, 1)$  is the efficiency of the RF energy harvesting circuitry [7], [8], at  $U_{1,i}$ . The data signal transmitted by  $U_{1,i}$  and  $U_{2,j}$  in the uplink phase are denoted by  $x_{1,i} \sim \mathcal{CN}(0, P_{1,i})$  and  $x_{2,j} \sim \mathcal{CN}(0, P_{2,j})$ , respectively, where  $\mathcal{CN}(\mu, \sigma^2)$  stands for a circularly symmetric complex Gaussian random variable with mean  $\mu$  and variance  $\sigma^2$ . Assuming that all the energy harvested at  $U_{1,i}$  is used for uplink information transmission, then the transmitted power limits by  $U_{1,i}$  and  $U_{2,j}$  on the uplink are given, respectively, by

$$P_{1,i} = \frac{E_{1,i}}{\tau_{1,i}}, \quad i = 1, \dots, M, \quad (3)$$

$$P_{2,j} = \frac{E_{2,j}}{\tau_{2,j}}, \quad j = 1, \dots, N, \quad (4)$$

where  $E_{2,j}$  is the energy drawn by  $U_{2,j}$  from its dedicated energy supply (battery) within its assigned  $\tau_{2,j}$  fraction of time for information transmission. Therefore, the received signal at the BS in the uplink phase during  $\tau_{1,i}$  and  $\tau_{2,i}$  can be expressed, respectively, by

$$y_{1,i} = g'_{1,i} x_{1,i} + n_{1,i}, \quad i = 1, \dots, M, \quad (5)$$

$$y_{2,j} = g'_{2,j} x_{2,j} + n_{2,j}, \quad j = 1, \dots, N, \quad (6)$$

where  $n_{1,i} \sim \mathcal{CN}(0, \sigma^2)$  and  $n_{2,j} \sim \mathcal{CN}(0, \sigma^2)$  denote the noise at the BS within  $\tau_{1,i}$  and  $\tau_{2,j}$ , respectively. From (2)-(6), the achievable uplink throughput of  $U_{1,i}$  and  $U_{2,j}$  in bits/second/Hz is given by

$$\begin{aligned} R_{1,i}(\tau_0, \tau_{1,i}) &= \tau_{1,i} \log_2 \left( 1 + \frac{g_{1,i} P_{1,i}}{\Gamma \sigma^2} \right) \\ &= \tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right), \end{aligned} \quad (7)$$

$$\begin{aligned} R_{2,j}(E_{2,j}, \tau_{2,j}) &= \tau_{2,j} \log_2 \left( 1 + \frac{g_{2,j} P_{2,j}}{\Gamma \sigma^2} \right) \\ &= \tau_{2,j} \log_2 \left( 1 + \theta_j \frac{E_{2,j}}{\tau_{2,j}} \right), \end{aligned} \quad (8)$$

respectively, where  $\gamma_i = \frac{\eta_i h_{1,i} g_{1,i} P_B}{\Gamma \sigma^2}$ ,  $\theta_j = \frac{g_{2,j}}{\Gamma \sigma^2}$ , for  $i = 1, \dots, M$ ,  $j = 1, \dots, N$  and  $\Gamma$  denotes the signal to noise ratio gap.

In this paper, we propose two optimal resource allocation schemes for WPCN systems with two types of nodes, namely,

RF energy harvesting and legacy (battery-powered) nodes, which are different in the way Type II (legacy) nodes exploit their dedicated energy supplies. Under the first formulation, called Optimal Policy under per slot (Instantaneous) energy Constraint (OPIC), the consumed energy per slot by each legacy node ( $E_{2,j}$ ) is optimized subject to maximum allowable energy consumption per slot, denoted  $E_{max}$ . Under the second formulation, called Optimal Policy under Average energy Constraint (OPAC), we relax the strong “per slot energy requirement” of OPIC. In OPAC, the energy consumption of Type II nodes is limited to a pre-specified fixed value per slot, denoted by  $\bar{E}$ , in an attempt to limit the overall system energy consumption.

### III. OPTIMAL POLICY UNDER INSTANTANEOUS ENERGY CONSTRAINT (OPIC)

In this section, we formulate two optimization problems for WPCN with two types of nodes and establish their convexity which facilitates efficient solution using standard techniques. First, we target maximizing sum system throughput (problem **P1**). Second, motivated by the fundamental fairness-throughput trade-off, we cast the problem into a maxmin formulation (problem **P2**).

#### A. Sum Throughput Maximization

The motivation behind introducing this scheme is to characterize the maximum sum throughput for a WPCN with two types of nodes compared to the performance of the baseline WPCN with only energy harvesting nodes introduced in [4]. Towards this objective, we find the optimal transmission durations ( $\tau_0$  for harvesting, as well as Type I and Type II nodes) and the optimal consumed energy by each Type II node per slot ( $E_{2,j}$ ) that maximize the system sum throughput subject to a constraint on the total energy consumption per slot (Instantaneous), denoted by  $E_{max}$ , and the total transmission block time constraint. For fair comparison with [4], we take  $E_{max}$  to be the total per slot energy consumption of [4] under similar conditions, i.e. number of nodes of each type, channel gains, etc. Having the total per slot energy consumption available for each slot is a strong assumption that we relax in the next section. Nevertheless, we adopt it in this formulation in order to assess the best performance achievable by WPCN with two types of nodes under the same amount of resources available to the reference system in [4]. Therefore, from (7) and (8), the problem of sum throughput maximization can be

formulated as

**P1** :

$$\begin{aligned} \max_{\tau, E_2} \quad & \sum_{i=1}^M \tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right) + \sum_{j=1}^N \tau_{2,j} \log_2 \left( 1 + \theta_j \frac{E_{2,j}}{\tau_{2,j}} \right) \\ \text{s.t.} \quad & \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1, \\ & \sum_{i=1}^M \eta_i P_B h_{1,i} \tau_0 + \sum_{j=1}^N E_{2,j} \leq E_{max}, \\ & \tau \geq 0, \\ & E_2 \geq 0, \end{aligned} \tag{9}$$

where  $\tau = [\tau_0, \tau_{1,1}, \dots, \tau_{1,M}, \tau_{2,1}, \dots, \tau_{2,N}]$  and  $E_2 = [E_{2,1}, E_{2,2}, \dots, E_{2,N}]$ .

**Theorem 1.** *P1 is a convex optimization problem.*

*Proof:*  $\tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right)$  is the perspective function of the concave function  $\log_2(1 + \gamma_i \tau_0)$  which preserves the concavity of  $R_{1,i}$  with respect to  $(\tau_0, \tau_{1,i})$ . Also,  $\tau_{2,j} \log_2 \left( 1 + \theta_j \frac{E_{2,j}}{\tau_{2,j}} \right)$  is the perspective function of the concave function  $\log_2(1 + \theta_j E_{2,j})$  which preserves the concavity of  $R_{2,j}$  with respect to  $(\tau_{2,j}, E_{2,j})$ . Since the non-negative weighted sum of concave functions is also concave [9], then the objective function of **P1**, which is the non-negative weighted summation of concave functions, i.e.,  $R_{1,i}$  and  $R_{2,j}$  for  $i = 1, \dots, M$  and  $j = 1, \dots, N$ , is a concave function in  $(\tau, E_2)$ . In addition, all constraints of **P1** are affine in  $(\tau, E_2)$ . This establishes the proof. ■

Given the sum throughput maximization objective in **P1**, OPIC allocates more energy to the nodes with better channel power gains and, hence, more uplink transmission time. This, in turn, leads to unfair rate allocation among different users as will be shown in Fig. 3 in the simulation section.

#### B. Maxmin Fairness Formulation

Motivated by the fairness limitations of **P1**, we formulate a second optimization problem targeting the fairness in the well-known maxmin sense [10] subject to the same constraints of **P1** as follows.

$$\begin{aligned} \mathbf{P2} : \quad & \max_{\tau, E_2} \min_{i,j} (R_{1,i}(\tau_0, \tau_{1,i}), R_{2,j}(E_{2,j}, \tau_{2,j})) \\ \text{s.t.} \quad & \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1, \\ & \sum_{i=1}^M \eta_i P_B h_{1,i} \tau_0 + \sum_{j=1}^N E_{2,j} \leq E_{max}, \\ & \tau \geq 0, \\ & E_2 \geq 0. \end{aligned} \tag{10}$$

Along the lines of the proof of Theorem 1, problem **P2** convexity can be established. Details are omitted due to space

limitations. An equivalent optimization problem to **P2** can be formulated as

$$\begin{aligned}
\mathbf{P2}^* : \quad & \max_{t, \tau, E_2} t \\
\text{s.t.} \quad & \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1, \\
& \sum_{i=1}^M \eta_i P_B h_{1,i} \tau_0 + \sum_{j=1}^N E_{2,j} \leq E_{max}, \\
& \tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right) \geq t, \quad i = 1, \dots, M, \\
& \tau_{2,j} \log_2 \left( 1 + \theta_j \frac{E_{2,j}}{\tau_{2,j}} \right) \geq t, \quad j = 1, \dots, N, \\
& \tau \geq 0, \\
& E_2 \geq 0,
\end{aligned} \tag{11}$$

where  $t$  is an auxiliary variable that denotes the minimum throughput achieved by each user.

#### IV. OPTIMAL POLICY UNDER AVERAGE ENERGY CONSTRAINT (OPAC)

Motivated by the strong requirement of knowing the per slot total energy consumption of the system with only RF energy harvesting nodes in Section III and with the purpose of having a more practically viable problem formulation, we relax the aforementioned requirement. We introduce an alternative formulation that, instead, relies on a fixed energy budget for type II nodes allowed to consume per transmission block (i.e., slot), denoted by  $\bar{E}$ .

##### A. Sum Throughput Maximization

In this subsection, we formulate the sum throughput maximization problem for the OPAC scheme. From (7) and (8), the problem can be formulated as

$$\begin{aligned}
\mathbf{P3} : \quad & \max_{\tau} \sum_{i=1}^M \tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right) + \sum_{j=1}^N \tau_{2,j} \log_2 \left( 1 + \theta_j \frac{\bar{E}}{\tau_{2,j}} \right) \\
\text{s.t.} \quad & \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1, \\
& \tau \geq 0.
\end{aligned} \tag{12}$$

The objective function of problem **P3** is optimized over transmission durations ( $\tau_0$  for harvesting, as well as Type I and Type II nodes) subject to the total transmission block time constraint with a fixed ( $\bar{E}$ ) for type II nodes specified to satisfy an average energy consumption constraint on the network.

**Theorem 2.** ***P3** is a convex optimization problem and the optimal time allocations are given by*

$$\tau_0^* = \begin{cases} \frac{x_1^* - \bar{E}A_2 - 1}{x_1^* + A_1 - 1}, & \text{if } (x_1^* - 1) \geq \bar{E}A_2, \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

$$\tau_{1,i}^* = \begin{cases} \frac{\gamma_i (x_1^* - \bar{E}A_2 - 1)}{(x_1^* - 1)(x_1^* + A_1 - 1)}, & \text{if } (x_1^* - 1) \geq \bar{E}A_2, \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

$$\tau_{2,j}^* = \begin{cases} \frac{\theta_j \bar{E}}{x_1^* - 1}, & \text{if } (x_1^* - 1) \geq \bar{E}A_2, \\ \frac{\theta_j}{A_2}, & \text{otherwise,} \end{cases} \tag{15}$$

for  $i = 1, \dots, M$  and  $j = 1, \dots, N$ , where  $A_1 = \sum_{i=1}^M \gamma_i$ ,  $A_2 = \sum_{j=1}^N \theta_j$  and  $x_1^* > 1$  is the solution of  $f(x_1) = A_1$ , where

$$f(x) = x \ln(x) - x + 1. \tag{16}$$

*Proof:* Please refer to the Appendix.  $\blacksquare$

Based on Theorem 2, it is clear that the optimal time allocated to each user for uplink information transmission depends on its distance to the BS, i.e., the near users (with better channel power gains) to the BS are allocated more uplink transmission time than the far users, which demonstrates the doubly near-far phenomenon [4]. Moreover, it is observed that  $\tau_{2,j}^*$  is proportional to  $\bar{E}$ , i.e., as the amount of allocated energy per transmission block for each legacy node increases, the uplink allocated time for legacy nodes increases and that allocated for RF energy harvesting nodes decreases. Taking into consideration the above two observations, the sum throughput maximization results in an unfair achievable throughput among different users.

##### B. Maxmin Fairness Formulation

Targeting fairness among users, we adopt, once more, the maxmin fairness formulation within the OPAC WPCN paradigm as follows.

$$\begin{aligned}
\mathbf{P4} : \quad & \max_{\tau} \min_{i,j} (R_{1,i}(\tau_0, \tau_{1,i}), R_{2,j}(\bar{E}, \tau_{2,j})) \\
\text{s.t.} \quad & \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} \leq 1, \\
& \tau \geq 0.
\end{aligned} \tag{17}$$

Along the lines of Theorem 2, problem **P4** convexity can be established. Details are omitted due to space limitations. Moreover, an equivalent optimization problem to **P4** can be formulated as in **P2**<sup>\*</sup>.

#### V. NUMERICAL RESULTS

In this section, we provide numerical results showing the merits of the formulated optimization problems and the associated trade-offs. Motivated by the convexity of the formulated problems, we use standard optimization tools, e.g., CVX [9], to obtain the optimal solutions. We consider the channel power gains are modeled as  $h_{1,i} = g_{1,i} = 10^{-3} \rho_{1,i}^2 d_{1,i}^{-\alpha}$  for  $i = 1, \dots, M$  and  $g_{2,j} = 10^{-3} \rho_{2,j}^2 d_{2,j}^{-\alpha}$  for  $j = 1, \dots, N$ , where  $d_{1,i}$  denotes the distance between  $U_{1,i}$  and the BS,  $d_{2,j}$  denotes the distance between  $U_{2,j}$  and BS and  $\alpha$  denotes the pathloss exponent.  $\rho_{1,i}$  and  $\rho_{2,j}$  are the standard Rayleigh short term fading; therefore  $\rho_{1,i}^2$  and  $\rho_{2,j}^2$  are exponentially distributed random variables with unit mean. We consider

same parameters as in [4], we use  $P_B = 20$  dBm,  $\sigma^2 = -160$  dBm/Hz,  $\eta_i = 0.5$  for  $i = 1, \dots, M$ ,  $\Gamma = 9.8$  dB and the bandwidth is set to be 1 MHz. Moreover, each throughput curve shown later is obtained by averaging over 1000 randomly generated channel realizations. We compare the performance of our system with two types of nodes in OPIC and OPAC with the performance of the baseline WPCN with only Type I nodes [4] subject to same amount of available resources.

In Fig. 2, we compare the average maximum achievable sum throughput for the 3 studied systems vs. the pathloss exponent. We consider the same scenario for all schemes with  $N = 1$ ,  $M = 1$ ,  $d_{1,1} = 10m$  and  $d_{2,1} = 5m$  where the baseline WPCN system with energy harvesting nodes only is considered to have two users with same distances  $d_{1,1}$  and  $d_{2,1}$  given above. Our objective is to fairly compare the three systems, baseline WPCN, OPIC and OPAC, with same total amount of energy resources. Towards this objective, at each pathloss exponent value, the total harvested energy for each channel realization at the baseline WPCN is assumed to be  $E_{max}$  in **P1** (i.e., a per slot energy constraint). Also,  $\bar{E}$  for Type II nodes is computed using the optimal derived closed form time allocations (for **P3**) such that the average energy consumptions over the 1000 channel realizations are equal in both the baseline WPCN and the OPAC system.

A number of observations are now in order. First, we note that the average maximum sum throughput of the three studied systems monotonically decreases as the pathloss exponent increases. Second, as expected, the average maximum sum throughput achieved by **P1** is the highest due to the fact that OPIC allocates more energy to the user with better channel power gains that is, the legacy node in our scenario, to maximize the sum throughput. Therefore, in our scenario, the average maximum sum throughput is obtained via allocating more energy to the legacy node than the energy harvesting node and, hence, reducing  $\tau_0$ . Unlike the baseline WPCN with energy harvesting nodes only where the amount of harvested energy by the farther user cannot be efficiently utilized for uplink data transmissions and cannot be reduced through reducing the  $\tau_0$  as in **P1** since the user closer to the BS is also an energy harvesting node which harvests its energy during that  $\tau_0$  fraction of time. Therefore, it is clear that the average maximum sum throughput of **P1** outperforms the one of baseline WPCN with energy harvesting nodes only. Third, the average maximum sum throughput obtained by **P3** is less than that achieved by **P1** due to the OPAC that yields fixed  $\bar{E}$  that guarantees equal “average” consumed energy.

In Fig. 3, Jain’s fairness index [11] is plotted for the three systems under consideration against the pathloss exponent considering the same scenario in Fig. 2. It is observed that the fairness index of both proposed schemes, namely OPIC and OPAC, is less than that of the baseline WPCN with energy harvesting nodes only. This, in turn, highlights one instance of the fundamental throughput-fairness trade-off, where the superior sum throughput performance in **P1** and **P3** compared to baseline WPCN came at the expense of a modest degradation

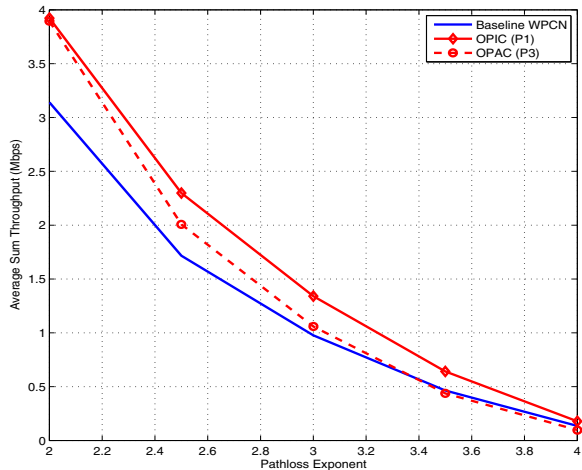


Fig. 2. Average maximum sum throughput for the 3 systems.

in the fairness.

Motivated by the inherent unfairness witnessed for the sum throughput maximization formulation for the 3 studied systems, Fig. 4 shows the average maxmin throughput comparison with the same set of parameters as in Fig. 2. It is observed that the average maxmin throughput achieved by **P2** and **P4** is larger than that of the baseline WPCN with energy harvesting nodes only. It is also observed that twice the average maxmin throughput of each system at each pathloss exponent value (which is the average sum throughput given that we have only two users) is less than the average maximum sum throughput for the same system (Fig. 2). This, in turn, demonstrates the fundamental trade off between achieving maximum sum throughput and achieving fair throughput allocations among different users.

Fig. 5 shows the effect of scaling the system with larger number of nodes on the network performance via comparing the average maximum sum throughput of **P1** and **P3** with the one of the baseline WPCN with energy harvesting nodes only. Towards this objective, we consider The network has six users with same distance  $d = \frac{10}{6}m$  for the three systems under consideration. It is observed that as the number of Type II nodes ( $N$ ) in both **P1** and **P3** increases, the average maximum sum throughput increases since increasing  $N$  reduces the allocated time for energy harvesting ( $\tau_0$ ) and, hence, the average maximum sum throughput increases via assigning that reduction in ( $\tau_0$ ) for uplink data transmission through Type II nodes. Therefore, it is clear that the maximum achievable sum throughput is obtained by the extreme case of  $N = 6$  and  $M = 0$  ( $\tau_0^* = 0$ ).

## VI. CONCLUSION

This paper studies a generalized problem setting for wireless powered communication networks. In which, the network has two types of nodes, energy harvesting nodes assumed to have RF energy harvesting capability and legacy nodes

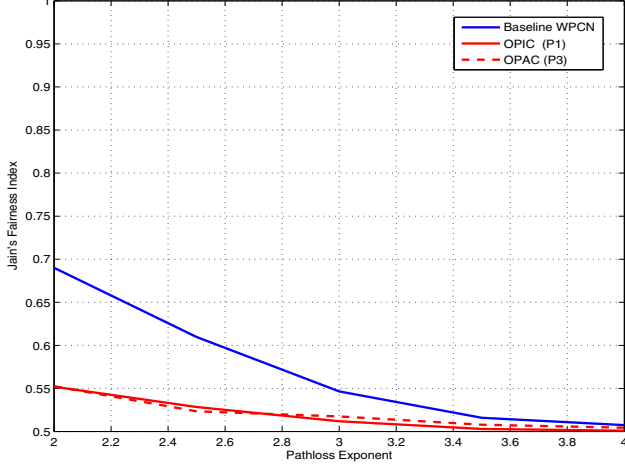


Fig. 3. Jain's fairness index for the 3 systems.

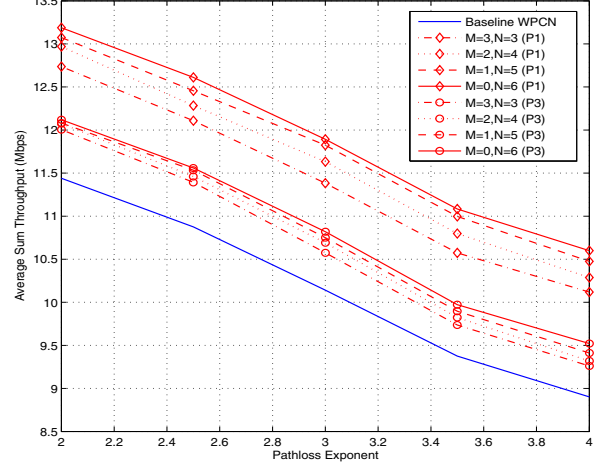


Fig. 5. Average maximum sum throughput for the 3 systems.

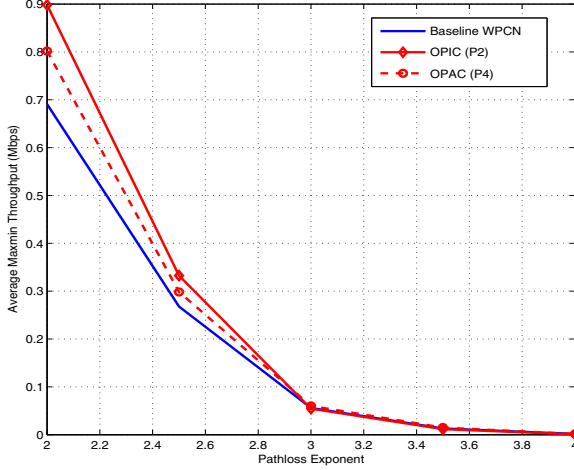


Fig. 4. Average maxmin throughput for the 3 systems.

assumed not to have RF energy harvesting capability and equipped with dedicated energy supplies. A BS first broadcasts an energizing signal over the downlink so that each energy harvesting user could harvest a certain amount of energy for uplink information transmission, then all users send their information independently to the BS using TDMA. We provide two optimal resource allocation schemes, namely OPIC and OPAC, subject to different energy constraints on the system. Furthermore, We formulate two optimization problems to investigate the maximum sum throughput and the maxmin throughput in both proposed schemes. Our numerical results reveal that both OPIC and OPAC schemes outperform the baseline WPCN with energy harvesting nodes only in terms of the maximum sum throughput and the maxmin throughput. They also demonstrate the fundamental trade off between achieving maximum sum throughput and achieving fairness

among different users.

## APPENDIX

Since  $\tau_{1,i} \log_2 \left( 1 + \gamma_i \frac{\tau_0}{\tau_{1,i}} \right)$  is the perspective function of the concave function  $\log_2 (1 + \gamma_i \tau_0)$  which preserves the concavity of  $R_{1,i}$  with respect to  $(\tau_0, \tau_{1,i})$ . Also,  $\frac{\partial^2}{\partial \tau_{2,j}^2} \left( \tau_{2,j} \log_2 \left( 1 + \theta_j \frac{\bar{E}}{\tau_{2,j}} \right) \right) = \frac{-\theta_j^2 \bar{E}^2}{\ln(2) \tau_{2,j} (\tau_{2,j} + \theta_j \bar{E})^2} < 0$  for  $\tau_{2,j} \in [0, 1]$ , therefore  $R_{2,j}$  is also concave function in  $\tau_{2,j}$ . A non-negative weighted sum of concave functions is concave [9], then the objective function of **P3** which is the non negative weighted summation of concave functions, i.e.,  $R_{1,i}$  and  $R_{2,j}$  for  $i = 1, \dots, M$  and  $j = 1, \dots, N$ , is concave function in  $\tau = [\tau_0, \tau_{1,1}, \dots, \tau_{1,M}, \tau_{2,1}, \dots, \tau_{2,N}]$ . Furthermore, all constraints in **P3** are affine in  $\tau$ , thus it is clear that **P3** is convex optimization problem and its Lagrangian is given by

$$\mathcal{L}(\tau, \lambda) = R_{sum}(\tau) - \lambda \left( \tau_0 + \sum_{i=1}^M \tau_{1,i} + \sum_{j=1}^N \tau_{2,j} - 1 \right), \quad (18)$$

where  $R_{sum}(\tau) = \sum_{i=1}^M R_{1,i}(\tau_0, \tau_{1,i}) + \sum_{j=1}^N R_{2,j}(\bar{E}, \tau_{2,j})$  and  $\lambda$  is the non negative Lagrangian dual variable associated with the constraint in (1). Hence, the dual function can be expressed as

$$G(\lambda) = \max_{\tau \in \mathcal{S}} \mathcal{L}(\tau, \lambda), \quad (19)$$

where  $\mathcal{S}$  is the feasible set specified by  $\tau \geq 0$ . It can be easily shown that there exists a  $\tau$  that strictly satisfies all constraints of **P3**. Hence, according to Slater's condition [9], strong duality holds for this problem; therefore, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient

for the global optimality of **P3**, which are given by

$$\tau_0^* + \sum_{i=1}^M \tau_{1,i}^* + \sum_{j=1}^N \tau_{2,j}^* \leq 1, \quad (20)$$

$$\lambda^* \left( \tau_0^* + \sum_{i=1}^M \tau_{1,i}^* + \sum_{j=1}^N \tau_{2,j}^* - 1 \right) = 0, \quad (21)$$

$$\frac{\partial}{\partial \tau_0} R_{sum}(\tau^*) - \lambda^* = 0, \quad (22)$$

$$\frac{\partial}{\partial \tau_{1,i}} R_{sum}(\tau^*) - \lambda^* = 0, \quad i = 1, \dots, M, \quad (23)$$

$$\frac{\partial}{\partial \tau_{2,j}} R_{sum}(\tau^*) - \lambda^* = 0, \quad j = 1, \dots, N, \quad (24)$$

where  $\tau^*$  and  $\lambda^*$  denote, respectively, the optimal primal and dual solutions of **P3**. Since  $R_{sum}(\tau)$  is a monotonic increasing function in  $\tau$ , therefore  $\tau_0^* + \sum_{i=1}^M \tau_{1,i}^* + \sum_{j=1}^N \tau_{2,j}^* = 1$  must hold. From (22), (23) and (24), we have

$$\sum_{i=1}^M \frac{\gamma_i}{1 + \gamma_i \frac{\tau_0^*}{\tau_{1,i}^*}} = \lambda^* \ln(2), \quad (25)$$

$$\ln \left( 1 + \gamma_i \frac{\tau_0^*}{\tau_{1,i}^*} \right) - \frac{\gamma_i \frac{\tau_0^*}{\tau_{1,i}^*}}{1 + \gamma_i \frac{\tau_0^*}{\tau_{1,i}^*}} = \lambda^* \ln(2), \quad i = 1, \dots, M, \quad (26)$$

$$\ln \left( 1 + \frac{\bar{E}\theta_j}{\tau_{2,j}^*} \right) - \frac{\frac{\bar{E}\theta_j}{\tau_{2,j}^*}}{1 + \frac{\bar{E}\theta_j}{\tau_{2,j}^*}} = \lambda^* \ln(2), \quad j = 1, \dots, N. \quad (27)$$

Therefore, from (26) and (27) we have

$$\frac{\gamma_1 \tau_0^*}{\tau_{1,1}^*} = \frac{\gamma_2 \tau_0^*}{\tau_{1,2}^*} = \dots = \frac{\gamma_M \tau_0^*}{\tau_{1,M}^*} = \frac{\bar{E}\theta_1}{\tau_{2,1}^*} = \frac{\bar{E}\theta_2}{\tau_{2,2}^*} = \dots = \frac{\bar{E}\theta_N}{\tau_{2,N}^*}. \quad (28)$$

Taking into consideration that  $\tau_0^* + \sum_{i=1}^M \tau_{1,i}^* + \sum_{j=1}^N \tau_{2,j}^* = 1$  and (28). Hence,  $\tau_{1,i}^*$  and  $\tau_{2,j}^*$  can be expressed, respectively, by

$$\tau_{1,i}^* = \frac{\gamma_i \tau_0^* (1 - \tau_0^*)}{A_1 \tau_0^* + \bar{E}A_2}, \quad i = 1, \dots, M, \quad (29)$$

$$\tau_{2,j}^* = \frac{\theta_j \bar{E} (1 - \tau_0^*)}{A_1 \tau_0^* + \bar{E}A_2}, \quad j = 1, \dots, N, \quad (30)$$

where  $A_1 = \sum_{i=1}^M \gamma_i$  and  $A_2 = \sum_{j=1}^N \theta_j$ . From (25), (26) and (27), it follows that

$$x_1 \ln(x_1) - x_1 + 1 = A_1, \quad (31)$$

where  $x_1 = 1 + \gamma_i \frac{\tau_0^*}{\tau_{1,i}^*} = 1 + \frac{\bar{E}\theta_j}{\tau_{2,j}^*}$ . From (29) and (30), it is clear that  $x_1 > 1$  if  $A_1 > 0$ ,  $A_2 > 0$  and  $0 < \tau_0^* < 1$ . According to [4, Lemma 3.2], there exists a unique solution

$x_1^* > 1$  for (31). Thus from (29)-(31), the optimal time allocations are given by

$$\tau_0^* = \frac{x_1^* - \bar{E}A_2 - 1}{x_1^* + A_1 - 1}, \quad (32)$$

$$\tau_{1,i}^* = \frac{\gamma_i (x_1^* - \bar{E}A_2 - 1)}{(x_1^* - 1)(x_1^* + A_1 - 1)}, \quad i = 1, \dots, M, \quad (33)$$

$$\tau_{2,j}^* = \frac{\theta_j \bar{E}}{x_1^* - 1}, \quad j = 1, \dots, N. \quad (34)$$

If  $(x_1^* - 1) < \bar{E}A_2$ , then the total block time will be assigned to the Type II nodes for uplink information transmissions. Therefore, from (20) and (27), the optimal time allocations are given by

$$\tau_0^* = 0, \quad (35)$$

$$\tau_{1,i}^* = 0, \quad i = 1, \dots, M, \quad (36)$$

$$\tau_{2,j}^* = \frac{\theta_j}{A_2}, \quad j = 1, \dots, N. \quad (37)$$

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