Optimizing Cooperative Cognitive Radio Networks Performance With Primary QoS Provisioning

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***Abstract*— We consider the problem of optimizing the performance of a cooperative cognitive radio user subject to constraints on the quality-of-service (QoS) of the primary user (PU). In particular, we design the probabilistic admission control parameter of the PU packets in the secondary user (SU) relaying queue and the randomized service parameter at the SU under non-work-conserving (non-WC) and WC cooperation policies. In the non-WC policy, two constrained optimization problems are formulated; the first problem is maximizing the SU throughput while the second problem is minimizing the SU average delay. In both problems, a constraint is imposed on the maximum allowable average delay of the PU. We show the equivalence of the two problems and develop a low-complexity line search algorithm to find the optimal parameters. Subsequently, the idea of optimizing the SU average delay is developed for the more complex WC policy, for its superior resource utilization and performance. Due to the sheer complexity of this optimization problem, we formulate another problem whose solution yields a suboptimal upper bound on the optimal SU delay. Afterwards, a practical WC-policy-based algorithm is designed in order to closely approach the optimal value of the SU delay. We show, through numerical results, that the proposed cooperation policies represent the best compromise between enhancing the SU QoS and satisfying the PU QoS requirements. Furthermore, the superior performance of the suboptimal WC policy over the non-WC policy is illustrated. Finally, the merits of the WC-policy-based algorithm are demonstrated through extensive simulations.**

***Index Terms*— Cognitive relaying, stable throughput region, average packet delay.**

Manuscript received May 23, 2016; revised September 15, 2016; accepted October 12, 2016. Date of publication October 26, 2016; date of current version April 14, 2017. This work was made possible by grants number NPRP 4-1034-2-385 and NPRP 5-782-2-322 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors. This work was presented at the IEEE Wireless Communications and Networking Conference, Doha, Qatar, Apr. 2016. The associate editor coordinating the review of this paper and approving it for publication was Y. Li.

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Digital Object Identifier 10.1109/TCOMM.2016.2621744

I. INTRODUCTION

THE concept of cognitive radio was stimulated by the problem of severe underutilization of the licensed spectrum, in addition to the spectrum scarcity [1]–[3]. The cognitive radio technology aims at exploiting the spectrum holes and, hence, efficiently utilizing the precious wireless spectrum. Cognitive radio networks consist of licensed primary users (PUs), who may not transmit data the whole time, and unlicensed secondary users (SUs) having sensing capability of the spectrum in order to detect and exploit the spectrum holes for the transmission of their packets. The coexistence of SUs with PUs is subject to the condition that a certain level of QoS is guaranteed to the PUs.

The notion of cooperative wireless communications hinges on the broadcast nature of the wireless medium. A data transmission between a source and a destination might be received and decoded by intermediate nodes that could act as "relays" [4], [5]. One of the advantages of cooperative communications is improving the performance of wireless networks since the relay can retransmit packets that are not successfully decoded by the destination. A significant part of the literature was dedicated to studying the idea of cooperative communications from the perspective of the physical (PHY) layer, e.g., [6]–[8]. For example, cooperative transmission protocols were proposed in [6] for a system that consists of N half-duplex partners and one cell site in a delay-limited coherent fading channels. The performance of the proposed protocols was assessed through the Zheng-Tse diversity-multiplexing tradeoff framework [9]. In [7], the authors proposed a symbol error rate analysis and characterized the optimal power allocation for a decode and forward cooperation protocol in wireless networks. The authors in [8] designed a mechanism for a multi-node decode-and-forward relay selection that exploits the partial channel state information (CSI) at the source and the relays. The objective of the cooperative protocol was to achieve higher bandwidth efficiency and assure full diversity order.

Leveraging cooperative communications within the context of cognitive radio networks promises considerable performance gains. It was studied in the literature at the medium access control (MAC) layer. Recently, there has been growing interest in cognitive relaying networks where the SU can assist the PU in delivering its packets to the destination, e.g., [10]–[14]. Such cooperation would be beneficial to both the PU and the SU. Cognitive relaying networks allow the PU to reliably transmit its packets to the destination through the

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SU when the data is lost over the direct link. As a result, the number of time slots in which the SU can access the channel and transmit its own packets increases. For example, a cognitive interference channel was studied in [10] where the SU acts as a relay for the PU traffic. A power allocation scheme at the SU was designed in order to maximize the stable throughput of the secondary link for a fixed throughput of the primary link. It should be noted that the optimization problem did not impose any constraints on the average packet delay encountered by the PU. In [11], two cooperative cognitive multiple-access protocols were proposed in a network that consists of M source terminals, a relay node, and a common destination node. The performance gains of the proposed protocols over conventional relaying strategies were demonstrated in terms of the maximum stable throughput region and the delay performance. In [12], the stable throughput region was characterized for a cooperative cognitive network with a fixed scheduling probability. Specifically, the secondary link was allowed to share the channel along with the primary link, and the secondary node cooperatively relayed the successfully decoded PU packets that were not received by the primary destination.

A. Motivation

The motivation of this paper is the emergence of opportunistic real-time (ORT) traffic in cognitive radio networks, in general, and cooperative cognitive radio networks in particular, e.g., when the PUs and SUs are using multimedia applications, video streaming or voice over Internet Protocol (VoIP), demanding high throughput and stringent delay requirements. In these applications, the challenge lies in efficient utilization of available resources such that the QoS of the SU is maximized while guaranteeing the target QoS of the PU.

A protocol-level cooperative communication protocol was proposed in [15] for a wireless multiple-access system with probabilistic transmission success. The system comprised N users and a common destination. Each user was considered as a source and, at the same time, a potential relay. For the two-user case, user 1 had one queue for its own packets while user 2 had two queues; one for its own packets and the other for the relayed packets from user 1. The proposed cooperation policy assigned higher priority to user 1 to access the channel and transmit its packets. The shortcoming of applying this policy in the framework of cooperative cognitive radio networks was giving strict priority to user 1 (PU) packets, possibly yielding average PU packet delay much stricter than required (i.e., over designing the system for the PU). In other words, according to this policy, user 2 (or SU) packets may experience severe delay while PU packets can tolerate higher delays.

Scheduling polices can be classified as work conserving (WC) or non-work-conserving (non-WC). Work conserving scheduling algorithms always keep the resources busy if there are packets to be scheduled. In contrast, non-WC scheduling algorithms may leave the resources idle despite the presence of packets ready for transmission. In general, non-WC scheduling algorithms are inferior to their WC counterparts. However, they are much easier to analyze and optimize. For example, a non-WC cooperation policy for cognitive relaying was proposed in [13], for a cooperative cognitive radio network with two tunable parameters; the probabilistic relaying parameter (i.e., probabilistic admission control of the PU packets in the SU relaying queue), and the randomized service parameter at the SU (i.e., probabilistic selection between two queues; one for the SU packets and the other for the relayed PU packets). The fundamental delay-throughput tradeoff was studied and two optimization problem were formulated; the first problem was minimizing the average packet delay encountered by the PU subject to stability constraints for all the queues in the system. The second problem was minimizing the average packet delay encountered by the SU subject to the same constraints. It should be noted that the problem of optimizing the SU performance did not take into consideration the average delay of the PU. Thus, the optimal values of the randomized service and probabilistic relaying parameters resulted in severe delay for the PU packets. It is needless to mention that the essence of cognitive radio networks is maintaining a certain level of QoS for the PU while serving the SU.

In [14], the authors characterized the stable throughput region of the system in [13] when the relaying queue at the SU has limited capacity. In addition, the packet admission and queue selection probabilities were dependent on the relaying queue length at each time slot.

In summary, we propose utilizing the portion of time at which the PU has delay-insensitive traffic (e.g., web browsing) to provide better quality to the SU delay-sensitive traffic (e.g., video streaming) in exchange for some incentives. Therefore, the network resources are efficiently utilized and the cognitive user performance is optimized with primary user QoS provisioning.

B. Summary of Results

Unlike [13]–[15], this paper optimizes the QoS of real-time applications of SUs while preserving the QoS of PUs in cooperative cognitive radio networks for both non-WC as well as WC cooperation policies. The main contributions of this paper are as follows. In the first part of the paper, we investigate a cooperative cognitive radio network that operates under a non-WC cooperation policy. More specifically, we formulate two distinct optimization problems. The first problem maximizes the SU throughput while the second problem minimizes the average packet delay encountered by the SU. Both metrics are optimized subject to a constraint on the average delay encountered by the PU packets. Although the formulation of each problem yields a non-convex optimization problem, the first problem is transformed into a set of linear programs, whereas the second problem is transformed into a set quasiconvex optimization problems. In addition, we prove that the problem of optimizing the SU throughput is equivalent to optimizing the SU delay for the studied system. The numerical results show that the proposed optimal non-WC cooperation policy guarantees that the throughput and the average packet delay of the SU are enhanced, while honoring the average PU packet delay constraint.

In the second part of the paper, we study the problem of optimizing the SU packet delay subject to a constraint on the PU packet delay for a WC cooperation policy towards more efficient resource utilization. The cooperation policy considered in the first part of the paper leads to a non-WC policy because it is susceptible to wasting idle time slots. However, the mathematical analysis of its average packet delay is mathematically tractable. On the other hand, the derivation of closed-form expressions for the average packet delay of the WC policy is complex because the analysis involves the interaction of three dependent queues [16]. In order to alleviate these hurdles, another optimization problem is proposed whose solution provides an upper bound on the optimal SU delay. Afterwards, the numerical results show that the proposed suboptimal WC policy outperforms the non-WC policy studied in the first part of the paper. Finally, a practical WC-policy-based algorithm is proposed in order to closely approach the optimal solution of the target optimization problem.

It is worth mentioning that our work in [17] considers only the non-WC cooperation policy. This paper extends the work in [17] by analyzing the WC cooperation policy for the studied system.

C. Organization

The rest of this paper is organized as follows. The system model is given in Section II. The SU throughput and the average SU packet delay are characterized in Section III. The problem of optimizing the SU performance subject to a constraint on the PU delay is formulated and solved in Section IV for the non-WC cooperation policy. In Section V, the problem of minimizing the SU delay subject to the same constraint for a WC cooperation policy is investigated and a WC-policy-based algorithm is proposed in order to approach the optimal SU delay. Numerical results are presented and discussed in Section VI. Finally, the paper is concluded in Section VII.

II. System Model

We consider the cooperative cognitive radio network depicted in Fig. 1. The network comprises two users (e.g., two mobile stations), a PU and a SU, and a common destination (e.g., a base station). The PU is equipped with a queue, Q_p, for the primary user packets. On the contrary, the SU has two queues, Q_s and Q_{sp}. Q_s is intended for the secondary user packets, whereas Q_{sp} is intended for the packets that are overheard, decoded and enqueued from the PU. All queues are assumed to be of infinite length. The assumption of infinite queue length is reasonable when the queue size is much larger than the packet size.

We assume a time-slotted system where the transmission of a packet takes exactly one time slot. The packet arrival processes at Q_p and Q_s are modelled as Bernoulli random processes with rates λ_p and λ_s packets per time slot, respectively, where $0 \leq \lambda_p \leq 1$ and $0 \leq \lambda_s \leq 1$. The packet arrival processes are assumed independent from each other and packet arrivals at each queue are independent and identically distributed (i.i.d.) across time slots. The evolution of the length of the j^{th} queue is characterized as

$$Q_j^{t+1} = \left(Q_j^t - Y_j^t\right)^+ + X_j^t, \text{ for } j \in \{p, sp, s\}, \quad (1)$$

where $(x)^+ = \max(x, 0)$. Q_j^t denotes the number of packets of the j^{th} queue at time slot t. X_j^t and Y_j^t are binary random

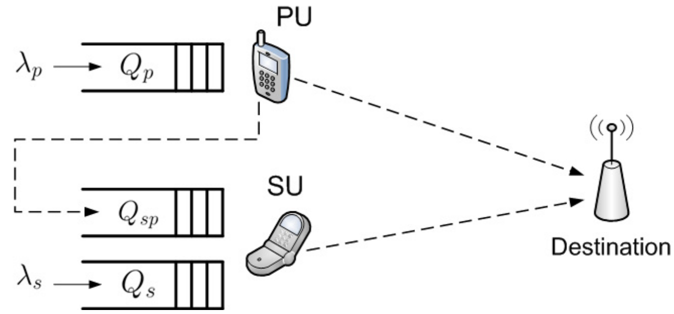

Fig. 1. The system model. The dashed lines represent communication links between nodes.

variables that represent the number of packets that arrive at or depart from the j^{th} queue at time slot t, respectively. A positive acknowledgment packet (ACK) is sent by a receiving node that successfully decodes a packet and heard by all other nodes in the network. In the cooperative cognitive radio network illustrated in Fig. 1, an ACK is sent from either the destination or the SU. The length of an ACK is assumed to be very short compared to the slot duration. It is also assumed that the errors as well as the delay in the acknowledgment feedback channel are negligible. This assumption is justified by employing low rate codes in the feedback channel.

The prime causes of the degradation of wireless link quality are multipath fading, additive noise, and signal attenuation. We assume that the random processes modeling the channel gains and noise are stationary. The probability of wireless link outage is the probability that the transmission rate of a source exceeds the instantaneous link capacity. For fixed-rate transmission over the primary and secondary links, the link outage probability is inversely proportional to the average signal-to-noise ratio (SNR) at the receiver. Therefore, the link outage occurs when the average SNR is below the threshold at which the receiver can decode the incoming packets without errors. Throughout this paper, the quality of wireless links is abstracted by the likelihood that a node correctly decodes a packet. The probability of successful packet reception, i.e., the probability of no link outage, between the PU and the destination, the SU and the destination, and the PU and the SU are denoted by h_{pd}, h_{sd} and h_{ps}, respectively.

Similar to [11], [13], and [14], the SU is assumed to perfectly know the state of the PU of whether it is backlogged or idle and, hence, there is no interference in our system. A possible approach to accomplish this objective is via sensing the communication channel by the SU in order to detect the time slots at which the PU is idle. This can be achieved by using detectors that have high detection probability at the SU. If the SU causes interference to the PU due to a misdetection, the interference structure could be leveraged in the detection process. Nevertheless, this is beyond the scope of this paper.

We adopt the following cooperation policy at the MAC layer.

A. When the PU Is Backlogged

Q_p immediately transmits the head-of-line (HOL) packet to the destination since it is the spectrum owner. Three potential cases arise:

- If the packet is successfully decoded by the destination, an ACK is broadcasted and the packet is dropped from the system, regardless of whether the SU successfully decodes it or not.
- If the packet is successfully decoded by the SU, but is not decoded by the destination, the packet is stored in Q_{sp} with probability a. If admitted, the SU broadcasts an ACK and the packet is dropped from Q_p.
- If neither the SU nor the destination decodes the packet, then it is kept in Q_p for future retransmission.

B. When the PU Is Idle

- The channel is accessed by the SU and a packet is transmitted either from Q_s with probability b or from Q_{sp} with probability $1-b$.
- If the destination successfully decodes the packet, an ACK is broadcasted and the packet is dropped from the system. Otherwise, the packet is kept in its respective queue for future retransmission.

The aforementioned cooperation policy is non-WC [18]. In general, a WC conserving system is superior to its non-WC counterpart. The reason lies behind the possibility that the non-WC system might have packets in its queues, yet the slot is wasted. A typical case occurs when the SU accesses the channel and an empty queue is selected for transmission while the other queue is non-empty. It should be noted that we focus in the first part of the paper on the aforementioned non-WC cooperation policy, due to its mathematical tractability. However, we relax this assumption and study the WC cooperation policy in Section V despite its reported complexity [16] attributed to the interaction of three dependant queues.

III. BACKGROUND: THROUGHPUT AND AVERAGE DELAY CHARACTERIZATION

In this section, we characterize the service rates for various queues as well as the arrival rate for the relay queue at the SU with the aid of different probabilistic events for the cooperative cognitive radio network depicted in Fig. 1. Next, the stability of the queues of the network is established. Finally, the expressions for the average packet delay of the PU and SU are presented.

A packet departs Q_p in two cases: 1) if it is decoded by the destination, i.e., the direct link is not in outage, or 2) if it is not decoded by the destination, yet, is decoded by the SU and admitted by Q_{sp}. Thus, the service rate of Q_p, μ_p, is given by

$$\mu_p = h_{pd} + \left(1 - h_{pd}\right) h_{ps} a. \tag{2}$$

On the other hand, a packet in Q_s is served when Q_p is empty, which occurs with probability $\left(1 - \frac{\lambda_p}{\mu_p}\right)$, Q_s is selected for transmission, which occurs with probability b, and there is no channel outage between the SU and the destination, which occurs with probability h_{sd}. Therefore, the service rate of Q_s, μ_s, is given by

$$\mu_s = b h_{sd} \left(1 - \frac{\lambda_p}{\mu_p}\right). \tag{3}$$

Similarly, the service rate of Q_{sp}, μ_{sp}, is given by

$$\mu_{sp} = (1-b)\, h_{sd} \left(1 - \frac{\lambda_p}{\mu_p}\right). \tag{4}$$

Furthermore, a packet is buffered at Q_{sp} when Q_p is non-empty, the direct link is in outage but the link between the PU and SU is not in outage, and the packet is admitted by Q_{sp}. Consequently, the packet arrival rate to Q_{sp}, λ_{sp}, is defined as

$$\lambda_{sp} = a \left(1 - h_{pd}\right) h_{ps} \frac{\lambda_p}{\mu_p}. \tag{5}$$

The stability of a queue is characterized by Loynes' theorem [19]. When the arrival and service processes of a queue are stationary, the queue is stable if and only if the packet arrival rate is strictly less than the packet service rate. Otherwise, the queue is unstable. Accordingly, the stability of the queues for the studied network is characterized by the following inequalities

$$\lambda_p < \mu_p, \quad \lambda_s < \mu_s, \quad \lambda_{sp} < \mu_{sp}. \tag{6}$$

The average delay experienced by the PU packets and the SU packets can be characterized by applying Little's law [20] as follows

$$D_p = \frac{N_p + N_{sp}}{\lambda_p}, \qquad D_s = \frac{N_s}{\lambda_s}, \tag{7}$$

where N_p, N_{sp} and N_s are the average queue lengths of Q_p, Q_{sp} and Q_s, receptively. N_p is obtained by direct application of the Pollaczek-Khinchine formula [20] on Q_p, a discrete-time M/M/1 queue with Bernoulli arrival rate λ_p and geometrically distributed service rate μ_p; it is given by

$$N_p = \frac{\lambda_p - \lambda_p^2}{\mu_p - \lambda_p}. \tag{8}$$

On the other hand, the expressions for N_{sp} and N_s, in terms of μ_p, are given by

$$N_{sp} = \frac{\lambda_p\left(\mu_p - h_{pd}\right)\left(\overline{b} h_{sd} \overline{\mu_p} \lambda_p - \left(\mu_p - h_{pd}\right)\mu_p \lambda_p - h_{pd}\lambda_p + \mu_p^2\right)}{\mu_p\left(\mu_p - \lambda_p\right)\left(\overline{b} h_{sd}\left(\mu_p - \lambda_p\right) - \lambda_p\left(\mu_p - h_{pd}\right)\right)}, \tag{9}$$

$$N_s = \frac{\left(\mu_p - \lambda_p\right)\left(b h_{sd}\left(\mu_p - \lambda_p\right) - \lambda_s \mu_p\right)}{b h_{sd} \lambda_p \lambda_s \overline{\mu_p} + \left(\lambda_s - \lambda_s^2\right)\left(\mu_p - \lambda_p\right)\mu_p}, \tag{10}$$

where $\overline{b} = 1 - b$ and $\overline{\mu_p} = 1 - \mu_p$. The proof of these expressions directly follows the approach in [16]. In particular, N_{sp} and N_s are evaluated by applying the moment generating function approach and analyzing the interaction of the joint lengths of the dependent queues Q_p and Q_{sp}, and Q_p and Q_s, respectively. Please refer to Theorem 5 in [13, Sec. IV] for the details of the proof.

IV. OPTIMIZING THE SECONDARY USER PERFORMANCE FOR A NON-WC COOPERATION POLICY

In this section, the problem of optimizing the SU QoS under constraints on the QoS of the PU is formulated and solved for the non-WC cooperation policy. In the first part, the SU throughput is optimized subject to a constraint on the average PU delay. In the second part, the average SU delay is optimized subject to the same constraint.

A. Optimizing the Secondary User Throughput

In this subsection, we investigate the problem of maximizing the SU throughput subject to a constraint on the average PU packet delay, D_p. Note that introducing a constraint on D_p is stricter than the stability constraint and, hence, implies the stability of Q_p, i.e., the queue length is guaranteed not to grow to infinity. Therefore, there is no need for a Q_p stability constraint in the sought formulation. Consequently, the target constrained optimization problem is formulated as

$$\begin{aligned} \textbf{P1:} \quad & \max_{a,b} \quad b\, h_{sd}\left(1-\frac{\lambda_p}{\mu_p}\right) \\ & \text{s.t.} \quad 0 \le a \le 1, \\ & \qquad 0 \le b \le 1, \\ & \qquad \mu_p = h_{pd} + \left(1-h_{pd}\right) h_{ps} a, \\ & \qquad \frac{N_p + N_{sp}}{\lambda_p} \le \psi, \\ & \qquad (8),\ (9), \end{aligned} \tag{11}$$

where the objective function is simply the SU packet service rate, μ_s. ψ specifies the maximum average packet delay that the PU can tolerate. In real systems, the delay sensitivity of the PU applications should map to the value of ψ accordingly. **P1** is non-convex since the Hessian of the objective function is not negative semidefinite. Our goal is to convert **P1** to a set of linear programs that can be solved for the optimal in an iterative manner as shown next.

Towards this objective, we go through a number of steps. First, the range of possible values of μ_p is defined as

$$h_{pd} \le \mu_p \le h_{pd} + \left(1-h_{pd}\right) h_{ps}. \tag{12}$$

This inequality can be readily verified from (2). The service rate of the PU packets, μ_p, depends on the packet admission probability, a. Since $0 \le a \le 1$, we can accordingly specify the lower and upper bounds on μ_p as shown in (12). Second, we fix μ_p and then run **P1** iteratively for every possible value of μ_p. Therefore, the only variable in the reformulated optimization problem is b, while μ_p, and consequently a, would be constant in each iteration where the optimization problem is solved. It is evident from (3) that μ_s is an affine function in b. On the other hand, it can be shown through (8) and (9) that D_p is a quasiconvex function in b since it is a convex over concave function.

As a result, the solution set of the constraint on the delay encountered by the PU packets is the ψ-sublevel set of the quasiconvex function D_p, which can be represented as the 0-sublevel set of the convex function ϕ_ψ that is given by

$$\begin{aligned} \phi_\psi &= N_p + N_{sp} - \lambda_p \psi, \\ &= \lambda_p\left(\mu_p - h_{pd}\right)\left(\overline{b} h_{sd} \overline{\mu_p} \lambda_p - \left(\mu_p - h_{pd}\right)\mu_p \lambda_p - h_{pd}\lambda_p + \mu_p^2\right) \\ &\quad - \mu_p\left(\mu_p - \lambda_p\right)\left(\overline{b} h_{sd}\left(\mu_p - \lambda_p\right) - \lambda_p\left(\mu_p - h_{pd}\right)\right)\left(\lambda_p \psi - N_p\right), \end{aligned} \tag{13}$$

where N_p is given by (8). Note that ϕ_ψ is an affine function of b. Thus, **P1** can be cast as the following optimization problem

for $\mu_p = h_{pd} : \delta : h_{pd} + \left(1-h_{pd}\right) h_{ps}$ **do**

$$\begin{aligned} \textbf{P2:} \quad g_2\left(\mu_p\right) = & \max_{b} \quad h_{sd}\left(1-\frac{\lambda_p}{\mu_p}\right) \\ & \text{s.t.} \quad 0 \le b \le 1, \\ & \qquad \phi_\psi \le 0, \\ & \qquad (13). \end{aligned} \tag{14}$$

end for

return $\max_{\mu_p} g_2\left(\mu_p\right)$,

where δ is a pre-specified increment value for μ_p.

Since the objective and constraint functions of **P2** are all affine, **P2** is a linear program for each iteration on μ_p [21]. A closed-form expression of the solution of **P2** is characterized by the following lemma.

*Lemma 1: For a given μ_p, the optimal solution of **P2**, $b^*\left(\mu_p\right)$, is given by* (15)*, as shown at the bottom of this page.*

Proof: It is evident that the objective function of **P2** monotonically increases with b, where $0 \le b \le 1$. However, $b^*\left(\mu_p\right)$ must satisfy the constraint $\phi_\psi \le 0$, which can be rewritten, via simple algebraic manipulations, as $b \le f(\mu_p, \psi)$, where $f(\mu_p, \psi)$ is the second term of the min expression in (15). Combining these two statements yields the result of the lemma. ■

We can see from (15) that the optimal value of b depends on the parameters of the problem. For example, the PU direct channel gain, h_{pd} can be high enough to satisfy the condition on the PU delay. In this case, the optimal value of the parameter b is given by 1 and no relaying is required from the SU. In contrast, when the direct channel of the PU cannot support the target delay ψ, the SU gives sufficient priority to the relaying queue that causes the PU delay constraint to be satisfied with equality.

The solution of **P1** is a low complexity line search over μ_p in the interval $\left[h_{pd},\ h_{pd} + \left(1-h_{pd}\right) h_{ps}\right]$. The number of search points depends on the step size δ. For each search point, a closed form expression for $b^*\left(\mu_p\right)$, is obtained from (15) and

$$b^*\left(\mu_p\right) = \min\left(1,\ 1 - \frac{\lambda_p^2\left(\mu_p - h_{pd}\right)\left(\frac{-h_{pd}}{\mu_p} - \left(\mu_p - h_{pd}\right)\right) + \lambda_p \mu_p\left(\mu_p - h_{pd}\right) - \left(\lambda_p \psi - N_p\right)\left(\mu_p - h_{pd}\right)\left(\lambda_p^2 - \lambda_p \mu_p\right)}{-h_{sd}\left(\frac{\lambda_p^2}{\mu_p}\left(\mu_p - h_{pd}\right)\left(1-\mu_p\right) - \left(\lambda_p \psi - N_p\right)\left(\lambda_p^2 - 2\lambda_p \mu_p + \mu_p^2\right)\right)}\right). \tag{15}$$

the objective function $g_2(\mu_p)$ of **P2** is evaluated. Afterwards, the algorithm searches for the maximum of $g_2(\mu_p)$ over all the considered values of μ_p. Finally, given the optimal μ_p, we can calculate a^* from (2). Therefore, the total number of function evaluations required for the algorithm is given by $1+2\left(1-h_{pd}\right)h_{ps}/\delta$.

B. Optimizing the Secondary User Delay

In this subsection, we shift our attention to opportunistic spectrum access in networks supporting real-time traffic, i.e., ORT traffic, which have received attention only recently [3]. Towards this objective, we investigate the problem of minimizing the average delay encountered by SU packets, D_s, subject to a constraint on the average PU packet delay, D_p.

We follow the same analysis presented in the previous subsection. Note that the minimization of D_s guarantees the stability of Q_s unless the problem is infeasible. Therefore, Q_s and Q_p stability conditions will be redundant and, hence, omitted from the problem formulation. This step reduces the complexity of the optimization problem. Consequently, the target constrained optimization problem is formulated as

$$\begin{aligned}\textbf{P3:}\quad & \min_{a,b} \quad \frac{N_s}{\lambda_s}\\ & \text{s.t.} \quad 0 \le a \le 1,\\ & \qquad 0 \le b \le 1,\\ & \qquad \mu_p = h_{pd} + \left(1-h_{pd}\right)h_{ps}a,\\ & \qquad \frac{N_p+N_{sp}}{\lambda_p} \le \psi,\\ & \qquad (8),\ (9),\ (10),\end{aligned} \tag{16}$$

where the objective function is the SU packet delay, D_s. **P3** is non-convex. However, we can exploit the structure of **P3** to convert it to a set of quasiconvex optimization problems that can be solved for the optimal in an iterative manner as shown next. Following the same approach applied in the previous subsection, **P3** can be solved iteratively as follows

for $\mu_p = h_{pd} : \delta : h_{pd} + \left(1-h_{pd}\right)h_{ps}$ **do**

$$\begin{aligned}\textbf{P4:}\quad g_4(\mu_p) = & \min_{b} \quad \frac{N_s}{\lambda_s}\\ & \text{s.t.} \quad 0 \le b \le 1,\\ & \qquad \phi_\psi \le 0,\\ & \qquad (10),\ (13).\end{aligned} \tag{17}$$

end for

return $\max_{\mu_p} g_4(\mu_p)$.

Once again, we can see that this optimization problem is a low complexity line search in the interval $\left[h_{pd},\ h_{pd}+\left(1-h_{pd}\right)h_{ps}\right]$. It can be shown through (10) that D_s is quasiconvex in b. Since the objective function of **P4** is quasiconvex and the constraints are convex, **P4** is a quasiconvex optimization problem for each iteration on μ_p [21], and its solution is characterized by the following lemma.

*Lemma 2: For a given μ_p, the optimal solution of **P4**, $b^*(\mu_p)$, is equal to the optimal solution of **P2**.*

Proof: In pursuance of solving **P4**, we delve into the relationship between the optimization problems **P2** and **P4**. In the former problem, it is obvious that we maximize an objective function that monotonically increases with b. However, in the latter problem, we minimize an objective function that monotonically decreases with b. This can be readily verified by evaluating the first derivative of D_s with respect to b. It can be shown that

$$\frac{\partial D_s}{\partial b} = \frac{-h_{sd}\mu_p\left(\lambda_p\lambda_s^2\left(1-\mu_p\right)\left(\mu_p-\lambda_p\right)+\left(\lambda_s-\lambda_s^2\right)\left(\mu_p-\lambda_p\right)^3\right)}{\lambda_s\left(\lambda_s\mu_p\left(\mu_p-\lambda_p\right)-h_{sd}\left(\mu_p-\lambda_p\right)^2 b\right)^2}, \tag{18}$$

which is negative definite irrespective of the choice of b and, hence, D_s monotonically decreases with b. Taking into consideration that both problems have the same constraints, it can be asserted that **P2** and **P4** are equivalent optimization problems; the feasible sets and the optimal solutions of both problems are identical. In other words, the problem of maximizing the SU throughput is equivalent to the problem of minimizing the SU packet delay for the adopted system model and cooperation policy. This completes the proof of the lemma. ■

V. Towards a Work-Conserving Cooperation Policy

In this section, the focal point of our discussion is investigating the problem of optimizing the average delay experienced by the SU subject to a constraint on the average delay experienced by the PU for a WC policy in the cooperative cognitive radio network depicted in Fig. 1. As stated in Section II, a WC policy outperforms a non-WC policy due to the efficient utilization of time slots, i.e., an empty queue is never selected for packet transmission as long as the other queue is non-empty. In this section, we aim to quantify this performance gain and construct a practically viable WC cooperation policy that approaches the optimal performance. This section is divided into three parts. First, the proposed WC cooperation policy is introduced. Next, the problem of optimizing the SU delay subject to a constraint on the PU delay for the WC policy is introduced and a suboptimal solution is provided. Nevertheless, we show that the resulting SU and PU delay from the suboptimal solution is superior to that of the non-WC policy. Finally, a practical WC-policy-based algorithm is proposed in order to closely approach the optimal solution of the target optimization problem.

A. Work-Conserving Cooperation Policy

When the PU is backlogged, the cooperation policy is the same as the one considered in Section II. On the other hand, when the PU is idle, the cooperation policy is altered to make it WC as follows

- The SU accesses the channel and a packet is transmitted either from Q_s with probability b or Q_{sp} with probability $1-b$.
- If the queue chosen above happens to be empty, the other queue is immediately selected for packet transmission.

Therefore, there is no possibility for a time slot to be wasted because an empty queue is never selected for packet transmission as long as the other queue is non-empty.
- If the destination decodes the packet successfully, it sends an ACK and the packet is dropped from the system. Otherwise, the packet is kept in its respective queue for future retransmission.

The proposed WC policy achieves better slot utilization, compared to the non-WC policy, since there is no chance of a slot going idle while there are packets in the system.

Let N_p^{wc}, N_{sp}^{wc} and N_s^{wc} denote the average queue length of Q_p, Q_{sp} and Q_s under the WC policy, respectively. For the proposed WC policy, the problem of minimizing the average delay experienced by the SU, D_s^{wc}, subject to a constraint on the average delay experienced by the PU, D_p^{wc}, is formulated as

$$\begin{aligned} \textbf{P5:} \quad \min_{a,b} \quad & \frac{N_s^{wc}}{\lambda_s} \\ \text{s.t.} \quad & 0 \le a \le 1 \\ & 0 \le b \le 1 \\ & \frac{N_p^{wc} + N_{sp}^{wc}}{\lambda_p} \le \psi, \end{aligned} \tag{19}$$

where

$$N_p^{wc} = N_p, \tag{20}$$

and N_p is the average queue length of Q_p under the non-WC policy and is given by (8). Note that (20) follows from the fact that the non-WC and WC policies follow the same rules when the PU is backlogged. The proposed WC policy creates interaction between two queues in the system, namely Q_s and Q_{sp}, which were independent in the non-WC system. This interaction, added to the dependence in the non-WC system of the service processes of Q_s and Q_{sp} on the length of Q_p yields interaction between three queues which is highly complex to analyze [16] and motivates our next discussion towards circumventing this major hurdle.

B. Suboptimal WC Cooperation Policy

Since the delay analysis of three interacting queues operating under the WC cooperation policy proposed in Subsection V-A is notoriously complex [16], it is difficult to get closed-form expressions for N_{sp}^{wc} and N_s^{wc}. In this subsection, we find a suboptimal solution to **P5**, and then prove that this solution outperforms the optimal solution of **P3** of the non-WC policy. First, let us consider a virtual queue Q_{ss} that is a result of merging Q_{sp} and Q_s together. Let N_{ss}^{wc} denote the average length of Q_{ss}, i.e., $N_{ss}^{wc} = N_{sp}^{wc} + N_s^{wc}$. Therefore, N_{ss}^{wc} is characterized as

$$N_{ss}^{wc} = \frac{\sigma\xi - \tau\upsilon}{\xi - \tau}, \tag{21}$$

where

$$\sigma = \frac{\lambda_p(1-\lambda_p)+\lambda_s(1-\lambda_s)-\lambda_p\lambda_s}{h_{pd}-\lambda_p-\lambda_s} + \frac{\lambda_p(1-\lambda_p)}{h_{pd}+ah_{ps}(1-h_{pd})-\lambda_p},$$

$$\begin{aligned} \tau &= \frac{h_{sd}-h_{pd}}{\lambda_p+\lambda_s-h_{pd}}, \\ \upsilon &= \frac{\lambda_s(\lambda_s-1)\left(h_{pd}+ah_{ps}(1-h_{pd})\right)-ah_{ps}(1-h_{pd})\lambda_p}{\left(\lambda_s+ah_{ps}(1-h_{pd})\right)\left(h_{pd}+ah_{ps}(1-h_{pd})\right)}, \\ \xi &= \frac{h_{sd}+ah_{ps}(1-h_{pd})}{\lambda_s+ah_{ps}(1-h_{pd})}, \end{aligned} \tag{22}$$

and the proof of (21) can be found in the Appendix.

Next, we construct the objective function and the PU delay constraint of the new optimization problem. In particular, we minimize the average queue length of Q_{ss}, namely N_{ss}^{wc}, rather than the average SU packet delay of the WC policy, D_s^{wc}. Moreover, the constraint imposed on the average PU packet delay of the WC policy, D_p^{wc}, is substituted with the average PU packet delay of the non-WC policy, D_p, given by (7). Thus, the optimization problem is recast as

$$\begin{aligned} \textbf{P6:} \quad \min_{a,b} \quad & N_{ss}^{wc} \\ \text{s.t.} \quad & 0 \le a \le 1, \\ & 0 \le b \le 1, \\ & \frac{N_p + N_{sp}}{\lambda_p} \le \psi, \\ & (8),\ (9),\ (21), \end{aligned} \tag{23}$$

where N_{sp} is the average queue length of Q_{sp} under the non-WC policy. Recall that N_{ss}^{wc} includes the average length of the SU packet queue in addition to the average length of the SU relaying queue. Moreover, D_p (PU delay of the non-WC policy) is an upper bound on D_p^{wc} (PU delay of the WC policy) because the non-WC policy is susceptible to potentially wasting idle time slots, as explained in Section II. Consequently, the SU delay resulting from solving **P6** is an upper bound on the optimal SU delay that can be achieved if we could get closed-form expressions for D_p^{wc} and D_s^{wc} and solve **P5**. In other words, the solution of **P6** is a suboptimal solution to **P5**. The following lemma characterizes the solution of **P6**.

*Lemma 3: The optimal values a^* and b^* of **P6** are equal to those of **P3**.*

Proof: We follow the same footsteps of the proof of Lemma 2. By taking the first derivative of N_{ss}^{wc} in (21) with respect to a (note that N_{ss}^{wc} is a function of a only), it turns out to be negative definite and, hence, N_{ss}^{wc} monotonically decreases with a (the verification of this statement is straightforward but tedious and is therefore omitted due to space limitations). Since both optimization problems have monotonically decreasing objective functions. The optimal solution lies on the boundary of the feasible set of each set. Furthermore, since both problems have the same feasible set, i.e., the same constraints, they are equivalent problems and, hence, **P3** and **P6** have identical optimal values a^* and b^*. ∎

Owing to the fact presented by Lemma 3, the SU and PU delay resulting from solving **P6** cannot be higher than the SU and PU delay resulting from solving **P3** for the same values of a and b. The reason is that the non-WC policy adopted in **P3** is susceptible to potential waste of idle time slots. In other words, Lemma 3 verifies that the proposed suboptimal WC cooperation policy yields better performance than the optimal

Algorithm 1 Finding the values of a and b that closely approach the optimal SU delay for the proposed WC policy

```
Solve the optimization problem P6 and get a_ub and b_ub.
Set a = a_ub and Set b = b_ub.
while b ≤ 1 do
  Measure D_p^wc over a sufficient number of time slots.
  if D_p^wc ≤ ψ then
    Set b = b + Δb.
    Continue to the next iteration.
  else
    Set b = b − Δb.
    Terminate the loop.
  end if
end while
```

non-WC cooperation policy proposed in the first part of the paper. The performance gain of the proposed suboptimal WC policy is illustrated through the numerical results presented in Section VI.

C. WC-Policy-Based Algorithm

The previous subsection provides an upper bound on the optimal SU packet delay of **P5** for the proposed WC cooperation policy. This is attributed to the fact that using the values of a and b resulting from solving **P6** causes the constraint on D_p^{wc} in **P5** to be satisfied with strict inequality. That is why there is some room to further minimize D_s^{wc} without violating the imposed constraint on D_p^{wc}. In this subsection, a WC-Policy-based algorithm is motivated to find the values of a and b so that we can closely approach the optimal SU delay, $D_s^{wc^*}$, for the proposed WC policy. The results of the previous subsection are exploited to introduce the proposed algorithm.

The algorithm outline is as follows, where Δb is a pre-specified increment value for b, and $[a_{ub}, b_{ub}]$ is the solution of **P6**.

It should be noted that, for a given value of a, increasing the value of b gives more priority to the SU packet queue, Q_s, over the SU relaying queue, Q_{sp}, when both queues are non-empty (if one of the queues is empty, the other one is immediately selected for packet transmission due to the WC property of the proposed policy). This leads, consequently, to a corresponding decrease in the SU delay. However, the incremental increase in the value of b is subject to a constraint that the resulting PU delay is not greater than ψ, the maximum delay that the PU can tolerate. The essence and role of the algorithm are clearly visualized through the simulation results demonstrated in Section VI. It should also be noted that without the solution of **P6**, the iteration on the value of b would have started from 0 since $0 \leq b \leq 1$. However, exploiting the solution of **P6** enables us to start the iteration from b_{ub}, a value at which we obtain a tighter upper bound on the optimal value of the SU delay, and, hence, the required number of iterations needed to approach the optimal SU delay, $D_s^{wc^*}$, decreases considerably. Thus, the proposed WC-policy-based algorithm is practical because it reduces to a low complexity line search in the interval $[b_{ub}, 1]$. Furthermore, the numerical results show that the SU delay resulting from the proposed WC-policy-based algorithm is lower than the suboptimal WC policy and the optimal non-WC policy without violating the constraint on the PU delay.

The proposed adaptive algorithm is inspired from rate adaptation algorithms where the modulation and coding index is adjusted based on the feedback from the receiver. In the proposed algorithm, the feedback message is derived from the delay of the primary user message measured at the destination node, i.e., D_p^{wc}. This delay can be measured from the application layer and then the destination node transmits a binary message to the SU to indicate whether it should increase or decrease the value of the parameter b as indicated in Algorithm 1.

VI. Numerical Results

A. Non-WC Cooperation Policy

In this subsection, we evaluate the performance of the proposed optimal non-WC cooperation policy for the cognitive radio network depicted in Fig. 1. We compare the proposed policy to a baseline cooperation policy (BL), coined "unconstrained partial cooperation policy" [13]. In BL, the SU probabilistically cooperates with the PU in delivering its packets in Q_{sp} with no constraint on the delay encountered by the PU packets. BL can be formulated using the same objective functions of **P1** and **P3**, yet, subject only to queues stability constraints. It should be emphasized that the BL yields better throughput and packet delay for the SU because it optimizes these performance metrics subject to the less stringent queue stability constraints. However, this, in turn, gives rise to poor PU performance in terms of arbitrarily large delays, as reported in [13], since there is no delay constraint on the PU packets. On the other hand, the proposed cooperation policy aims at optimizing the performance of the SU subject to a more stringent constraint, that is, the PU packet delay. Therefore, unlike [13], the proposed optimization problems balance the tradeoff between protecting the QoS of the PU and enhancing the QoS of the SU.

For the numerical results presented next, the following system parameters are used. The successful packet reception probabilities between the nodes of the network are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$. Note that we set $h_{sd} > h_{pd}$ since the cooperation of the SU in delivering the PU packets does not make sense when $h_{sd} \leq h_{pd}$ and it would be better to transmit the PU packets to the destination through the direct link in such a case. Also, we have selected $h_{ps} > h_{pd}$ to give the SU a better chance of decoding the PU packets than the primary destination and fill up its relaying queue. Furthermore, we solve **P1** and **P3** for different constraint values on the PU delay, $D_p \leq \psi$ where $\psi = 10$ and 20.

In the first part of this subsection, we investigate the maximum stable throughput, i.e., the boundary of the stable throughput region, for the proposed optimal non-WC cooperation policy. In Fig. 2, we plot the stable throughput region of the system for different constraint values on the average delay experienced by the PU packets, D_p. Under the BL scheme, the SU enjoys higher throughput since there is no constraint on D_p. However, the PU experiences huge packet delay,

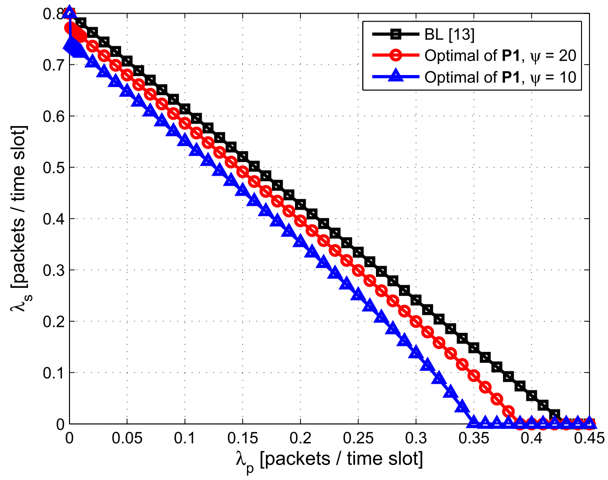

Fig. 2. The stable throughput region. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

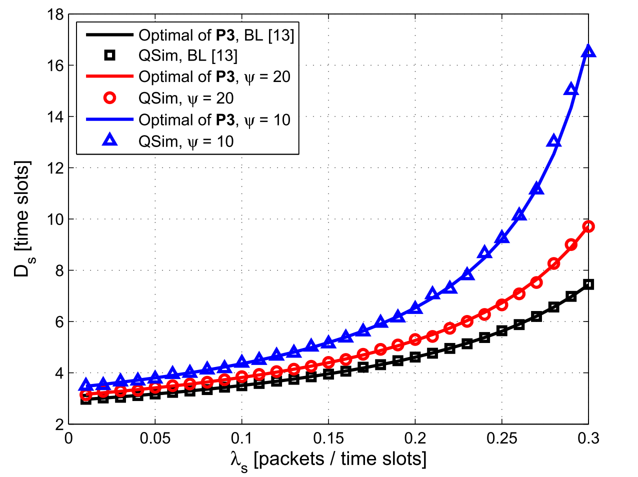

Fig. 3. The delay-throughput tradeoff at the SU, $\lambda_p = 0.2$. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

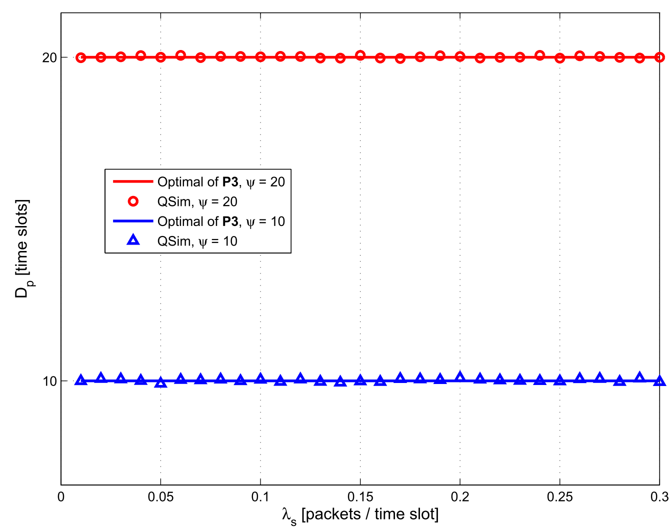

Fig. 4. The average PU packet delay versus the arrival rate of the SU packets, $\lambda_p = 0.2$. The corresponding PU delay of the BL takes arbitrarily large values [13] and, hence, it is not plotted. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

i.e., $D_p \to \infty$. Note that the violation of the QoS requirements of PUs (spectrum owners) is a serious problem in cognitive radio networks. On the contrary, in the proposed optimal cooperation policy, when the PU delay constraint is introduced in **P1**, e.g, $D_p \leq 20$, the average packet delay experienced by the PU is guaranteed not to exceed $\psi = 20$. Moreover, the system does not lose much in terms of the stable throughput region. Hence, the proposed problem formulation optimizes the SU throughput, but, at the same time, maintains a certain level of QoS for the PU. However, protecting the PU QoS comes at the expense of a decrease in the SU stable throughput compared to the BL. Furthermore, when the constraint on D_p in **P1** becomes tighter, i.e., ψ decreases, the stable throughput region shrinks in order to satisfy this constraint.

Next, we shift our attention to the delay performance of the PUs and SUs for the proposed optimal non-WC cooperation policy. We set $\lambda_p = 0.2$ in Figs. 3 and 4,, and $\lambda_s = 0.2$ in Fig. 5. The average delays are computed via the optimal solution of **P3** and the queue simulation (QSim). The packet delays are averaged over 10^5 time slots in the QSim. In each time slot, when simulating a packet transmission, a bernoulli random variable is generated with the corresponding successful packet reception probability. The packet reception is considered successful if the random variable is equal to 1 and unsuccessful otherwise. The results of the optimal solution of **P3** coincide with those obtained from the QSim as shown in Figs. 3 to 5.

Fig. 3 depicts the delay-throughput tradeoff at the SU for different constraint values on the average delay experienced by the PU packets, D_p. It is obvious that D_s monotonically increases with λ_s for all cooperation policies. Furthermore, we can see that the SU delay of the BL is lower than the SU delay introduced by the proposed optimal cooperation policy. However, the corresponding PU packet delay of the BL takes arbitrarily large values, i.e., $D_p \to \infty$. Unlike BL, the proposed optimal cooperation policy minimizes the SU delay and guarantees that the PU packet delay remains bounded, i.e., less than or equal to ψ, as shown in Fig. 4. In other words, assuring a certain level of QoS for the PU while enhancing the SU QoS comes at the expense of an increase in the SU delay compared to the BL. We can also see from Fig. 3 that when the constraint on D_p becomes tighter, i.e., ψ decreases, the resulting value of the objective function of **P3**, D_s, increases. The reason for this behavior lies behind the strict constraint on the PU delay that forces the system to choose Q_{sp} more often and, hence, a lower probability of choosing Q_s is obtained from the solution of **P3**, i.e., lower b, in order to satisfy the constraint. Therefore, D_s increases.

Fig. 4 shows the average delay of the PU packets, D_p, versus the arrival rate of the SU packets, λ_s, for different constraint values on D_p. It should be noted that the PU delay of the BL takes arbitrarily large values, i.e., $D_p \to \infty$, since there is no delay constraint on the PU packets in the BL. In the proposed optimal cooperation policy, on the other hand, it is evident that the PU delay constraint is always satisfied with equality, i.e., $D_p = \psi$. In other words, the constraint on D_p

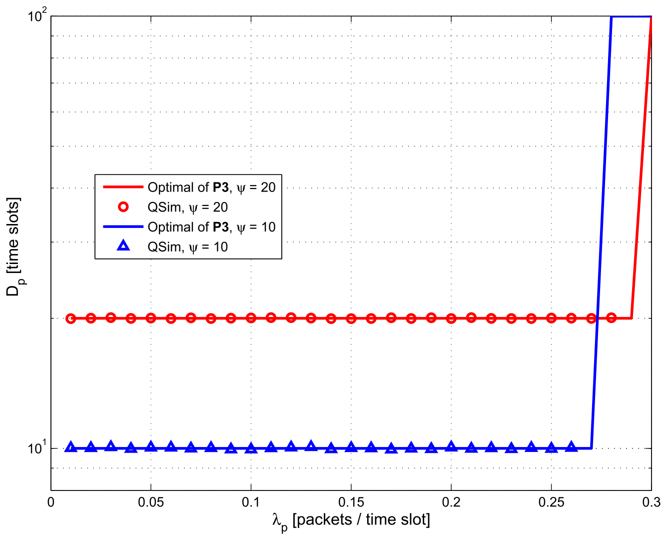

Fig. 5. The delay-throughput tradeoff at the PU, $\lambda_s = 0.2$. The corresponding PU delay of the BL takes arbitrarily large values [13] and, hence, it is not plotted. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

is satisfied at the boundary of the feasible set of **P3** in order to reach the minimum value of the objective function, i.e., the minimum SU delay.

Fig. 5 captures the delay-throughput tradeoff at the PU for different constraint values on the average delay encountered by the PU packets, D_p. We can see that there is a maximum value for λ_p after which **P3** becomes infeasible, e.g., the sudden jump of D_p at $\lambda_p = 0.29$ when $\psi = 20$, and at $\lambda_p = 0.27$ when $\psi = 10$. In other words, there are no values for a and b that stabilize the queues of the system under the PU delay constraint. Furthermore, when the constraint on D_p becomes tighter, the value of λ_p at which the system reaches the unstable state becomes smaller. Note that these values of λ_p for the different constraint values on D_p are consistent with the stable throughput region plotted in Fig. 2 when $\lambda_s = 0.2$.

B. Suboptimal WC Policy & WC-Policy-Based Algorithm

In this subsection, we assess the performance of the proposed suboptimal WC cooperation policy and WC-policy-based algorithm for the network depicted in Fig. 1. The channel success probabilities are $h_{pd} = 0.3$, $h_{ps} = 0.4$, $h_{sd} = 0.8$. The constraint on the PU delay is $D_p \leq \psi$ where $\psi = 10$. Moreover, we set $\lambda_p = 0.2$. In the queue simulations, the packet delays are averaged over 10^5 time slots.

Fig. 6 and Fig. 7 show, respectively, the average delay of the SU and the PU packets, D_s and D_p, versus the arrival rate of SU packets, λ_s, for three different cooperation polices, namely optimal non-WC policy (**P3**), suboptimal WC policy (**P6**), and WC-policy-based algorithm. It is obvious that the suboptimal WC policy introduces lower average SU and PU delay compared to the non-WC policy. This is due to the efficient utilization of time slots, i.e., an empty queue is never selected for packet transmission as long as the other queue is non-empty. We can see from Fig. 6 that the WC-policy-based algorithm yields higher PU delay than the suboptimal

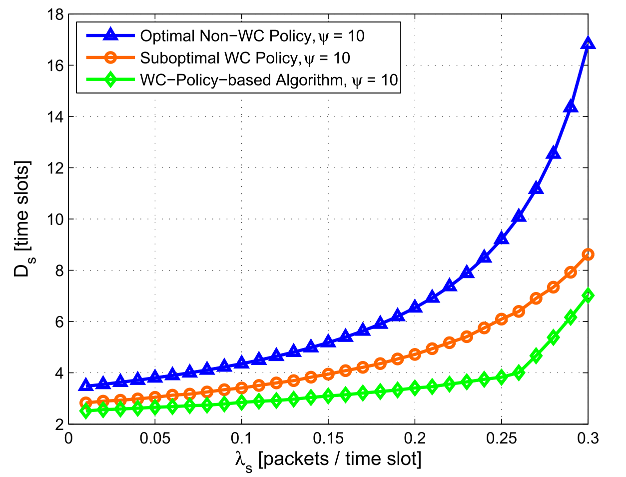

Fig. 6. The average delay of the SU packets versus the arrival rate of the SU packets under different cooperation policies, $\lambda_p = 0.2$ and $\psi = 10$. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

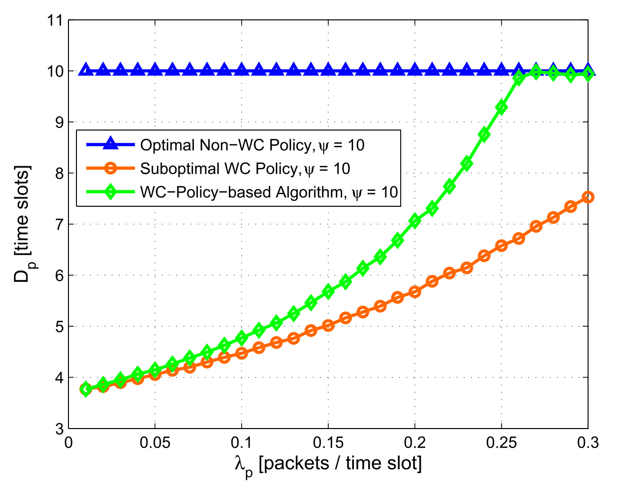

Fig. 7. The average delay of the PU packets versus the arrival rate of the SU packets under different cooperation policies, $\lambda_p = 0.2$ and $\psi = 10$. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

WC policy. Nevertheless, the PU delay achieved by the WC-policy-based algorithm does violate the PU delay constraint. We can also see from Fig. 7 that the WC-policy-based algorithm yields lower SU delay than the suboptimal WC policy. The WC-policy iteratively trades the decrease in the SU delay for an increase in the PU delay until the constraint on the PU delay is violated. According to our formulation, the measure of optimality is achieving the minimum SU delay for a given upper bound on the PU delay. Hence, the WC policy is superior to the suboptimal WC policy.

Finally, Fig. 8 illustrates how the proposed WC-policy-based algorithm works at $\lambda_s = 0.3$. The first iteration of the algorithm corresponds to the solution given by the suboptimal WC policy obtained from solving **P6**, i.e. a = 1 and b = 0:68. The algorithm incrementally increase b, and then measures the resulting average delay experienced by PU packets over a sufficient number of time slots. In each iteration, the PU delay increases while the SU delay decreases. The iterations continue until the constraint on the PU delay is violated. In our

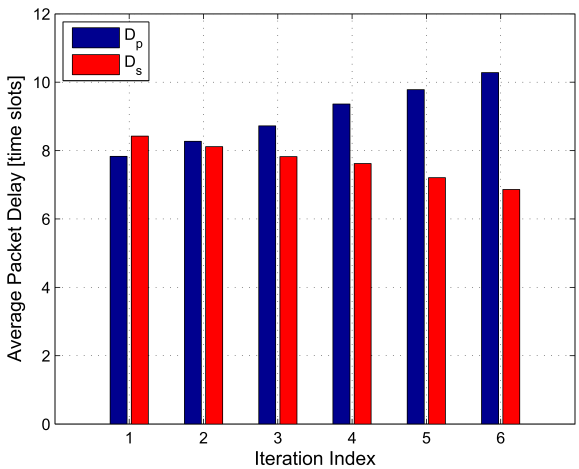

Fig. 8. The average packet delay of the PU and SU packets, stemmed from the proposed WC-policy-based algorithm, over different iterations, $\lambda_p = 0.2$, $\lambda_s = 0.3$ and $\psi = 10$. The parameters of the wireless channel are $h_{pd} = 0.3$, $h_{ps} = 0.4$, and $h_{sd} = 0.8$.

simulation this happened at the sixth iteration. The algorithm then terminates and returns the solution that yielded the lowest value for the SU delay without violating the constraint on the PU delay, i.e., the solution corresponding to iteration 5.

VII. Conclusion

In this paper, we have studied cooperative cognitive radio networks with the objective of optimizing the QoS of a SU while sustaining a target QoS for the PU. In the first part of the paper, we have focused on a non-WC system due to its mathematical tractability. We have identified the optimum cooperation policy for such system that maximizes the SU traffic–and equivalently minimizes its delay–subject to a constraint on the maximum packet delay that the PU can tolerate. We have demonstrated through numerical simulations that the stable throughput region of the proposed cooperation policy approaches that of the unconstrained partial cooperation policy. Furthermore, the average PU packet delay of the proposed policy is much lower than the one of the unconstrained partial cooperation policy. However, this comes at the expense of an increase in the average SU packet delay.

In the second part of the paper, we have studied a WC cooperation policy and investigated the problem of optimizing the SU packet delay subject to a constraint on the PU packet delay. Due to the sheer complexity of deriving closed-form expressions for the average delay of the PU and SU packets for a WC policy, we have proposed a suboptimal WC policy that outperforms the non-WC policy proposed in the first part of the paper. Furthermore, we have proposed a novel theoretically-founded WC-policy-based algorithm in pursuance of approaching the optimal SU delay of the target optimization problem.

Appendix

The proof hinges on the approach of the moment generating function of the joint length of two dependent queues [16]. In order to deal only with two interacting queues, we merge Q_{sp} and Q_s at the SU into one virtual queue denoted by Q_{ss}. The moment generating function of the joint queue lengths of Q_p and Q_{ss} is expressed as

$$\begin{aligned} G(w,z) &= \lim_{t\to\infty} \mathbf{E}\left(w^{Q_p^t} z^{Q_{ss}^t}\right) \\ &= \lim_{t\to\infty} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} w^i z^j\, \mathbf{P}\left(Q_p^t = i,\, Q_{ss}^t = j\right), \end{aligned} \tag{24}$$

where $\mathbf{E}(.)$ is the expectation operator and $\mathbf{P}(.)$ is the probability operator. From (24), it can be shown that N_{ss}^{wc}, the average queue length of Q_{ss}, is defined as

$$N_{ss}^{wc} = \frac{\partial G(w,z)}{\partial z}\bigg|_{w=z=1} = \lim_{t\to\infty} \sum_{j=1}^{\infty} j\mathbf{P}\left(Q_{ss}^t = j\right). \tag{25}$$

Furthermore, the following identities can be readily verified from the definition of the moment generating function in (24). These identities are vital to complete the proof.

$$\begin{aligned} G(w,z)\Big|_{w=z=1} &= 1, \\ G(w,z)\Big|_{w=0,\, z=1} &= \lim_{t\to\infty} \mathbf{P}\left(Q_p^t = i\right) \\ &= 1 - \frac{\lambda_p}{h_{pd} + a h_{ps}\left(1-h_{pd}\right)}, \\ \frac{\partial G(z,z)}{\partial z}\bigg|_{z=1} &= \lim_{t\to\infty}\left(\sum_{i=1}^{\infty} i\mathbf{P}\left(Q_p^t = i\right) + \sum_{j=1}^{\infty} j\mathbf{P}\left(Q_{ss}^t = j\right)\right) \\ &= N_p^{wc} + N_{ss}^{wc}, \end{aligned} \tag{26}$$

where $N_p^{wc} = N_p$ is given by (8) and is obtained by direct application of the Pollaczek-Khinchine formula [20] on Q_p that is a discrete-time M/M/1 queue with Bernoulli arrival rate λ_p and geometrically distributed service rate $\mu_p = h_{pd} + a h_{ps}\left(1 - h_{pd}\right)$. Taking into consideration the queue length evolution characterized in (1), we have

$$\begin{aligned} &\mathbf{E}\left(w^{Q_p^{t+1}} z^{Q_{ss}^{t+1}}\right) \\ &= \mathbf{E}\left(w^{Q_p^t - Y_p^t + X_p^t} z^{Q_{ss}^t - Y_{ss}^t + X_{ss}^t}\right) \\ &= \left(1 - \lambda_p + \lambda_p w\right)\left(1 - \lambda_s + \lambda_s z\right) \mathbf{E}\left(w^{Q_p^t - Y_p^t} z^{Q_{ss}^t - Y_{ss}^t}\right), \end{aligned} \tag{27}$$

where the binary random variables X_j^t and Y_j^t represent the number of packets that arrive at (or depart from) the j^{th} queue at time slot t, respectively. Note that the second line in (27) follows from the fact that the packet arrival processes at Q_p and Q_s are independent from each other, independent from all other events, and are modelled as Bernoulli random processes with rates λ_p and λ_s, respectively. Now, we study the distinct potential evolutions of the queue states of Q_p and Q_{ss} as follows

- $Q_p^t = 0$, $Q_{ss}^t = 0$
  Both queues are empty and, hence, no packet departs from either queue, i.e., $Y_p^t = 0$ and $Y_{ss}^t = 0$.
- $Q_p^t > 0$, $Q_{ss}^t = 0$
  Since Q_{ss} is empty, no packet departure occurs, i.e., $Y_{ss}^t = 0$. On the other hand, a packet departs from Q_p if the destination successfully decodes the packet received

over the direct link, or the SU successfully decodes the packet, yet the destination does not decode it, and it is admitted into Q_{sp}. Otherwise, the packet is kept in Q_p for future retransmission. Thus, Y_p^t is given by

$$Y_p^t = \begin{cases} 1 & \text{with probability } h_{pd} + ah_{ps}\left(1-h_{pd}\right) \\ 0 & \text{with probability } \left(1-h_{pd}\right)\left(1-ah_{ps}\right). \end{cases} \tag{28}$$

- $Q_p^t = 0,\ Q_{ss}^t > 0$
  Since Q_p is empty, no packet departure occurs, i.e., $Y_p^t = 0$. On the other hand, a packet departs from the virtual queue Q_{ss}, i.e., a packet departs either from Q_s or Q_{sp}, if the destination successfully decodes the packet. Otherwise, the packet is kept in its respective queue for future retransmission. Thus, Y_{ss}^t is given by

$$Y_{ss}^t = \begin{cases} 1 & \text{with probability } h_{sd} \\ 0 & \text{with probability } 1-h_{sd}. \end{cases} \tag{29}$$

  It should be carefully noted that the effect of b (probability of selecting Q_s to transmit its packets to the destination) does not appear in Y_{ss}^t equation. This is by virtue of the WC property introduced in the proposed cooperation policy. If the selected queue happens to be empty, the other queue is immediately selected for transmission. In other words, there is always packet departure as long as the virtual queue Q_{ss} is not empty.
- $Q_p^t > 0,\ Q_{ss}^t > 0$
  Since the PU has a higher priority over the SU to transmit whenever it has packets in its queue, this case boils down to the case of $Q_p^t > 0,\ Q_{ss}^t = 0$.

In the light of the aforementioned possibilities, (27) can be rewritten as

$$\begin{aligned}
&\mathbf{E}\left(w^{Q_p^{t+1}} z^{Q_{ss}^{t+1}}\right) \\
&= \left(1-\lambda_p+\lambda_p w\right)\left(1-\lambda_s+\lambda_s z\right)\Bigg(\mathbf{E}\left(\mathbf{1}\left(Q_p^t = 0,\ Q_{ss}^t = 0\right)\right) \\
&\quad + \left(\frac{h_{pd}+ah_{ps}\left(1-h_{pd}\right)}{w} + \left(1-h_{pd}\right)\left(1-ah_{ps}\right)\right) \\
&\quad \times \mathbf{E}\left(w^{Q_p^t} \cdot \mathbf{1}\left(Q_p^t > 0,\ Q_{ss}^t = 0\right)\right) \\
&\quad + \left(\frac{h_{sd}}{z} + (1-h_{sd})\right)\mathbf{E}\left(z^{Q_{ss}^t} \cdot \mathbf{1}\left(Q_p^t = 0,\ Q_{ss}^t > 0\right)\right) \\
&\quad + \left(\frac{h_{pd}+ah_{ps}\left(1-h_{pd}\right)}{w} + \left(1-h_{pd}\right)\left(1-ah_{ps}\right)\right) \\
&\quad \times \mathbf{E}\left(w^{Q_p^t} z^{Q_{ss}^t} \cdot \mathbf{1}\left(Q_p^t > 0,\ Q_{ss}^t > 0\right)\right)\Bigg),
\end{aligned} \tag{30}$$

where $\mathbf{1}\left(k \in S\right)$ is an indicator function that indicates whether a random variable k belongs to a set S. It is defined as

$$\mathbf{1}\left(k \in S\right) = \begin{cases} 1 & \text{with probability } \mathbf{P}\left(k \in S\right) \\ 0 & \text{with probability } \mathbf{P}\left(k \notin S\right). \end{cases} \tag{31}$$

When taking the limit of (30) as $t \to \infty$, we get

$$\begin{aligned}
G(w,z) = &\left(1-\lambda_p+\lambda_p w\right)\left(1-\lambda_s+\lambda_s z\right) \\
&\times \frac{f_1(w,z)\,G(0,0) + f_2(w,z)\,G(0,z)}{z\, f_3(w,z)},
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
f_1(w,z) &= h_{sd} w\left(z-1\right), \\
f_2(w,z) &= h_{sd} w - z\left(h_{pd} + ah_{ps}\left(1-h_{pd}\right) z\right) \\
&\quad + wz\left(h_{pd} + ah_{ps}\left(1-h_{pd}\right) - h_{sd}\right), \\
f_3(w,z) &= w - \left(1-\lambda_p+\lambda_p w\right)\left(1-\lambda_s+\lambda_s z\right) \\
&\quad \times \left(h_{pd}+ah_{ps}\left(1-h_{pd}\right) z + \left(1-ah_{ps}\right)\left(1-h_{pd}\right) w\right).
\end{aligned} \tag{33}$$

Lastly, following the same algebraic manipulations as in [16], with the aid of the identities presented in (25) and (26), gives the average queue length of Q_{ss} as shown in (21).

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