A Feedback- Soft Sensing-Based Cognitive Access Scheme with Feedback Erasures

Ahmed M. Arafa[§*], Karim G. Seddik[‡], Ahmed K. Sultan[†], Tamer ElBatt[*] and Amr A. El-Sherif[†]

[§]Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA.
[*]Wireless Intelligent Networks Center (WINC), Nile University, Smart Village, Egypt.
[‡]Electronics Engineering Department, American University in Cairo, AUC Avenue, New Cairo 11835, Egypt.
[†]Department of Electrical Engineering, Alexandria University, Alexandria 21544, Egypt.
Email: arafa@umd.edu, kseddik@aucegypt.edu, salatino@alumni.stanford.edu, telbatt@ieee.org, amr.elsherif@ieee.org

***Abstract*—In this paper, we examine a cognitive spectrum access scheme in which a secondary user exploits the primary feedback information. We consider an overlay model in which the secondary user accesses the channel by certain access probabilities that are function of the spectrum sensing metric. In setting our problem, we assume that the secondary user can receive the primary link's feedback automatic repeat request (ARQ), but through an erasure channel. This means that the primary feedback may either be received correctly or is erased with a certain erasure probability. We study the cognitive radio network from a queuing theory point of view. Access probabilities are determined by solving a secondary throughput maximization problem subject to a constraint on the primary queues' stability. Fortunately, our problem is convex and can be solved using standard optimization techniques. Our scheme yields improved results in the secondary throughput than the non-feedback based access scheme attributed to the efficient utilization of the primary user's *unerased* feedback messages.**

I. Introduction

Cognitive radio technology is a communication paradigm that emerged in order to solve the spectrum scarcity problem by allowing unlicensed (or secondary) users to exploit the under-utilized spectrum of the licensed (or primary) users. Coexistence of such secondary users (SU) along with primary users (PU) is allowed provided that minimal or no harm is caused upon the primary network, and that a minimum quality of service is guaranteed for primary users. In a typical cognitive radio setting, the cognitive transmitter senses the primary activity and decides on accessing the channel based on the sensing outcome. This setting is problematic in the sense that cognitive users are not aware of their impact on the primary network, besides the usual sensing errors.

Toward alleviating such issues, *soft* sensing [1] was introduced as a way to enhance the sensing reliability. The idea is to use the value of the test statistic as a confidence measure for the sensing outcome. This value is then used to specify a channel access probability for the secondary network. Access probabilities as a function of the sensing metric are obtained by solving an optimization problem formulated to maximize the secondary throughput given a constraint on the primary queue stability. Soft sensing was first introduced in [2]. However, the focus was on physical layer power adaptation to maximize the capacity of the secondary link.

On the other hand, other works have focused on enhancing the SU's awareness of the primary environment through overhearing the primary ARQ feedback, and adjusting either its transmit power or access probabilities accordingly. For instance, in [3], the SU observes the automatic repeat request (ARQ) from the primary receiver. The ARQs reflect the PU's achieved packet rate. The cognitive radio's objective is to maximize the secondary throughput under the constraint of guaranteeing a certain packet rate for the PU. In [4], the authors use a partially observable Markov decision process (POMDP) to devise an optimized admission control policy. Secondary power control on the basis of the primary link feedback is investigated in [5].

The ARQ mechanism introduces redundancy in the system, in the form of copies of the same message transmitted in subsequent time slots. The idea of exploiting this redundancy is investigated in [6] where several protocols are proposed, in which the secondary transmitter collects side-information about the primary message in the first primary transmission, which is exploited to relay the primary message, if a retransmission occurs. While in [7], the authors introduce a mechanism where the secondary receiver can perform interference cancellation during the whole primary ARQ window by decoding the primary message, thus enhancing its own outage performance. In particular, they investigate a Backward Interference Cancellation mechanism in which the secondary receiver buffers the secondary transmissions that underwent outage due to primary interference, and attempts to recover them once the knowledge about the primary message becomes available due to decoding operation in a future instant.

An SU that can *perfectly* overhear, and make use of, the primary feedback messages is expected to have better throughput and cause lower primary queuing delay relative to the case where the primary feedback is not leveraged. This is attributed to the fact that by overhearing the primary feedback, the SU acquires knowledge about the PU's potential activity in the future which, in turn, aids in further optimizing its access decisions to the wireless channel in an attempt to avoid interference with the PU. In a recent work [8], we have investigated this case and the results were as expected. In this paper, however, we contend that an SU overhearing *imperfect* primary feedback (i.e. feedback messages that are exposed to erasures), can still have better throughput than an SU that simply ignores the primary feedback, provided that an appropriate model is used to deal with the erased feedback messages.

We consider a time-slotted system in which an SU is attempting to access the primary channel whenever sensed idle. By the end of each time slot, a feedback (ACK/NACK) is sent from the primary receiver to the primary transmitter to acknowledge the reception of packets. We assume that the SU can overhear this feedback in order to increase its degree of awareness of the primary network, but cannot overhear it as perfectly as the PU can. We adopt an erasure model just as in [9] where the concept of overhearing, and responding to, unreliable feedback is introduced. In particular, the primary feedback channel, as viewed by the SU, is modeled as an erasure channel, in which either the ACK/NACK messages are decoded reliably, or are declared in error (i.e. erased[1]). Our proposed scheme dealing with this issue is shown to yield improved results regarding the secondary throughput than a scheme that does not leverage the primary feedback at all.

II. System Model

We consider the uplink of a TDMA primary network consisting of M_p primary users, along which we have one SU trying to acquire access to the channel. let $\mathcal{M}_p = \{1, 2, ..., M_p\}$ denote the set of all primary users. Each PU has its dedicated time slot to send its data, while the SU's transmission strategy is overlay, in which it employs random access to leverage the sensed-idle time slots. We adopt a collision model in which whenever more than one transmission proceeds at a time, all packets involved are lost.

At the beginning of each time slot, the SU senses the channel using energy detection and, if found empty, it attempts accessing it with a certain access probability. Primary users access the channel in their dedicated time slots whenever they have packets to send. By the end of each time slot, a feedback is sent from the primary receiver to the primary transmitter to acknowledge the reception of packets. The SU is assumed to overhear this feedback to monitor its effect on the primary network. However, the SU cannot decode the primary feedback messages as perfectly as the primary users can. From the SU's view point, the primary feedback channel is modeled as an erasure channel, in which either the feedback messages are decoded reliably, or are declared in error (i.e. erased). In this work, we consider feedback imperfection only on the primary receiver-secondary user link. Imperfection on both feedback links will be investigated in an extended version.

The channel is modeled as a Rayleigh flat fading channel with additive white Gaussian noise (AWGN). The received signal at node j from node q at time slot t is given by

$$y_{qj}^t = \sqrt{G_q r_{qj}^{-\gamma}} h_{qj}^t x_q^t + n_j^t \tag{1}$$

[1]The erasure feedback channel can arise if the SU sets a SNR threshold for the feedback channel below which an erasure is declared and no decoding of the feedback bit is attempted; above that threshold, the feedback bit is always assumed to be correctly decoded.

where G_q is the transmitted power, r_{qj} is the distance between the two nodes, and γ is the path loss exponent. x_q^t is the transmitted signal, which is assumed to be drawn from any constant modulus constellation, M-ary PSK for instance, with zero mean and unit variance. h_{qj}^t is the channel coefficient between the two nodes, modeled as i.i.d. circularly symmetric complex Gaussian random variable with zero mean and unit variance. The noise term n_j^t is also modeled as i.i.d. circularly symmetric complex Gaussian random variable with zero mean and variance N_0. We assume the channel is stationary and independent from slot to slot.

For a transmission to be successful, not only there should be no collisions between packets, but the channel must not be in outage as well, i.e. the received SNR should not be smaller than a pre-specified threshold ζ. From the signal model in (1), the outage probability between nodes q and j is given by $P_{qj}^o = Pr\left\{|h_{qj}|^2 < \frac{\zeta N_0 r_{qj}^\gamma}{G_q}\right\} = 1 - \exp\left(-\frac{\zeta N_0 r_{qj}^\gamma}{G_q}\right)$.

Each PU has an infinite buffer for storing its incoming equal-length packets. The packet arrival processes at the primary queues are assumed to be Bernoulli i.i.d. with an average arrival rate of λ_q for user q. A slot duration is equal to the packet transmission time, and therefore, $0 \leq \lambda_q \leq 1$, $\forall q$. Arrival of packets at the primary queues can occur at any time during the time slot. However, a PU can only attempt transmitting its newly arrived packet in the next time slot.

In the sequel, we assume symmetry conditions, for simplicity of analysis and presentation, in which all primary users' transmit powers are equal and all distances between the SU and the primary users are equal. Therefore the subscript qj is dropped in the rest of the paper. Also, we assume that all λ_q's are the same for all primary users, and are equal to λ_p. Furthermore, in our model we consider secondary throughput analysis; the case in which the SU always has packets to send.

III. Background: Soft Sensing-Based Access

Instead of relying on hard decisions resulting from a binary hypothesis testing, and in order to enhance the decision authenticity, we focus on the concept of soft sensing introduced in [1]. Soft sensing is basically using the energy statistic $\|y_{ps}\|^2$ acquired from the energy detector as a measure of reliability, where p stands for PU and s stands for SU. The lower the value of $\|y_{ps}\|^2$ compared to the decision threshold η, the more certain the SU becomes that the PU is idle in the time slot in question. If the interval $[0, \eta]$ is divided into n subintervals as shown in Fig. 1, then for each subinterval $i \in [1, n]$, an access probability a_i is assigned. As a result:

- If $\|y_{ps}\|^2$ lies in the i^{th} subinterval, the SU attempts accessing the channel with probability a_i.
- While if $\|y_{ps}\|^2$ value is greater than η, the SU does not access the channel.

Under the previously mentioned system model assumptions, for the SU to successfully send its packets, the following events have to all take place simultaneously: it has to correctly identify the channel as idle, i.e. no false alarm occurs; it must gain access to the channel; its own link must not be in outage;

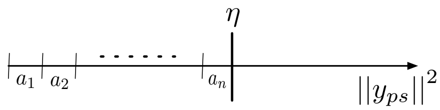


Fig. 1: Soft Sensing: Division of the interval $[0, \eta]$ into subintervals and their associated access probabilities.

and the PU's queue has to be empty of packets. Thus, the service process of the SU can be characterized as

$$Y_s^t = \mathbf{1}\left(\bigcup_{q \in \mathcal{M}_p}\left[A_q^t \bigcap \{Q_q^t = 0\} \bigcap \overline{O_{sd}^t} \bigcap \overline{\mathcal{A}} \bigcap P_s\right]\right) \quad (2)$$

where $\mathbf{1}(\cdot)$ denotes the indicator function ($\mathbf{1}(A) = 1$ if event A occurs, and 0 otherwise). A_q^t denotes the event that time slot t is assigned to the q^{th} PU, $\overline{O_{sd}^t}$ denotes the event that the link between the SU and its destination is not in outage, $\mathcal{A}$ is the event of false alarm, P_s is the event that the SU gains access to the channel, and $\{Q_q^t = 0\}$ denotes the event that the q^{th} PU queue is empty.

The joint event of no false alarm and gaining channel access when the PU is not present can be expressed, according to our signal model, as $Pr\left\{\overline{\mathcal{A}} \bigcap P_s\right\} = p_s^0 = \sum_{i \in [1,n]} p_i^0 a_i$, where $p_i^0 = \exp\left(-\frac{(i-1)\eta}{2n\sigma_0^2}\right) - \exp\left(-\frac{i\eta}{2n\sigma_0^2}\right)$, and σ_0^2 is the variance of the detector when no PU is present (please refer to [1] for detailed proofs of these formula).

IV. Imperfect Feedback-Based Access Scheme Analysis

A. Imperfect Feedback

Assuming a binary primary feedback system (ACK/NACK), the SU overhears this feedback through a binary erasure channel with erasure probability α (BEC(α)). Hence, either the SU overhears a clear ACK/NACK, or declares an erasure status. The rare case in which a NACK gets converted into an ACK or vice versa is considered of negligible probability. The SU reacts in response to these cases as follows:

- If an ACK/no feedback is overheard: SU uses soft sensing and attempts accessing the channel in the next time slot.
- If a NACK is overheard: SU backs-off; it refrains from accessing the channel in the next time slot, allowing for an interference-free retransmission of the lost PU packet.
- In the case of declaring an erasure status, which occurs with probability α, the SU *ignores* the erased feedback message and acts normally (i.e. uses soft sensing and attempts accessing the channel in the next time slot).

It can be inferred from the above model that the SU only responds to *reliable* overheard feedback. This scheme should provide the SU with more throuhput than a scheme that does not leverage the feedback. The rationale is as follows: if a NACK is overheard, this means that the PU will resend its lost packet in the next time slot with probability one. But this is when the SU backs-off. Therefore, sure collisions between primary and secondary packets will be avoided, and the PU will serve its packets more often, which in turn allows more idle time slots for the SU to exploit.

B. Markov Chain Model

The Markov chain modeling the primary queue evolution is shown in Fig. 2. As shown, there are two major classes of states where the PU can be: state class k_{ON}, denoting the PU's acquisition of k packets, and at the same time, the SU's ability to access the channel in the next time slot (SU ON mode); and state class k_{OFF}, denoting the PU's acquisition of k packets, but this time the SU is prohibited from accessing the channel in the next time slot due to overhearing a NACK in the previous time slot (SU OFF mode). Let π_k and ϵ_k represent the probability that the PU is in state k_{ON} and k_{OFF}, respectively. Arising from the fact that a PU would have zero packets if, and only if, it was successful in delivering its packet during the previous time slot, and therefore provoking an ACK feedback, it then follows that the SU cannot be in OFF mode while the PU has zero packets in its queue, hence, $\epsilon_0 = 0$.

As shown in Fig. 2, a downward transition from state $(k+1)_{ON}$ at time slot t, to state k_{ON} at time slot $t+1$ occurs if: the PU does not receive any packets during time slot t, which has a probability $1 - \lambda_p$; and at the same time succeeds in transmission, which has a probability $\Gamma_p = \frac{1}{M_p}(1 - P_{pd}^o)\left(1 - \sum_{i \in [1,n]} p_i^1 a_i\right)$, i.e. the PU got allocated to the time slot, its forward channel was not in outage (P_{pd}^o is the outage probability between a PU and its destination), and the SU did not miss detect the presence of the PU and gain access to the channel simultaneously. These two events are independent, and hence, their joint probability boils down to their product. It is worth noting here that Γ_p has the same value as the PU service rate μ_p in the baseline no feedback scheme introduced in [1], since they both denote the successful primary transmission probability in the same surrounding conditions. The probability that a PU stays in k_{ON}, for $k \geq 1$, is equal to $\lambda_p \Gamma_p + (1 - \lambda_p)(1 - \Gamma_p)\alpha$, which means that either the PU received an extra packet during time slot t and succeeded in transmission, or that it did not receive any extra packets during time slot t, failed in transmission, and more over, the NACK sent as a feedback was overheard as an erased packet, and therefore ignored, by the SU.

On the other hand, an upward transition from state k_{OFF} at time slot t, to state $(k+1)_{OFF}$ at time slot $t+1$ occurs if: the PU receives a packet during time slot t, which has a probability λ_p; fails in transmission (which now should be considered in the absence of the SU) which has a probability $\delta = 1 - \frac{1}{M_p}(1 - P_{pd}^o)$, meaning that either the PU did not get allocated to the time slot, or it did but its forward channel was in outage; and finally, the NACK sent as a feedback was overheard correctly by the SU and was not erased, which has a probability $1 - \alpha$. The rest of the probabilities can be derived using similar arguments.

C. Secondary Throughput Analysis

In this subsection, we derive an expression for the SU throughput in the studied imperfect feedback-based regime. The SU service event will be just the same as in (2). Due to the feedback, it is only the value of $Pr\{Q_q^t = 0\}$ (which is

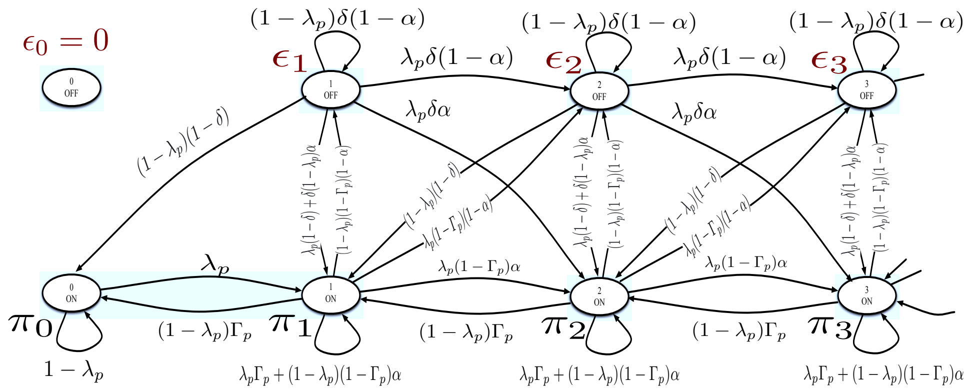


Fig. 2: Markov Chain modeling the PU queue evolution.

equivalent in our model to π_0) that is going to change. The PU Markov chain in Fig. 2 can be analyzed, using global balance equations, in order to get the value of π_0 which is given by

$$\pi_0 = \frac{\chi - \lambda_p}{(1-\delta)(1-\alpha) + \Gamma_p \alpha}, \quad (3)$$

where $\chi = \lambda_p \Gamma_p + (1-\lambda_p)(1-\delta) + (1-\lambda_p)(\Gamma_p - (1-\delta))\alpha$ (proof in Appendix).

We can now write the formula of the SU throughput by taking the expectation of the SU service event in (2) to give

$$\mu_s = E\left\{Y_s^t\right\} = \left(\frac{\chi - \lambda_p}{(1-\delta)(1-\alpha) + \Gamma_p \alpha}\right)(1 - P_{sd}^0)\left(\sum_{i \in [1,n]} p_i^0 a_i\right), \quad (4)$$

where $E\{\cdot\}$ is the expectation operator.

In this work, the stability of the PU queue is studied as the performance measure. Access probabilities are chosen such that the SU throughput is maximized provided that the PU's queue is stable. Stability can be loosely defined as keeping a quantity of interest bounded; in our case, the queue size. It is worth noting that for an irreducible and aperiodic Markov chain, the queue is stable if there exists a non-zero value for the probability of the queue being empty [10]. This condition is equivalent in our model to having $\pi_0 > 0$, which leads to $\lambda_p < \chi$. Therefore, the optimization problem is given by

$$\max_{a_i, i \in [1,n]} \quad \mu_s, \quad \text{subject to} \quad \lambda_p < \chi. \quad (5)$$

Fortunately, the optimization problem of (5) using (4) is convex, and therefore can be solved using any standard optimization technique [11] (proof omitted due to space limits).

D. Primary Delay Analysis

In this subsection, we only present final expressions for the average PU packet delay (proofs omitted due to space limits). For the scheme in Section III, one can show that

$$D_p = \frac{1 - \lambda_p}{\mu_p - \lambda_p}, \quad (6)$$

where D_p is the PU delay. While for the proposed feedback-based scheme, the delay is given by

$$D_p = \frac{(\Gamma_p - \chi)(\chi - \lambda_p)^2 + (1-\lambda_p)^2(1-\Gamma_p)\chi}{(1-\lambda_p)(1-\chi)((1-\delta)(1-\alpha) + \Gamma_p \alpha)(\chi - \lambda_p)} \quad (7)$$

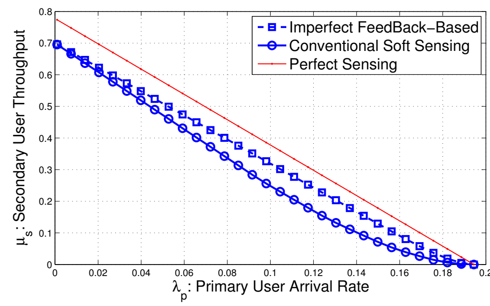


Fig. 3: SU throughput for various schemes vs. primary arrival rate at $\alpha = 0.1$ and $M_p = 4$ PUs.

V. Performance Results

In this section, we provide some performance results comparing our proposed access scheme with others. We consider a network of $M_p = 4$ primary users among which an SU is demanding service. The distance between the primary transmitters and receivers is set to 100 m, and is also set to 100 m between the secondary transmitter and receiver, while it is set to 150 m between any PU and the SU. The SNR threshold ζ is 10 dB, the transmit power is 100 mW, the path loss exponent $\gamma = 3.7$, and $N_0 = 10^{-11}$ W/Hz. The region below the energy threshold η is divided into $n = 4$ regions each having a different access probability. The SU overhears erased feedback messages with probability $\alpha = 0.1$.

In Fig. 3, the SU throughput is plotted against the primary arrival rate. Three schemes are compared: the proposed feedback-based scheme; a baseline no feedback (conventional) soft sensing scheme, where the SU does not exploit the primary feedback messages; and a perfect sensing scheme (an upper bound), which is a genie-aided scheme in which the SU is perfectly acquainted of the idle primary time slots, and therefore accesses the channel during them with probability one. We can see that our feedback-based scheme outperforms the no feedback one. This is attributed to the increased awareness of the SU about the primary environment after overhearing, and reacting to, the *unerased* feedback messages.

In Fig. 4, the average primary queuing delay is plotted against primary arrival rate. We can see that our proposed feedback-based access scheme also outperforms the conventional soft sensing scheme for all possible values of λ_p.

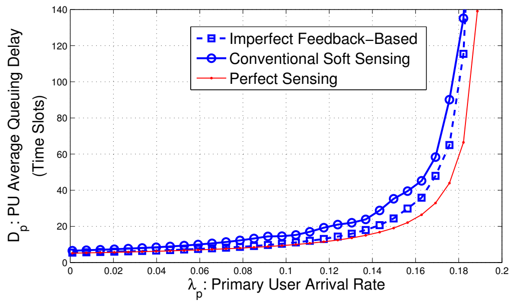
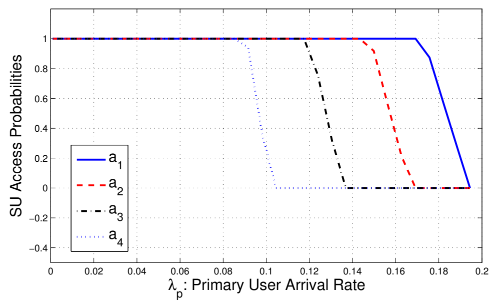
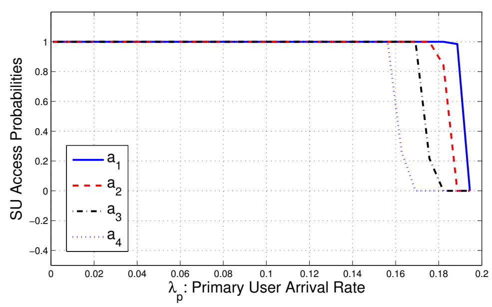


Fig. 4: Average PU queuing delay for various schemes vs. primary arrival rate at $\alpha = 0.1$ and $M_p = 4$ PUs.



(a) Conventional Soft Sensing.



(b) Feedback-Based Soft Sensing.

Fig. 5: SU access probabilities. M_p =4 PUs, and $\alpha = 0.1$.

SU access probabilities for both the no feedback and the feedback-based schemes are plotted against primary arrival rate in Fig. 5(a) and Fig. 5(b), respectively. It can be easily noticed that in our scheme, the SU is able to access the channel more frequently than in the conventional one, which explains why it can attain higher throughput.

In order to see how our system performs with different erasure probabilities, we provide some plots against α at a primary arrival rate of $\lambda_p = 0.08$. In Fig. 6, the SU throughput is plotted against α. As expected, as α varies from 0 to 1, the SU throughput of our proposed scheme keeps degrading until reaching its minimum (which is the same value of the conventional soft sensing scheme's throughput) at $\alpha = 1$; a case in which the SU ignores all overheard feedback messages as they are all erased. Therefore, our proposed scheme's worst conditions are the conventional soft sensing scheme's best.

Another plot is provided in Fig. 7. This time, the average primary queuing delay is plotted against α. Note that the primary delay is not an optimization variable to be minimized

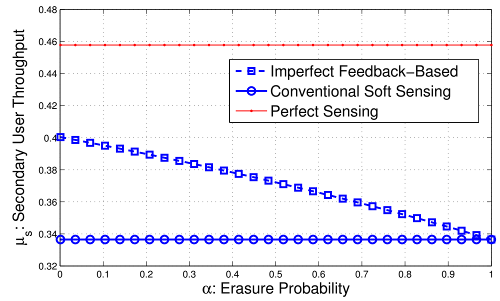


Fig. 6: SU throughput for various schemes vs. feedback erasure probability at $\lambda_p = 0.08$ and $M_p = 4$ PUs.

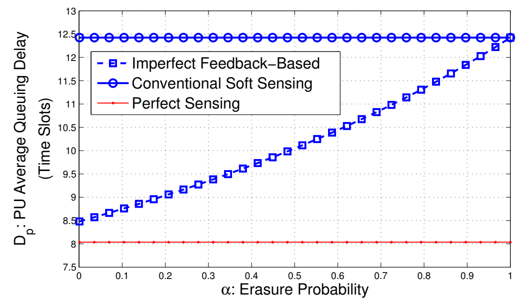


Fig. 7: Average PU queuing delay for various schemes vs. feedback erasure probability at $\lambda_p = 0.08$ and $M_p = 4$ PUs.

(and thus, its values in our proposed scheme are not guaranteed to be lower than the values of the conventional soft sensing one). However, as α varies from 0 to 1, it is kept lower than the conventional soft sensing's one. An important notice is that at $\alpha = 0$, the SU overhears perfect primary feedback, which will surely cause the primary delay to be lower than that in the conventional soft sensing scheme. While at $\alpha = 1$, the SU ignores the overheard feedback (as it is always erased), which causes no difference than the conventional soft sensing scheme. Other than those two values of α, the primary delay of our scheme is not guaranteed to be lower than that in the conventional soft sensing scheme. In fact, for higher values of the arrival rate, the PU delay of our scheme may be even higher at some values of α between 0 and 1; a price that is to be compromised in order to increase the secondary throughput.

VI. Conclusions

We examined a cognitive spectrum access scheme in which an SU exploits the primary feedback information, which is received through an erasure channel. We considered an overlay model in which the SU accesses the channel by certain access probabilities that are function of the spectrum sensing metric. We studied the cognitive radio network from a queuing theory point of view in which access probabilities are determined by solving a secondary throughput maximization problem subject to a constraint on the primary queues' stability. The problem is convex and, hence, can be solved efficiently. Our scheme yielded improved results in the SU throughput than the baseline no feedback scheme. This is attributed to the efficient utilization of the PU's *unerased* feedback messages.

APPENDIX

Referring to the Markov chain in Fig. 2, we can write the balance equation around state 0_{ON} as

$$\pi_0\lambda_p = \pi_1\bar{\lambda}_p\Gamma_p + \epsilon_1\bar{\lambda}_p\bar{\delta}, \tag{8}$$

where the notation $\bar{x} = 1 - x$. Writing the balance equation around state 1_{OFF} we get

$$\epsilon_1(1 - \bar{\lambda}_p\delta\bar{\alpha}) = \pi_1\bar{\lambda}_p\bar{\Gamma}_p\bar{\alpha},$$

therefore, we have

$$\pi_1 = \epsilon_1\frac{1 - \delta\bar{\lambda}_p\bar{\alpha}}{\bar{\lambda}_p\bar{\Gamma}_p\bar{\alpha}}. \tag{9}$$

Substituting by (9) in (8), we get

$$\epsilon_1 = \frac{\lambda_p\bar{\Gamma}_p\bar{\alpha}}{\chi}\pi_0, \tag{10}$$

where $\chi = \lambda_p\Gamma_p + \bar{\lambda}_p\bar{\delta} + \bar{\lambda}_p(\Gamma_p - \bar{\delta})\alpha$. Using (10) in (9):

$$\pi_1 = \frac{\lambda_p(1 - \bar{\lambda}_p\delta\bar{\alpha})}{\bar{\lambda}_p\chi}\pi_0. \tag{11}$$

Writing the balance equation around state 1_{ON}, we have

$$\pi_1(1 - \lambda_p\Gamma_p - \bar{\lambda}_p\bar{\Gamma}_p\alpha) = \pi_0\lambda_p + \epsilon_1(\lambda_p\bar{\delta} + \bar{\lambda}_p\delta\alpha) + \pi_2\bar{\lambda}_p\Gamma_p + \epsilon_2\bar{\lambda}_p\bar{\delta}.$$

Using (8) to substitute for the term $\pi_0\lambda_p$, we get

$$\pi_1\bar{\Gamma}_p(1 - \bar{\lambda}_p\alpha) = \epsilon_1(\bar{\delta} + \bar{\lambda}_p\delta\alpha) + \pi_2\bar{\lambda}_p\Gamma_p + \epsilon_2\bar{\lambda}_p\bar{\delta}. \tag{12}$$

Using (10) and (11) into (12), we now have

$$\pi_2\bar{\lambda}_p\Gamma_p + \epsilon_2\bar{\lambda}_p\bar{\delta} = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0. \tag{13}$$

Writing the balance equation around state 2_{OFF}, we get

$$\epsilon_2(1 - \bar{\lambda}_p\delta\bar{\alpha}) = \epsilon_1\lambda_p\delta\bar{\alpha} + \pi_1\lambda_p\bar{\Gamma}_p\bar{\alpha} + \pi_2\bar{\lambda}_p\bar{\Gamma}_p\bar{\alpha},$$

but since from (10) and (11) we have

$$\epsilon_1\lambda_p\delta + \pi_1\lambda_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0,$$

therefore

$$\epsilon_2\frac{1 - \bar{\lambda}_p\delta\bar{\alpha}}{\bar{\alpha}} - \pi_2\bar{\lambda}_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0. \tag{14}$$

From (13) and (14) we can get the following

$$\epsilon_2 = \frac{\bar{\lambda}_p\bar{\alpha}}{1 - \bar{\lambda}_p\bar{\alpha}}\pi_2. \tag{15}$$

Therefore, using (15) in (13) we get

$$\epsilon_2 = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2 . \frac{\bar{\lambda}_p\bar{\Gamma}_p\bar{\alpha}}{\bar{\chi}^2}\pi_0, \quad \pi_2 = (\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi})^2 . \frac{(1 - \bar{\lambda}_p\bar{\alpha})\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0. \tag{16}$$

From the symmetry of the upcoming states in the Markov chain, expressions (15) and (16) can be generalized for any ϵ_k and π_k with $k \geq 2$, since all the upcoming balance equations will give the same result.

We can now use the normalization condition, $\sum_{k=0}^{\infty}(\pi_k + \epsilon_k) = 1$, to get the value of π_0. First, we will divide the summation as follows

$$\sum_{k=0}^{\infty}(\pi_k + \epsilon_k) = \pi_0 + \underbrace{(\pi_1 + \epsilon_1)}_{A} + \underbrace{\sum_{k=2}^{\infty}(\pi_k + \epsilon_k)}_{B} = 1. \tag{17}$$

Simplifying the term B: since, for $k \geq 2$, we have

$$\pi_k + \epsilon_k = \psi^k\frac{\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0, \quad \text{where} \quad \psi = \frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}.$$

Hence,

$$B = \frac{\bar{\Gamma}_p\pi_0}{\bar{\chi}^2}\sum_{k=2}^{\infty}\psi^k = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{\lambda_p}{\chi - \lambda_p}\right)\pi_0 \tag{18}$$

The last summation converges only if $\psi < 1$, that is equivalent to $\lambda_p < \chi$. This is actually the stability condition for the PU queue. After some manipulations, the term A is given by

$$A = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{\chi + \bar{\Gamma}_p}{\bar{\Gamma}_p}\right)\pi_0. \tag{19}$$

Using (18) and (19), the final result becomes

$$A + B = \frac{\lambda_p\left(\bar{\Gamma}_p + \bar{\delta} + (\Gamma_p - \bar{\delta})\alpha\right)}{\chi - \lambda_p}\pi_0. \tag{20}$$

Using (20) in (17), π_0 is finally given by

$$\pi_0 = \frac{\chi - \lambda_p}{\bar{\delta} + (\Gamma_p - \bar{\delta})\alpha}, \tag{21}$$

which satisfies the balance equation given in (8).

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