Exploiting Spatial Spectrum Holes in Multiuser MIMO Systems

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***Abstract*—In this paper, a modified spectrum sensing algorithm for Cognitive Radio is proposed for the detection of *spatial spectrum holes*. We consider two scenarios, namely, the static and the dynamic scenarios. In both scenarios, by exploiting a priori information about primary users (PUs) activity, a better sensing for spatial spectrum holes in MU-MIMO can be achieved. A Maximum A Posteriori (MAP) detector with relaxed constraints is considered where both the block sparsity structure of the signal being sensed and the knowledge of a priori information about the PUs activity are considered. In the dynamic case, the PUs activity is modeled as a two-state Markov chain.**

***Index Terms*—MAP Detector, MU-MIMO, Spatial Spectrum Holes**

I. Introduction

With the highly increased demand of wireless communications applications and services, the policy of fixed spectrum assignments has been proved to be an inefficient way of utilizing the available radio spectrum resources. According to the Federal Communications Commission (FFC), 90% of most of the fixedly assigned bands capacity is not utilized [1], leading to virtual spectrum scarcity. To alleviate this problem, Cognitive Radio (CR) has emerged as a possible solution for this problem of inefficient utilization of radio spectrum. CRs strategy is to adaptively and dynamically access the unutilized spectrum bands, known as spectrum holes, which arise from sparse activity. The main task of a CR is to sense the spectrum to discover the spectrum holes, upon which the CR can adaptively assign these free spectrum holes to prospective secondary users (SU).

Many spectrum sensing techniques have been developed, such as energy detectors, matched-filter detectors, and feature detectors [2]. However, traditional techniques of spectrum sensing do not ultimately utilize the available spectrum as they overlook the opportunity of exploiting the fact that in multi-user multi-input multi-output (MU-MIMO) systems the number of antennas at the primary receivers have to be greater than or equal to PUs transmitters for reliable decoding [3]. By making use of this fact, SUs can transmit their data along with the PUs as long as the number of active PUs is less than the number of primary receiving antennas. Consequently, there still exist spatial dimensions that create spatial spectrum holes, which a CR can assign to its users without degrading the performance of the PU network.

Recently, compressive sensing (CS) has been used as an efficient tool, in terms of the required number of measurements to be sensed, to detect or recover sparse signals. It has been adopted in many papers, such as [4] and [5], for wideband spectrum sensing in CR networks. Also, ℓ_q regularization, Least Absolute Shrinkage and Selection Operator (LASSO), and Group LASSO have been extensively studied lately as novel strategies based on CS and they have found applications in CR and spectrum cartography [6]. In [7], the authors utilized the sparsity of signals along with the finite alphabet constraint to deal with the multiuser detection problem in CDMA system when the activity of the users is unknown. They proposed optimal MAP and suboptimal MAP detectors based on the idea of incorporating the users inactivity by augmenting zero symbol in the alphabet constraint and made use of the compressive sensing techniques for decoding the users data.

Accounting for the block sparsity pattern of OFDM signals, the authors in [3] used CS tools to detect spatial spectrum holes in the uplink of a MU-MIMO OFDM cell. However, only sensing the spatial spectrum holes in a static environment was addressed and a dynamic environment, where users change their activity states with time, was not considered, which is the more realistic situation.

In this paper, an algorithm for spatial spectrum holes sensing is considered in a dynamic environment exploiting the a priori knowledge of the PUs used constellation and their previously detected activity states. Also, we utilize the idea of augmenting zero symbols in the constellation alphabet introduced in [7]; we adapt this idea for our problem settings of spatial spectrum holes sensing and expand it to deal with the dynamic nature of the PUs in an OFDM system. Simulation results show that the proposed algorithm outperforms the traditional compressive sensing based method that makes use only of the block sparse structure of the signal [3].

The rest of the paper is organized as follows. Section II presents the system model. In Section III, the sensing strategy is presented for both the static and the dynamic environments. Section III ends with a summary for the proposed sensing algorithm. Section IV presents the simulation results for the proposed sensing strategy. Finally, conclusions are drawn in Section V .

Notations: Throughout the paper we refer to vectors with

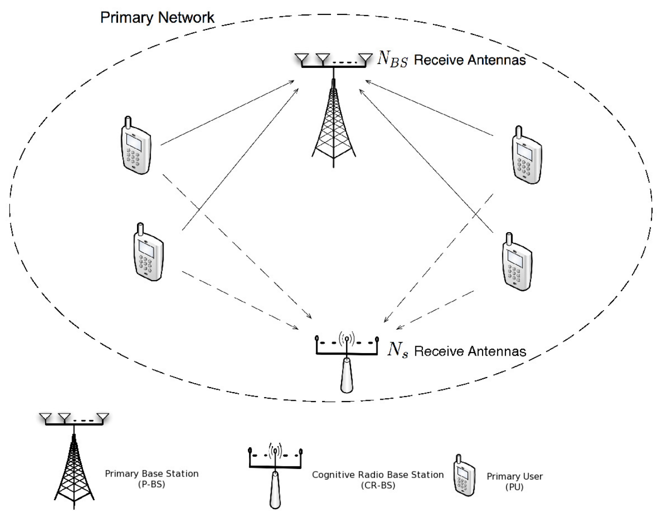

Fig. 1: System model

bold lower cases such as $\mathbf{g}$. $\mathbf{diag}(\mathbf{g})$ refers to the diagonal matrix whose diagonal elements are the elements of the vector $\mathbf{g}$. Due to size limitations, we will sometimes refer to $\mathbf{diag}(\mathbf{g})$ as $\mathbf{D}(\mathbf{g})$. Hermitian of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^{\mathcal{H}}$.

II. System Model

An uplink scenario of a single MIMO OFDM cell is considered where a set of single-antenna primary users communicate with a single primary base station (P-BS) in a multiuser MIMO settings as shown in Fig. 1[1]. Next, we will describe the system models for the primary and the secondary networks.

A. Primary User Network

The primary user network has a maximum of N_P active users, sharing the same set of subcarriers, which can simultaneously send their data to their primary base station (P-BS) equipped with $N_{BS} \geq N_P$ receiving antennas. The condition $N_{BS} \geq N_P$ guarantees that the P-BS will be able to separate the signals from the different users. The number of active PUs N_a can be much less than N_{BS} and this allows for some spatial spectrum holes that can be occupied by the SUs since the P-BS can separate the interference caused by these SUs on the primary network. Each PU sends its OFDM symbol $\mathbf{x}_i$ on L subcarriers with an average transmitted power constraint $\mathrm{E}\left[\mathbf{x}_i\mathbf{x}_i^{\mathcal{H}}\right] \leq L$. It is assumed that all PUs symbols $\mathbf{x}_i$'s are carved from the same constellation for simplicity of presentation but the idea can be easily extended to other scenarios of different constellations.

B. Secondary User Network

The secondary user network has a cognitive radio base station (CR-BS) with N_S receive antennas in order to sense the instantaneous number of active primary users, hence the free spatial transmission dimensions. These free dimensions can be assigned to the prospective SUs so that they can transmit their data simultaneously with the primary users without highly degrading the performance of the primary network. Our focus in this paper will be on the detection of the number of active primary users sharing the same frequency resources (same set of subcarriers). Note that the number of SUs plus the PUs must not exceed the available transmission dimensions of the primary network (i.e., it must be less than N_{BS} for the P-BS to be able to separate the signals of the PUs and the interference from the SUs).

It is assumed that the CR-BS has perfect knowledge of the channels between the PUs and the N_S CR base station sensing antennas. It is a reasonable assumption because this channel knowledge can be achieved by channel estimation via the reference signals already being transmitted by the PUs. We assume a slowly fading channel model so that the channel estimates from the previous PU active slots are still valid.

The received signal on the k-th subcarrier at the j-th CR-BS antenna can be written as

$$\mathbf{y}_j(k) = \sum_{i=1}^{N_P} \mathbf{h}_{ji}(k)\mathbf{x}_i(k) + \mathbf{n}_j(k), \quad k = 1, \cdots, L, \tag{1}$$

where $\mathbf{y}_j$ is an $L \times 1$ vector, $\mathbf{x}_i$ is the transmitted OFDM symbol from the i-th PU, $\mathbf{h}_{ji}(k)$ is the channel gain on the k-th subcarrier between the j-th CR-BS antenna and the i-th PU and $\mathbf{n}_j$ is a vector of independent, identically distributed (i.i.d.) complex Gaussian noise samples received at the j-th CR-BS antenna with zero mean and a covariance matrix $\sigma^2\mathbf{I}$. Collecting the data from the L subcarriers we get

$$\mathbf{y}_j = \sum_{i=1}^{N_P} \mathrm{diag}(\mathbf{h}_{ji})\mathbf{x}_i + \mathbf{n}_j. \tag{2}$$

Collecting the received signals from all of the CR-BS antennas we get

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{3}$$

where

$$\mathbf{H} = \begin{pmatrix} D(\mathbf{h}_{11}) & D(\mathbf{h}_{12}) & \cdots & D(\mathbf{h}_{1N_P}) \\ D(\mathbf{h}_{21}) & D(\mathbf{h}_{22}) & \cdots & D(\mathbf{h}_{2N_P}) \\ \vdots & \vdots & \vdots & \vdots \\ D(\mathbf{h}_{N_S1}) & D(\mathbf{h}_{N_S2}) & \cdots & D(\mathbf{h}_{N_SN_P}) \end{pmatrix}, \tag{4}$$

$\mathbf{y} = [\mathbf{y}_1^T\mathbf{y}_2^T \cdots \mathbf{y}_{N_S}^T]$, $\mathbf{x} = [\mathbf{x}_1^T\mathbf{x}_2^T \cdots \mathbf{x}_{N_P}^T]$, and $\mathbf{n} = [\mathbf{n}_1^T\mathbf{n}_2^T \cdots \mathbf{n}_{N_S}^T]$ are the concatenation of received vectors $\mathbf{y}_j$'s at all CR-BS antennas, the concatenation of all the PUs symbols vectors $\mathbf{x}_i$'s, and the concatenation of the received noise vectors $\mathbf{n}_j$'s at all CR-BS antennas, respectively.

III. Sensing Strategy

A. Static environment

Now, our goal is to detect the activity of each PU by sensing the concatenated vector $\mathbf{x}$. If the i-th PU is active then its corresponding vector, $\mathbf{x}_i$, can have Q^L possible alphabet vectors, where Q is the constellation size. If the i-th PU is

[1] MU-MIMO is a supported transmission mode in the LTE and LTE-Advanced mobile standards.

inactive then its corresponding vector, $\mathbf{x}_i$, will be a zero vector. Assuming that the probability that a PU is active equals p_a then the probability of a zero data vector equals $(1-p_a)$, while the probability of any possible non-zero vector equals $\frac{p_a}{Q^L}$ (assuming equiprobable data vectors). Then the optimal detector in the Bayes risk sense is a *maximum a posteriori probability* (MAP) detector, due to the non-equiprobable nature of the possible data vectors, given by

$$\hat{\mathbf{x}}^{MAP} = \underset{\substack{\mathbf{x}=[\mathbf{x}_1^T \cdots \mathbf{x}_{N_P}^T] \\ \mathbf{x}_i \in \mathcal{A}}}{\arg\max} \; p(\mathbf{y}|\mathbf{x})p(\mathbf{x}), \tag{5}$$

where $\mathcal{A} = \{$zero vector, Q^L possible alphabet vectors$\}$.

The prior probability of the concatenated vector $\mathbf{x}$, $p(\mathbf{x})$, can be expressed as

$$\begin{aligned} p(\mathbf{x}) &= \prod_{i=1}^{N_P} \Pr[\mathbf{x}_i] \\ &= \left(\frac{p_a}{Q^L}\right)^{\frac{\sum_{i=1}^{N_P}\|\mathbf{x}_i\|_0}{L}} (1-p_a)^{N_P - \frac{\sum_{i=1}^{N_P}\|\mathbf{x}_i\|_0}{L}}, \end{aligned} \tag{6}$$

and $p(\mathbf{y} \mid \mathbf{x})$ is a complex Gaussian distribution with mean $\mathbf{x}$ and covariance matrix $\sigma^2\mathbf{I}$. Then we have

$$\ln p(\mathbf{x}) = \frac{\sum_{i=1}^{N_P}\|\mathbf{x}_i\|_0}{L} \ln\left(\frac{p_a}{Q^L}\right) + \left(N_P - \frac{\sum_{i=1}^{N_P}\|\mathbf{x}_i\|_0}{L}\right) \ln(1-p_a). \tag{7}$$

The MAP detector reduces to

$$\hat{\mathbf{x}}^{MAP} = \underset{\substack{\mathbf{x}=[\mathbf{x}_1^T \cdots \mathbf{x}_{N_P}^T] \\ \mathbf{x}_i \in \mathcal{A}}}{\arg\min} \; \frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|_2^2 + \lambda \sum_{i=1}^{N_P} \|\mathbf{x}_i\|_0, \tag{8}$$

where $\lambda = \frac{1}{L} \ln \frac{1-p_a}{p_a \,/\, Q^L}$.

Since we have $(Q^L+1)^{N_P}$ possible combinations for the vector $\mathbf{x}$, the MAP detector requires exhaustive search with exponential complexity of order $(Q^L+1)^{N_P}$, which is too large to be implemented in a real time CR since L, the number of subcarriers, is usually large (for example, the resource block size in LTE is $L = 12$ which is the smallest unit that can be assigned to a user). Therefore, we can relax our problem constraints in (8) by ignoring the alphabet constraint and using the ℓ_2-norm instead of the ℓ_0-norm leading to a convex optimization problem regularized by λ. Note that λ should be positive to maintain the convexity of the problem [8], and since p_a is small due to the low activity factor (i.e., $p_a < \frac{1}{2}$), λ is always positive.

$$\hat{\mathbf{x}}^{LR} = \arg\min_{\mathbf{x}} \frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|_2^2 + \lambda \sum_{i=1}^{N_P} \sqrt{L}\sqrt{\|\mathbf{x}_i\|_2^2}. \tag{9}$$

Note that with the assumption that all the PUs use the same constellation[2] and for the special case that the power on each subcarrier equals one, $\|\mathbf{x}_i\|_0$ equals $\|\mathbf{x}_i\|_2^2$, and the sub-optimality of the detector comes only from the relaxed alphabet constraints.

[2]A constant modulus constellation is assumed.

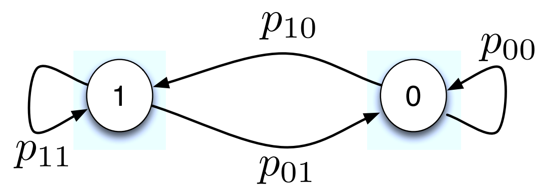

Fig. 2: Markov chain model of the PU activity

B. Dynamic environment

Now, we consider the dynamic scenario where the activity of each PU can be modeled as a two-state Markov chain [9] [10]. The two states are the active state "1" and the inactive state "0" with small transition probabilities, p_{01} and p_{10}, as shown in Fig. 2. PUs can assist the CR to obtain this transition probabilities; if this is not the case and there is no cooperation from the PU-BS these transition probabilities can be estimated as in [11].

We can rewrite (9) to adapt it for the dynamic case as

$$\hat{\mathbf{x}}^{Dyn} = \arg\min_{\mathbf{x}} \frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|_2^2 + \sum_{i=1}^{N_P} \lambda_i \sqrt{L}\sqrt{\|\mathbf{x}_i\|_2^2}, \tag{10}$$

where $\lambda_i = \frac{1}{L} \ln \frac{1-P_i}{P_i\,/Q^L}$.

In the above, the parameter P_i is the probability of the i-th PU to be active. P_i at a certain instant depends on the previous state of the PU so $P_i = \Pr(i\text{-th PU active } \mid \text{previous detected state of activity of } i\text{-th PU})$ and it takes the values p_{11} or p_{10} depending on the previous state of the PU.

It is important to note again that the convexity is preserved under positive scaling [8], which is the case in (10). To assure that the scaling factors λ_i's always take non-negative values, P_i must be less than or equal to $Q^L/(Q^L+1)$, which is always the case since $Q^L >> 1$.

C. Summary of the proposed algorithm

- Step (1): Solve the convex problem in (9) if not considering the dynamic activity, or the problem in (10) if considering the dynamic activity.
- Step (2): Zero out all the elements of the estimated vector $\hat{\mathbf{x}}^{LR}$ or $\hat{\mathbf{x}}^{Dyn}$ whose values below the threshold to obtain a vector $\mathbf{x}^{th}$ whose elements are zeroes and ones. This threshold is set to differentiate between the activity state of each subcarrier separately and its value depends on the constellation used.
- Step (3): Get ℓ_0-norm of $\mathbf{x}_i$'s and compare it with $L/2$. If ℓ_0-norm of $\mathbf{x}_i$ for any i is greater than $L/2$ then user i state is declared as active, otherwise it is declared as inactive.

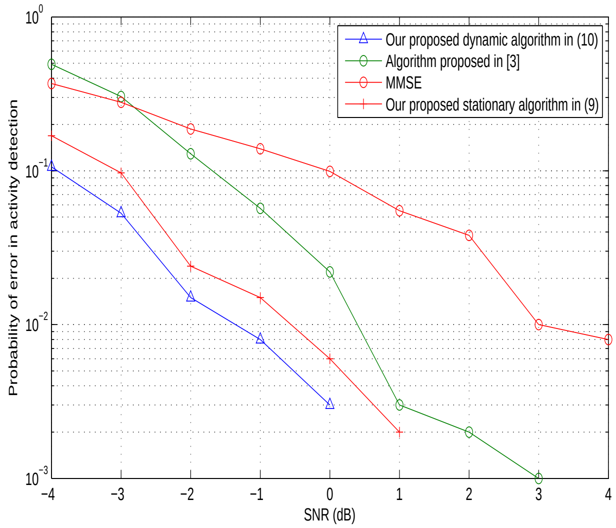

Fig. 3: The probability of error in PU activity detection for the different spatial spectrum holes detection algorithms.

IV. Simulation Results

In this section, we present the simulation results of our modified proposed spatial sensing technique that accounts for the dynamic activity of PUs and compare it to its counterpart that was proposed in [3].

We simulate an OFDM uplink system with $L = 12$ and $N_P = 8$, i.e., up to 8 different users can share the same 12 subcarriers[3]. The number of active PUs, N_a, changes dynamically according to a Markov model with transition probabilities: $p_{01} = p_{10} = 0.1$ and $p_{11} = p_{00} = 0.9$, and with initial probabilities: $p_0 = \Pr[\text{inactive state}] = 0.5$ and $p_1 = \Pr[\text{active state}] = 0.5$. All the transmitted symbols are carved from QPSK constellation. The threshold used equals 0.5. The channel between all PUs and all CR antennas are modeled as independent, multipath Rayleigh fading channels with 10 taps and each tap has a variance of 1/10.

Figure 3 shows the performance of our proposed detector in equation (9) supplying it with only the stationary probability of activity of the PUs, which in our settings equals 0.5. Figure 3 shows also the performance of the proposed detector in equation (10) supplying it with transition probabilities. We also show the performance of the CS-based detector of [3] along with the Minimum Mean Square Error MMSE detector.

It is clear from Fig. 3 that our proposed detectors have better performance, as explained above, due to considering both the block sparsity structure of the signal being sensed and the knowledge of the stationary probability of activity in the case of using equation (9), and the use of the previously detected states of PUs in the dynamic case using equation (10). It is also clear that the detector that exploits the previously detected states gives the best performance.

[3]A resource block in LTE and LTE-Advanced standards is a 12 contiguous subaccariers and it is the smallest unit that can be assigned to a user.

V. Conclusions

In this paper, an optimal MAP detector and suboptimal MAP-based detectors with relaxed constraints for detecting spatial spectrum holes were proposed. The proposed algorithms make use of the finite alphabet property of transmitted data as well as the a priori PU activity pattern to better detect the number of active primary users. It was shown that better detection performance can be achieved by exploiting the previously detected states of the PUs and the PUs activity patterns. The proposed algorithms outperform the previously proposed algorithms for detecting the spatial spectrum holes.

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