On the Effective Capacity of Delay Constrained Cognitive Radio Networks with Relaying Capability

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Abstract. In this paper we analyze the performance of a secondary link in a cognitive radio relaying system operating under a statistical quality of service (QoS) delay constraint. In particular, we quantify analytically the Effective Capacity improvement for the secondary user when it offers a packet relaying service to the primary user packets that are lost under the SINR interference model. Towards this objective, we utilize the concept of Effective Capacity introduced earlier in the literature as a metric to quantify the wireless link throughput under statistical QoS delay constraints, in an attempt to support real-time applications using cognitive radios. We study a two-link network, a single secondary link and a primary network abstracted to a single primary link, with and without relaying capability. We analytically prove that exploiting the packet relaying capability at the secondary transmitter improves the Effective Capacity of the secondary user. Finally, we present numerical results that support our theoretical findings.

1 Introduction

Over the past decade, there has been surge in demand for the wireless spectrum due to the bandwidth-hungry applications, e.g., multimedia communications. Moreover, there has been ample evidence that the wireless spectrum has been significantly underutilized. In [1], the cognitive radio (CR) concept has been first introduced as a promising technology due to its opportunistic, agile and efficient spectrum utilization merits. Cognitive radios enable secondary users (SUs) to co-exist with the primary (licensed) users (PUs) in the same frequency band

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without causing harmful interference. Three major cognitive radio paradigms have been introduced in the literature: underlay, overlay, and interweave [2].

Providing quality of service (QoS) guarantees has been a daunting challenge for wireless networks, in general, and for cognitive radio networks, in particular. The Effective Capacity (EC) concept originally proposed in [3] is a throughput performance metric for a wireless link under statistical QoS (delay) constraints. It is considered the wireless dual concept to the notion of "Effective Bandwidth" which was originally coined for wired networks in [4].

Introducing the relay nodes in cognitive networks has been studied in [5], the authors used cooperative relay node to assist the transmission of CRNs. In [6], proposed an adaptive cooperation diversity scheme including best-relay selection while ensuring the QoS of the primary user. In [7], the authors proposed a feedback-based random access channel scheme for cognitive relaying networks. However, delay constraints for opportunistic users with real-time communication requirements were not considered.

In [8], we quantified the EC gains and transmission power reduction attributed to exploiting the primary user feedback at the secondary transmitter. However, the SUs in [8] do not provide a relaying service to the unsuccessful primary packets. Previous work did not studied the effective capacity of the cognitive radios. However,the closest to our work is [9], where the EC for interference and delay constrained cognitive radio relaying channels is characterized. The system model in [9] hinges on the underlay cognitive radio paradigm, whereas our system exhibits the characteristics of interweave cognitive radios which mandates spectrum sensing and allows for SU-PU co-existence as long as the SINR is above an acceptable threshold. In addition, we add a relaying service at the SU rather than using dedicated relaying nodes as in [9].

Our main contribution in this paper is to show that a higher EC, and hence, a higher data rate can be sustained if the secondary user offers a packet relaying service to the primary user. We develop a queuing theoretic analysis to capture the gains of adding relaying capability to the cognitive radio network. We show analytically that adding a relaying capability to cognitive radio networks not only increases its EC but also helps the PU to evacuate its queue faster and, hence, giving more opportunity to the SU to transmit over the shared channel.

The rest of the paper is organized as follows. A background on the EC concept is given in Section 2. The system model and underlying assumptions are presented in Section 3. In Section 4, the EC problem for cognitive relaying networks is formulated and analyzed. Afterwards, the numerical results and discussion are presented in Section 5. Finally, we conclude the paper and point out potential directions for future research in Section 6.

2 Background: Effective Capacity

In [3], Wu and Negi introduced the notion of *effective capacity* (EC) of a wireless link as the maximum constant arrival rate that can be supported by a given channel service process while satisfying a statistical QoS requirement specified by the QoS exponent, denoted θ. The EC concept is a link layer modeling abstraction to incorporate QoS requirements, such as delay, into system performance analysis studies of wireless systems. Using EC as a performance metric enables us to evaluate the cognitive radio network throughput under statistical QoS constraint without performing queuing analysis.

If Q is defined as the stationary queue length, then θ is the decay rate of the tail distribution of the queue length Q, that is

$$\lim_{q\to\infty} \frac{\log \Pr(Q \geq q)}{q} = -\theta. \tag{1}$$

From (1), it is clear that the EC captures a probabilistic QoS constraint. Practically, θ, which depends on the statistical characterization of the arrival and service processes, establishes bounds on the delay (or buffer length). It has been established in [3] that the EC for a given QoS exponent θ is given by

$$-\lim_{t\to\infty} \frac{1}{\theta t} \log_e \mathbb{E}\left\{e^{-\theta S(t)}\right\} = -\frac{\Lambda(-\theta)}{\theta}, \tag{2}$$

where $\Lambda(\theta) = \lim_{t\to\infty} \frac{1}{t} \log_e \mathbb{E}\left\{e^{-\theta S(t))}\right\}$ is a function of the logarithm of the moment generating function of $S(t)$, $S(t) = \sum_{k=1}^{t} r(k)$ represents the time accumulated service process and $\{r(k), k= 1, 2, \cdots\}$ is the discrete, stationary and ergodic stochastic service process.

3 System Model

We consider a time slotted system as shown in Fig. 1. Where data is transmitted in frames of duration T seconds, that fits exactly in one time slot. The primary network traffic is abstracted to a single primary link. Hence, our analysis is valid for any number of primary users. Assuming one frequency channel, the primary transmitter will access the channel whenever it has a packet to send. On the other hand, the single SU attempts to access the medium with a certain policy, described later, based on the spectrum sensing outcome. The SU is assumed to have a packet to send at the beginning of each time slot (i.e. the SU queue is saturated). We assume that the SU uses the first N seconds out of the slot duration T for spectrum sensing.

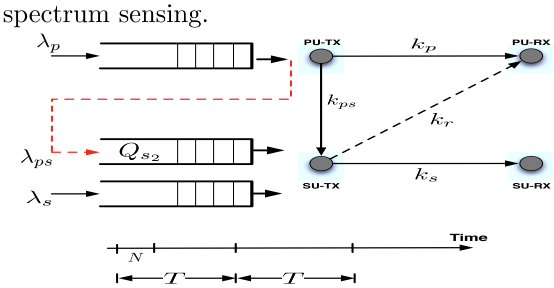

Fig. 1. Cognitive Relay System Model.

In the rest of this paper, we refer to the system where the SU offers a relay service to the "undelivered" primary packets, besides sending its own packets, as the "Cognitive Relay system". On the other hand, the baseline system with no relaying capability is referred to as "No-relay system". The EC of both systems is analyzed under the SINR interference model. According to the cognitive radio system adopted in this paper (which is a hybrid between underlay and interweave), the SU transmits its packets with a lower power level P_1 when the channel is sensed busy. However, if the medium is sensed idle, the SU transmits with a higher power level P_2. These power levels correspond to the SU transmission rates of r_1 and r_2 for busy and idle mediums, respectively. We assume non-perfect spectrum sensing. Hence, a miss-detection event occurs if the PU is active and the medium is sensed idle by the SU. On the other hand, a false alarm occurs when the medium is sensed busy while the primary user is not sending. Simple energy detection [10] is adopted as the spectrum sensing mechanism.

The discrete time secondary link input-output relations for idle and busy channels in the i^{th} symbol duration are given, respectively, by

$$y(i) = h(i)x(i) + n(i) \qquad i = 1, 2, \cdots \tag{3}$$

$$y(i) = h(i)x(i) + s_p(i) + n(i) \qquad i = 1, 2, \cdots , \tag{4}$$

where $x(i)$ and $y(i)$ represent the complex-valued channel input and output, respectively. $h(i)$ denotes the fading coefficient between the cognitive transmitter and receiver, $s_p(i)$ is the interference signal from the primary network on the SU and $n(i)$ is the additive thermal noise at the secondary receiver modeled as a zero-mean, circularly-symmetric complex Gaussian random variable with variance $\mathbb{E}\{|n(i)|^2\} = \sigma_n^2$. The channel bandwidth is denoted by B. The channel input is subject to the following average energy constraints: $\mathbb{E}\{|x(i)|^2\} \leq P_1/B$ or $\mathbb{E}\{|x(i)|^2\} \leq P_2/B$ for all i's, when the channel is sensed to be busy or idle, respectively. The fading coefficients are assumed to have arbitrary marginal distributions with finite variances, that is, $\mathbb{E}\{|h(i)|^2\} = \mathbb{E}\{z(i)\} = \sigma^2 < \infty$, where $|h(i)|^2 = z(i)$. Finally, we consider a block-fading channel model and assume that the fading coefficients stay constant for a block of duration T seconds (i.e., one frame duration) and change independently from one block to another.

In the proposed model, we leverage a perfect error-free primary feedback channel. The primary receiver sends a feedback at the end of each time slot to acknowledge the reception of packets. Typically, the PU receiver sends an ACK if a packet is correctly received, however, a NACK is sent if a packet is lost. Failure of reception is attributed to primary channel outage. In case of an idle slot, no feedback is sent. The SU is assumed to overhear and decode this primary feedback perfectly and to act as follows: if an ACK/no feedback is overheard, the SU behaves normally and starts sensing the channel in the next time slot. On the other hand, if a NACK is overheard by the SU, yet, it can successfully decode the PU's data packet, then the SU stores it in the relay queue and sends an ACK to the PU as explained in the next section.

We consider a cognitive relaying system where the SU plays a role in relaying the "undelivered" primary packets. We recall that the SU has two separate queues; the first queue stores packets to be relayed for the PU (Relay queue). The second queue stores the SU own packets (Secondary Queue). The SU senses the medium and accesses it with either P_1 or P_2 according to the sensing outcome, while giving the advantage to evacuate the relay queue first.

We assume that all four links in the studied system are subject to outage, that is, the outage probability in the primary link is denoted k_p, in the PU-TX and SU-TX link is denoted k_{ps}, in the SU-TX and PU-RX (relaying channel) is denoted k_r and in the secondary link is denoted k_s.

In our model, we assume that the PU occupies the wireless channel with a fixed prior probability ρ_p [9]. The channel sensing can be formulated as a hypothesis testing problem between the additive white Gaussian noise $n(i)$ and the primary signal $s_p(i)$ in noise. Noting that there are NB complex symbols in a duration of N seconds, this can be expressed mathematically as follows:

$$H_0 : y(i) = n(i), \qquad i = 1, \cdots, NB; \tag{5}$$

$$H_1 : y(i) = s_p(i) + n(i), \qquad i = 1, \cdots, NB. \tag{6}$$

Hence, it is straightforward to write down the probabilities of false alarm P_f and detection P_d as follows:

$$P_f = Pr(Y > \omega|H_0) = 1 - P\left(\frac{NB\omega}{\sigma_n^2}, NB\right); \tag{7}$$

$$P_d = Pr(Y > \omega|H_1) = 1 - P\left(\frac{NB\omega}{\sigma_{sp}^2 + \sigma_n^2}, NB\right), \tag{8}$$

where ω is the energy detector threshold, $Y = \frac{1}{NB}\sum_{i=1}^{NB}|y(i)|^2$ and $P(x,a)$ denotes the regularized lower gamma function defined as $P(x,a) = \frac{\gamma(x,a)}{\Gamma(a)}$ where $\gamma(x,a)$ is the lower incomplete gamma function. Note that the test statistic Y is chi-square distributed with $2NB$ degrees of freedom.

4 The Effective Capacity of the Relaying Secondary User under the SINR Model

In order to perform EC analysis for the cognitive radio relaying system, the primary activity has to be analytically quantified. Therefore, a queuing analysis for both, primary queue and relay queue, is conducted in this section. Afterwards, we will develop the system Markov chain that characterizes the cognitive user EC.

4.1 Primary User Queue Analysis

We assume that the primary packets arrive according to a Bernoulli arrival process. At each time slot, a new packet arrives with probability $0 \leq \lambda_p \leq 1$. The PU is assumed to send a packet in a time slot as long as its queue is non-empty. Hence, the PU access probability is expressed as $\Pr\{\text{PU accesses}\} = \rho_p$, where ρ_p denotes the probability of a non-empty primary queue. When the PU sends a packet one of three scenarios may arise:

- The packet is successfully received by the PU-RX. In this case, the packet is dropped from the secondary relay queue, if it was successfully received by the SU-TX.
- The packet is successfully received by the SU-TX but not received by the PU-RX. In this case, the packet is stored in the relay queue and dropped from the primary queue.

– The packet is neither received by the PU-RX nor the SU-TX. Hence, the PU will re-transmit it in the next time slot with probability one.

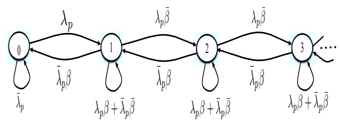

Fig. 2. Primary Queue Markov Chain.

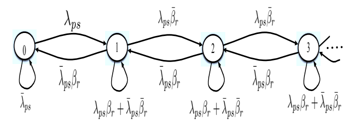

Fig. 3. Relay Queue Markov Chain.

The primary queue is modeled as a discrete-time Markov chain, where λ_p is the packets arrival rate at the PU. β is the service rate of this birth-death primary queue. The Markov chain state χ_n represents that there are n packets in the primary queue in this time slot. The events governing the transitions between states can be summarized as follows:

– $\Pr(\chi_{n+1}|\chi_n)$ means that a new packet is added to the queue due to either a new packet arrival while no packet is served within the same time slot.
– $\Pr(\chi_n|\chi_n)$ means that no new arrivals and no packet is serviced or new packet arrived while another one is successfully served by either the primary channel or the relay channel.
– $\Pr(\chi_n|\chi_{n+1})$ means that no new arrivals while a packet is successfully served by either the primary channel or the relay channel.

Applying the global balance equations at the states of the Markov chain we can characterize ρ_p as a function of β and λ_p as shown in the appendix.

4.2 Relay Queue Analysis

In order to characterize the Effective Capacity of the cognitive relaying user and complete the analysis, we need to characterize the non-empty probability of the relay queue. The arrival rate is λ_{ps} and the service rate is β_r. The relay queue can be also modeled by a birth-death queue, hence, we use similar steps to characterize ρ_r (details are given in the Appendix).

4.3 Modeling the Cognitive Radio Channel

Along the lines of [11], we develop a Markov chain capturing the dynamics of the cognitive radio channel where the state represents the sensing outcome (B-B, MD, FA, I-I) and the channel reliability (ON, OFF), as illustrated next.

Not knowing the channel conditions, the secondary transmitter sends at fixed rates. More specifically, the transmission rate is fixed at r_1 bits/s in the presence of active primary users while the transmission rate is r_2 bits/s when the PU is idle. We initially construct a state-transition model for cognitive transmissions by considering the cases in which the fixed transmission rates are smaller or greater than the instantaneous channel capacity values, and also incorporating the sensing decision and its correctness. In particular, if the fixed rate is smaller than the instantaneous channel capacity, we assume that reliable communication is achieved and the channel is in the ON state. Otherwise, we declare that outage has occurred and the channel is in the OFF state. Note that information has to be retransmitted in such a case.

4.4 State Transition Dynamics

The state transition dynamics of the SU are captured in the Markov chain depicted in Fig. 4. It is an eight states' Markov chain. Each state represents the sensing process outcome and the SU link ON or OFF as discussed next. Regarding the decision of channel sensing and its correctness, we have the following four possible cases: the channel is busy and detected busy (B-B), the channel is busy and detected idle (MD), idle and detected busy (FA), and, finally, the channel is idle and detected idle (I-I). In each case, we have two link outage possibilities, namely ON and OFF, depending on whether the transmission rate exceeds the instantaneous channel capacity or not. In order to identify these states, we have to first determine the instantaneous channel capacity in each time slot. Note that if the channel is detected busy, the secondary transmitter sends packets with power P_1. Otherwise, it transmits with a higher power, P_2. Considering the interference σ_{sp} caused by the primary users as additional Gaussian noise, we can express the instantaneous channel capacities in the above four cases as follows:

$$C_l = B \log_2(1 + SNR_l z(i)), \tag{9}$$

where SNR_l denotes the average signal-to-noise ratio (SNR) for each possible scenario l, where l = 1, 2, 3, 4. It is straightforward to write the SNRs in these four cases, that is $SNR_1 = \frac{P_1}{B(\sigma_n^2+\sigma_{sp}^2)}$, $SNR_2 = \frac{P_2}{B(\sigma_n^2+\sigma_{sp}^2)}$, $SNR_3 = \frac{P_1}{B\sigma_n^2}$ and $SNR_4 = \frac{P_2}{B\sigma_n^2}$. Note that in scenarios 1 and 3, the channel is detected busy and, hence, the transmission rate is r_1 while it is r_2 in scenarios 2 and 4.

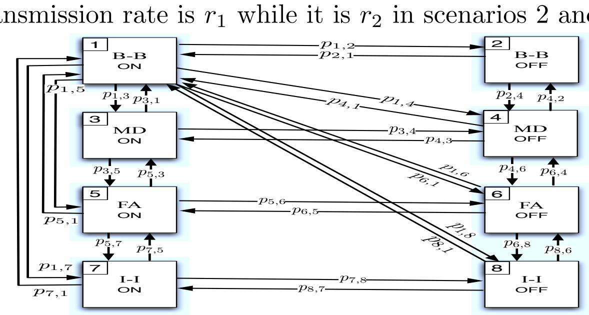

Fig. 4. The Markov chain model for the cognitive radio channel.

If these fixed rates are below the instantaneous capacity values, i.e., if $r_1 < C_1, C_3$ or $r_2 < C_2, C_4$, then the cognitive transmission is considered to be in the ON state where reliable communication is achieved. On the other hand, if $r_1 \geq C_1, C_3$ or $r_2 \geq C_2, C_4$, outage occurs and the secondary user transmission is in the OFF state. In those cases, reliable communication is not attained, and hence, the information has to be resent. It is assumed that a simple automatic repeat request (ARQ) mechanism is incorporated in the communication protocol to acknowledge the reception of data and to ensure that erroneous data is retransmitted. This state-transition model with eight states is depicted in Fig. 4. In states 1, 3, 5, and 7, the cognitive radio channel is in the ON state, and $r_1(T-N)$ bits in states 1 and 5, and $r_2(T-N)$ bits in states 3 and 7 are transmitted and successfully received. On the other hand, the transmission rate is zero in the OFF states.

The above Markov chain is fully characterized by its transition probability matrix $\mathbf{R}_{M\times M}$ defined as:

$$\mathbf{R}_{M\times M} = \left[p_{i,j}\right], 1 \leq i, j \leq M. \tag{10}$$

Given the EC expression in (2) and the state transition model in Fig. 4, the EC can be expressed as follows:[2]

$$EC(\theta) = \frac{\Lambda(-\theta)}{-\theta} = \max_{r1,r2} \frac{1}{-\theta} \log_e sp(\mathbf{\Phi}(-\theta)\mathbf{R}), \tag{11}$$

where the matrix $\mathbf{R}$ is the state transition matrix as defined above, and $sp(\mathbf{\Phi}(-\theta)\mathbf{R})$ is the spectral radius of the matrix $\mathbf{\Phi}(-\theta)\mathbf{R}$, that is, the maximum of the absolute of all eigenvalues of the matrix. Therefore, to reach a closed form expression for the EC, we need to get the eigenvalues of the matrix $\mathbf{\Phi}(-\theta)\mathbf{R}$. $\mathbf{\Phi}(-\theta)$ is a diagonal matrix defined as $\mathbf{\Phi}(-\theta) = diag(\phi_1(-\theta), \phi_2(-\theta), \cdots, \phi_M(-\theta))$ whose diagonal elements are the moment generating functions of the Markov process in each of the M states.

In order to fully characterize the EC, we first characterize the transition probability matrix $\mathbf{R}$ as follows.

$$\begin{aligned} p_{1,1} &= \rho_p P_d \Pr(r_1 < C_1(i+TB)|r_1 < C_1(i)) \\ &= \rho_p P_d \Pr(z(i+TB) > \alpha_1 | z(i) > \alpha_1), \end{aligned} \tag{12}$$

where $\alpha_1 = \frac{2^{\frac{r_1}{B}}}{SNR_1}$, the term $\Pr(r_1 < C_1(i+TB)|r_1 < C_1(i))$ represents the probability that the channel is ON (SU not in outage), ρ_p is the prior probability of the primary channel being busy, P_d is the probability of detection as in (8).

Note that $p_{1,1}$ depends, in general, on the joint distribution of $(z(i+TB), z(i))$. However, since fading changes independently from one block to another in the block-fading model, we can further simplify $p_{1,1}$ to

$$p_{1,1} = \rho_p P_d \Pr(z(i+TB) > \alpha_1) = \rho P_d \Pr(z(i) > \alpha_1)$$

Thus, we can immediately see that the transition probability $p_{1,1}$ does not depend on the original state. Hence, due to the block fading assumption, we can express

$$p_{i,1} = \rho_p P_d \Pr(z(i) \geq \alpha_1) \text{ for } i = 1, 2, \cdots, 8.$$

Similarly, $p_{i,2} = p_2 = \rho_p P_d \Pr(z < \alpha_1)$, $p_{i,3} = p_3 = \rho_p(1-P_d)\Pr(z \geq \alpha_2)$, $p_{i,4} = p_4 = \rho_p(1-P_d)\Pr(z < \alpha_2)$, $p_{i,5} = p_5 = (1-\rho_p)P_f \Pr(z \geq \alpha_3)$, $p_{i,6} = p_6 = (1-\rho_p)P_f \Pr(z < \alpha_3)$, $p_{i,7} = p_7 = (1-\rho_p)(1-P_f)\Pr(z \geq \alpha_4)$ and $p_{i,8} = p_8 = (1-\rho_p)(1-P_f)\Pr(z < \alpha_4)$, where $\alpha_2 = \frac{2^{\frac{r_2}{B}}}{SNR_2}$, $\alpha_3 = \frac{2^{\frac{r_1}{B}}}{SNR_3}$ and

[2] The proof can be found in [12, Ch.7].

$\alpha_4 = \frac{2^{\frac{r_2}{B}}}{SNR_4}$. Hence, the transition probability matrix is constructed as a unit rank matrix:

$$\mathbf{R} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdot & \cdot & p_{1,8} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{8,1} & p_{8,2} & \cdot & \cdot & p_{8,8} \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & \cdot & \cdot & p_8 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_1 & p_2 & \cdot & \cdot & p_8 \end{bmatrix}. \tag{13}$$

4.5 Characterizing the Effective Capacity

If we define Q as the stationary queue length, then θ is defined as the decay rate of the tail distribution of the queue length Q.

Hence, we have the following approximation for the buffer violation probability for large queue lengths, denoted by q_{max}

$$P(Q \geq q_{max}) \approx \exp^{-\theta q_{max}}. \tag{14}$$

Therefore, larger θ corresponds to more strict QoS constraints whereas smaller θ implies looser constraints. In certain settings, constraints on the queue length can be mapped to delay-QoS constraints.

In practical applications, the value of θ depends on the statistical characterization of the arrival and service processes, bounds on the delay or buffer lengths, and the target values of the delay or buffer length violation probabilities.

The effective capacity for a given QoS exponent θ is given by equation (1) where $S(t) = \sum_{k=1}^{t} r(k)$ represents the time accumulated service process and $\{r(k),\ k= 1, 2, \cdots\}$ is the discrete, stationary and ergodic stochastic service process.

Note that the service rate is $r = r_1(T - N)$ if the cognitive system is in state 1 or 5. Similarly, the service rate is $r = r_2(T - N)$ for states 3 and 7.

In OFF states, transmission rates exceed the instantaneous channel capacities and reliable communication is not possible. Hence, their service rates are effectively zero.

The state transition model for both systems under investigation, "Cognitive Relay system" and "No-relay system", is essentially the same. This is attributed to the fact that the cognitive channel (secondary link) has the same dynamics in both systems. However, the EC will be different due to the presence of the secondary relay queue in the cognitive relaying model. Next, we characterize the Effective Capacity of the cognitive relaying system using the state transition model described in the previous subsection.

For the cognitive radio channel with the state transition model described earlier, the spectral radius of $sp(\mathbf{\Phi}(-\theta)\mathbf{R})$ is the rank of this matrix. Hence, the normalized effective capacity in bits/s/Hz is given by

$$\begin{aligned} EC_{relay}(SINR, \theta) = {} & \frac{-(1-\rho_r)}{\theta TB} \log_e((p_1 + p_5)\exp^{-(T-N)\theta r_1} \\ & + (p_3 + p_7)\exp^{-(T-N)\theta r_2} + p_2 + p_4 + p_6 + p_8). \end{aligned} \tag{15}$$

On the other hand, the EC for the baseline "No-Relay system" is derived in [11] and is given by

$$EC_{no-relay}(SINR, \theta) = \frac{-1}{\theta TB} \log_e((p_1 + p_5) \exp^{-(T-N)\theta r_1} + (p_3 + p_7) \exp^{-(T-N)\theta r_2} + p_2 + p_4 + p_6 + p_8). \tag{16}$$

From both equations, (15) and (16), the EC in case of relay system is degraded by the probability of having an empty relay queue. It is obvious since the SU starts to send its own packets only if the relay queue is empty.

5 Numerical Results

In this section, we present numerical results that provide further insights about the effect of relaying on EC of CRNs. We show results for the relaying system and compare it with the baseline system where the SU has no relaying capability [11]. The numerical values used for the system parameters are as follows: $SNR_1 = 6.9$ db , $SNR_2 = 10$ db, $SNR_3 = 30.7$ db, $SNR_4 = 40$ db, $k_p = 0.6$, $k_{ps} = 0.2$, $k_r = 0.4$ if the PU is active,$k_r = 0.2$ if the PU is idle, $r_1 = 1000$ bps, $r_2 = 6000$ bps and $lambda_p = 0.38$. We also set $T = 0.1$ sec, $N = 0.026$ sec, $\lambda = 1.7$ and $B = 1000$ Hz. Note that the optimal values for r_1 and r_2 are obtained by simple numerical search such that, EC is maximized.

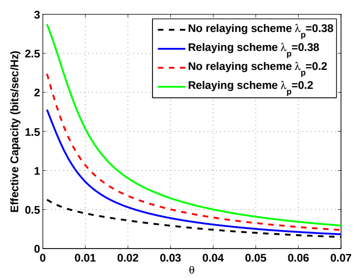

Fig. 5. SU EC of the Relay system and the no-relay system.

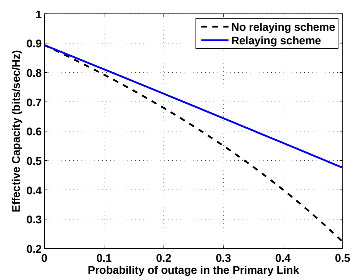

Fig. 6. EC of the Relay system and the no-relay system versus primary link outage probability.

In Fig. 5, we plot the SU EC for the cognitive relaying and the no-relay (baseline) system versus the, statistical QoS constraint, delay exponent θ for a sensing duration of $N = 0.026$ and for two values of the PU packet arrival rate. Clearly, as the delay exponent θ increases (stricter delay requirements), the effective capacity (the maximum rate that the channel can sustain in bit/sec/hertz) decreases. The same result can be easily distilled from the EC definition in (2). Moreover, it is shown that when the SU helps relaying the "unsuccessful" PU packets, for the set of outage probabilities given before, the secondary user attains higher EC. As θ increases, the performance gain decreases since stricter QoS constraints limits the secondary user throughput. Finally, it is intuitive to notice that the SU EC decreases as the primary user become more active in accessing the medium due to higher packet arrival rate, λ_p.

Next, we investigate the system behavior versus different link outage probabilities. It is worth noting that we only plotted the EC versus the outage probability values that preserve the system queues stability as explained in the appendix.

In Fig. 6, the EC is plotted versus k_p while fixing other outage probabilities like $k_{ps} = 0.2$ and $k_r = 0.4$. Similarly, we investigated the effect of k_{ps} and k_r on the cognitive user EC as shown in Fig. 7 and 8, while fixing other probabilities of outage. The No relay system EC remains constant over different outage probabilities for k_{ps} and k_r, while the Relay system gains a significant increase in terms of its EC which decays under high outage probabilities. In Fig. 8 relaying is not giving any chance to the SU to send his own packets when the outage probabilities k_r exceeds 0.92 due to multiple retransmissions. It is clear that we can always obtain higher EC by adding a relay capability to the SU and helping the PU to send more packets as well.

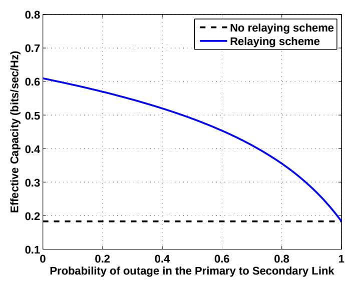

Fig. 7. EC of the Relay system and the no-relay system versus primary-secondary link outage probability

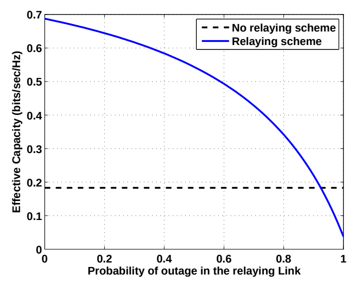

Fig. 8. EC of the Relay system and the no-relay system versus relaying link outage probability

6 Conclusion and Future Work

In this paper we study a two-link network, a primary network abstracted to a single primary link, a single secondary link with relaying capability. We show that exploiting the packet relaying capability at the secondary transmitter improves the EC of the secondary user. It is shown that under the "SINR Interference" model the SU can increase its chance to find an idle medium reaching a win-win situation with the PU sharing that medium. This work can be extended to investigate the case of multiple secondary users with cognitive relaying capabilities. In the future, we can also find a power allocation protocol to reduce the cognitive network power consumption under an EC lower bound constraint.

Appendix

Given the Markov chain in Fig. 2, to characterize the non-empty queue probability, we apply the global balance equation (GBE) on each state. Let β is the service rate of Q_p (let χ_i denote $\Pr(\chi_i)$). Applying the GBE at state 0:

$$\chi_0 \lambda_p = \chi_1 \bar{\lambda_p} \beta \;\rightarrow\; \chi_1 = \frac{\lambda_p}{\bar{\lambda_p}\beta} \chi_0. \tag{17}$$

Applying the balance equation at state 1,

$$\chi_1(\lambda_p \bar{\beta} + \bar{\lambda_p}) = \chi_0 \lambda_p + \chi_2 \bar{\lambda_p} \beta \rightarrow \chi_2 = \frac{1}{\beta} \left(\frac{\lambda_p \bar{\beta}}{\bar{\lambda_p} \beta} \right)^2 \chi_0. \tag{18}$$

Recursively, $\chi_i = \frac{1}{\beta}\left(\frac{\lambda_p\bar{\beta}}{\bar{\lambda_p}\beta}\right)^i \chi_0 \quad \forall i \geq 1$. Since $\sum_{i=1}^{\infty} \chi_i = 1$, we can calculate χ_0 with some manipulations. To ensure queue stability, we must have $\lambda_P\bar{\beta} \leq \bar{\lambda_p}\beta$, hence,

$$\chi_0 = \left[1 + \frac{\lambda_p}{(\bar{\lambda_p}\beta - \lambda_p\bar{\beta})}\right]^{-1}. \tag{19}$$

After some mathematical manipulations we can express χ_0 as:

$$\chi_0 = 1 - \frac{\lambda_p}{\beta}. \tag{20}$$

Finally we have $\rho_p = 1-\chi_0$. Where the service rate $\beta = 1-k_p \times k_{ps}$. Similarly, one can characterize, ρ_r. Then $\beta_r = (1-k_r|PUactive) \times \rho_p + (1-k_r|PUidle) \times (1-\rho_p)$. Hence we can write $\rho_r = \frac{\lambda_{Ps}}{\beta_r}$.

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