Full Duplex in Massive MIMO Systems: Analysis and Feasibility

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Abstract—In this paper, we study full duplex (FD) massive multiple-input multiple-output (MIMO) systems which are expected to result in a significant increase in the network's efficiency. We consider a base station equipped with massive MIMO which can operate either in FD or half duplex (HD). We start by studying the performance of the network when the number of antennas grows asymptotically. We derive bounds for both the uplink and downlink capacities. Afterwards, we find the necessary conditions to be satisfied for the base station to work in FD in terms of the number of transmitting antennas, co-channel interference and self-interference cancellation. Simulations results validate the derived capacity bounds and thresholds obtained for FD communication.

I. INTRODUCTION

Next generation communication networks are required to provide a significant increase in the achieved data rates. Accordingly, deploying different techniques that help give the necessary performance boost is essential. Theoretically speaking, full duplex (FD) can double the data rate achieved by half duplex (HD) transmission because it allows each node to simultaneously transmit and receive at the same time and frequency resources. However, the main hurdle that limits the capacity gain is the high self-interference (SI) caused by the node's transmission on the node's reception. Recently, the evolution in the SI cancellation has revived the attention to FD communication [1]–[3]. Additionally, large scale multiple-input multiple-output (MIMO) or massive MIMO [4]–[7] has recently emerged as a promising technique that increases the spectral efficiency and communication reliability than the traditional MIMO. The core idea of massive MIMO is to achieve network densification by largely increasing the number of active antennas. It is particularly fascinated that with a large number of antennas, the simplest form of user detection beamforming becomes optimal. However, it must be remarked that massive MIMO is feasible only in time division duplexing (TDD) systems because of the large number of channels that need to be estimated for user detection or beamforming [1].

[1]The advantage of TDD systems is that we only need to estimate the uplink channels and obtain the downlink channels assuming channel reciprocity.

Consequently, it is expected that utilizing both FD and massive MIMO can greatly enhance the communication network's capacity. However, as a result of the different factors that affect the FD performance, we must be very decisive when outlining the needed conditions to be satisfied for FD massive MIMO to be more efficient than HD massive MIMO. Actually, studying the performance of FD and massive MIMO networks has attracted much recent research work. In [8], an investigation of the effect of deploying massive MIMO in two-tier cellular networks is presented. Furthermore, a resource allocation problem is formulated to find the optimal users' biasing that maximizes the total system capacity. In [9], the utilization schemes of the spatial resources which can be used to enhance the uplink (UL) performance are considered for large SI on the UL reception. In [10], the ratio of the transmit antennas and receive antennas in FD massive MIMO is analyzed. It is shown that the optimal antenna ratio converges to the ratio between the numbers of DL users and UL users.

In this paper, we consider a single cell network with one massive MIMO base station (BS) which can operate in FD or HD and multiple downlink (DL) and UL users. To the best of our knowledge, all the previous work that addressed FD massive MIMO was concerned about deriving a ratio between the transmitting and receiving antennas that maximizes either the UL or DL FD capacity [10], [11]. Therefore, our motivation is to compare both the HD and FD massive MIMO performance and determine the necessary conditions to be satisfied under which it will be beneficial for the network to operate in the FD mode. Our contributions in this paper are summarized as follows:

- Investigating the difference between the different transmission modes in terms of the received power, co-channel interference (CCI), and multiuser interference (MUI).
- Analyzing the asymptotic network performance when the number of antennas grows.
- Deriving upper and lower bounds for the DL and the UL capacities, respectively, for both FD and HD communications.
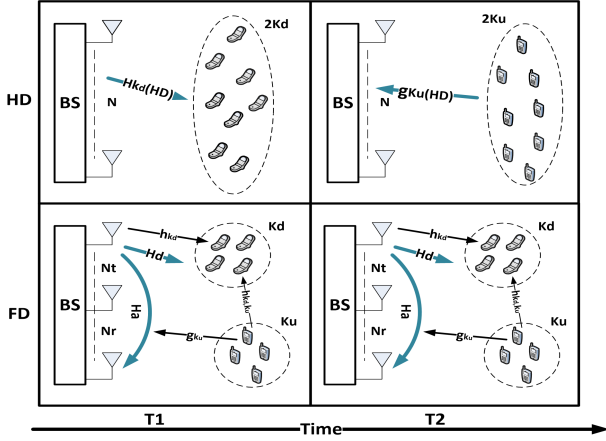
Fig. 1. System Model.

- Determining the necessary conditions to be satisfied for the FD mode to outperform the HD mode.

The remainder of this paper is organized as follows. The system model is presented in Section II. The asymptotic performance analysis is presented in Section III. Numerical results are presented in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

We consider a single cell network [2] with N antennas BS, which operates either in FD or HD, and serves multiple single antenna users. In the case of the FD transmission, the BS will assign N_t antennas for the DL transmission and N_r antennas for the UL reception, such that $N = N_t + N_r$. On the other hand, in the case of HD transmission, the BS will assign all of its N antennas either for transmission or reception. Additionally, when operating in FD, each time slot will be assigned to simultaneously serve K_d DL users and K_u UL users. However, when operating in HD, the system will operate in TDD, in which the first time slot will be assigned for the DL transmission of $2K_d$ DL users and the second time slot will be assigned for the UL transmission of $2K_u$ UL users. In other words, regardless of the transmission mode, the network will serve $2K_d$ DL users and $2K_u$ UL users in two successive time slots. The system model is shown in Fig. 1.

For massive MIMO deployment, it is assumed that $N_t \gg K_d$, and $N_r \gg K_u$. Furthermore, it is assumed that, in DL transmission, the BS deploys the ZF precoder. However, it must be mentioned that in case of deploying FD, the ZF precoder will account for the large SI from the BS transmitting antennas, and

hence, it will consider the BS receiving antennas as additional transmission directions on which the transmitter should limit the interference.

Based on the previous assumptions, we are going to illustrate the difference between both the FD and HD operations in the massive MIMO setting.

A. FD Transmission

As mentioned before, in the case of FD transmission, each time slot will simultaneously serve the transmission of K_d DL users and K_u UL user. Accordingly, the received signal-to-interference noise ratio (SINR) of the k_d^{th} DL user is given by

$$\Gamma_{DL|FD} = \frac{P_d\|\mathbf{h}_{k_d}\mathbf{w}_{k_d}\|^2}{\sigma^2 + \sum_{\substack{j=1 \\ j \neq k_d}}^{K_d} P_d\|\mathbf{h}_{k_d}\mathbf{w}_j\|^2 + \sum_{k_u=1}^{K_u} P_u\|h_{k_d,k_u}\|^2}, \quad (1)$$

where, P_d is the DL transmission power, $\mathbf{h}_{k_d} \in \mathbb{C}^{1 \times N_t}$ is the channel vector between the k_d^{th} DL user and the BS [3]. All channel coefficients are assumed to be independent and identically distributed (i.i.d.) zero mean complex Gaussian random variables with unit variance, i.e., Rayleigh flat fading channel model. $\mathbf{w}_{k_d} \in \mathbb{C}^{N_t \times 1}$ is the precoding vector for the k_d^{th} DL user's data. In this paper, we consider the ZF beamforming in which the precoding matrix $\mathbf{W}_{ZF} \in \mathbb{C}^{N_t \times (N_r + K_d)}$ is given by

$$\mathbf{W}_{ZF} = \left(\mathbf{H}_t^H \mathbf{H}_t\right)^{-1} \mathbf{H}_t^H, \quad (2)$$

where, $\mathbf{H}_t = \begin{bmatrix} \mathbf{H}_d & \mathbf{H}_a \end{bmatrix}^T \in \mathbb{C}^{(N_r + K_d) \times N_t}$ is the channel matrix which is composed of the DL channel matrix $\mathbf{H}_d = [\mathbf{h}_1 \ \mathbf{h}_2 \cdots \mathbf{h}_{K_d}]^T \in \mathbb{C}^{K_d \times N_t}$ and the channel between the transmitting and receiving antennas $\mathbf{H}_a \in \mathbb{C}^{N_r \times N_t}$. Furthermore, σ^2 is the additive white Gaussian noise variance. The second term in the denominator of the last expression is the MUI on the k_d^{th} DL user's transmission from other DL transmissions. Finally, the last term in the denominator of the last expression is the CCI on the k_d^{th} DL user's transmission from the UL transmissions, where P_u is the UL transmission power and h_{k_d,k_u} is the channel coefficient between the k_u^{th} UL user and the k_d^{th} DL user. Similarly, the received SINR from the k_u^{th} UL user is given by

$$\Gamma_{UL|FD} = \frac{P_u\|\mathbf{g}_{k_u}\|^2}{\sigma^2 + \sum_{\substack{i=1 \\ i \neq k_u}}^{K_u} P_u\|\mathbf{g}_i\|^2 + CP_d\|\mathbf{H}_a\mathbf{W}_a\|^2}, \quad (3)$$

[3]Throughout the paper, $\mathbf{h}_x$ denotes the channel vector between the x DL user and the BS

where, $\mathbf{g}_{k_u} \in \mathbb{C}^{N_r \times 1}$ is the channel vector between the k_u^{th} UL user and the BS, the second term in the denominator of the last expression is the MUI on the k_u^{th} UL user from other UL transmissions, the last term in the denominator of the last expression is the self-interference power, $0 \leq C \leq 1$ is the SI cancellation coefficient [9], [10] [4], and $\mathbf{W}_a$ includes all the precoding vectors corresponding to the BS receiving antennas. From(1)-(3), the total FD ergodic network DL and UL capacities per unit bandwidth are given, respectively, by

$$
\begin{aligned}
C_{DL|FD} &= 2K_d \mathbb{E}\{\log_2\left(1 + \Gamma_{DL|FD}\right)\}, \\
C_{UL|FD} &= 2K_u \mathbb{E}\{\log_2\left(1 + \Gamma_{UL|FD}\right)\}
\end{aligned} \quad (4)
$$

It must be mentioned that, from (4), it is assumed that, due to the large number of antennas, all the DL users' SINRs will have the same ergodic capacity. Similarly, we assume that all the UL users have the same ergodic capacity [14].

B. HD Transmission

As mentioned before, in the case of HD transmission, the first time slot will be assigned for $2K_d$ DL transmissions and the second time slot will be assigned for $2K_u$ UL transmissions. Accordingly, the received SINR at the k_d^{th} DL user is given by

$$
\Gamma_{DL|HD} = \frac{P_d \|\mathbf{h}_{k_d}^{HD} \mathbf{w}_{k_d}^{HD}\|^2}{\sigma^2 + \sum\limits_{\substack{j=1 \\ j \neq k_d}}^{2K_d} P_d \|\mathbf{h}_{k_d}^{HD} \mathbf{w}_j^{HD}\|^2}, \quad (5)
$$

where, $\mathbf{h}_{k_d}^{HD} \in \mathbb{C}^{1 \times N}$ is the channel vector between the k_d^{th} DL user and the BS in case of HD transmission, $\mathbf{w}_{k_d}^{HD} \in \mathbb{C}^{N \times 1}$ is the precoding vector for the k_d^{th} DL user's data in the case of HD transmission. The precoding matrix for the HD transmission is given by

$$
\mathbf{W}_{ZF}^{HD} = \left(\mathbf{H}_{HD}^H \mathbf{H}_{HD}\right)^{-1} \mathbf{H}_{HD}^H, \quad (6)
$$

where, $\mathbf{H}_{HD} \in \mathbb{C}^{(2K_d) \times N}$ is the total DL channel matrix for the HD transmission, the summation in the denominator of (5) is the MUI on the k_d^{th} DL user's transmission from other DL transmissions. Similarly, the received SINR at the k_u^{th} UL user is given by,

$$
\Gamma_{UL|HD} = \frac{P_u \|\mathbf{g}_{k_u}^{HD}\|^2}{\sigma^2 + \sum\limits_{\substack{i=1 \\ i \neq k_u}}^{2K_u} P_u \|\mathbf{g}_i^{HD}\|^2}, \quad (7)
$$

where, $\mathbf{g}_{k_u}^{HD} \in \mathbb{C}^{M \times 1}$ is the channel vector between the k_u^{th} UL user and the BS in the HD transmission, the summation in the denominator of (7) is the multiuser interference on the k_u^{th} UL

[4]As the value of C decreases, better SI cancellation is achieved

user from other UL transmissions in the HD transmission. From (5)-(7), the total HD ergodic network DL and UL capacities per unit bandwidth are given by

$$
\begin{aligned}
C_{DL|HD} &= 2K_d \mathbb{E}\{\log_2\left(1 + \Gamma_{DL|HD}\right)\}, \\
C_{UL|HD} &= 2K_u \mathbb{E}\{\log_2\left(1 + \Gamma_{UL|HD}\right)\}
\end{aligned} \quad (8)
$$

III. Asymptotic Analysis of Full Duplex and Half Duplex Capacity

In this section, we will derive upper bounds for the DL capacities for the FD mode and the HD mode. Additionally, we will derive lower bounds for the UL capacities of both the FD and the HD modes.

Before we proceed, we will recall some results from large random matrix theory. First, let $\mathbf{p} \triangleq [p_1 \cdots p_n]^T$ and $\mathbf{q} \triangleq [q_1 \cdots q_n]^T$ be mutually independent $n \times 1$ random vectors. The entries of $\mathbf{p}$ and $\mathbf{q}$ are i.i.d. zero-mean random variables such that $\mathbb{E}\{|p_i|^2\} = \sigma_p^2$ and $\mathbb{E}\{|q_i|^2\} = \sigma_q^2$, $i = 1, \cdots n$. Then,

$$
\frac{1}{n}\mathbf{p}^H \mathbf{p} \xrightarrow[n \to \infty]{a.s} \sigma_p^2, \quad \frac{1}{n}\mathbf{p}^H \mathbf{q} \xrightarrow[n \to \infty]{a.s} 0. \quad (9)
$$

Second, if $\mathbf{B} = \mathbf{A}\mathbf{A}^H$, where $\mathbf{A}$ is an $m \times n$ random matrix whose columns are zero-mean independent complex Gaussian vectors with covariance matrix $\mathbf{I}$. Then, for $n \geq m$, $\mathbf{B}$ is a central complex *Wishart* matrix with probability distribution $\mathbf{B} \sim \mathcal{W}(n, \mathbf{I})$, where n indicates the degrees of freedom. For a central Wishart matrix $\mathbf{B} \sim \mathcal{W}(n, \mathbf{I})$ with $n \geq m$,

$$
\mathbb{E}\big[\text{tr}\{\mathbf{B}^{-1}\}\big] = \frac{m}{n - m}. \quad (10)
$$

Finally, assume $\mathbf{B}$ is an $m \times n$ random matrix whose columns are zero-mean independent complex Gaussian vectors with identity covariance matrix. It is shown in [15] that as m grows without bound, we have

$$
\frac{1}{m}\mathbf{B}^H \mathbf{B} \xrightarrow{a.s} \mathbf{I}_n, \quad (11)
$$

where, $\mathbf{I}_n$ is an $n \times n$ identity matrix.

Based on the identity described in (11), the precoding matrices $\mathbf{W}_{ZF}$, and $\mathbf{W}_{ZF}^{HD}$ can be approximated, respectively, as

$$
\hat{\mathbf{W}}_{ZF} = \frac{\mathbf{H}_t^H}{N_r + K_d}, \quad \hat{\mathbf{W}}_{ZF}^{HD} = \frac{\mathbf{H}_{HD}^H}{2K_d}. \quad (12)
$$

Proposition 1 *An upper bound for the FD DL rate $\hat{C}_{DL|FD}$ is given by*

$$
\hat{C}_{DL|FD} = 2K_d \log_2 \left(1 + \left(\frac{N_t}{N_r + K_d}\right)^2 \lambda_d \Psi(\lambda_u)\right), \quad (13)
$$

where $\lambda_d = \frac{P_d}{\sigma^2}$, $\lambda_u = \frac{\sigma^2}{P_u}$ and $\Psi(\lambda_u)$ is given by

$$
\Psi(\lambda_u) = \frac{\lambda_u}{(K_u - 1)!} \Bigg[(-1)^{K_u - 2} e^{\lambda_u} \mathbf{E_i}(\lambda_u) \\
+ \sum_{l=1}^{K_u - 1} (l - 1)!(-1)^{K_u - l - 1} (\lambda_u)^{-l} \Bigg], \quad (14)
$$

where $\mathbf{E_i}(\lambda_u) = \int_{\lambda_u}^{\infty} e^{-x}/x \; dx$.

Proof: From (4), and by applying the Jensen's inequality to the concave function, we can get

$$
\begin{aligned}
C_{DL|FD} &\leq \hat{C}_{DL|FD} \\
&= 2K_d \log_2 \left(1 + \mathbb{E}\left\{\Gamma_{DL|FD}\right\}\right) \\
&= 2K_d \log_2 \left(1 + \left(\frac{N_t}{N_r + K_d}\right)^2 \lambda_d \right. \\
&\quad \times \left. E\left\{\frac{1}{1 + \frac{1}{\lambda_u}\sum_{k_u=1}^{K_u} \|h_{k_d,k_u}\|^2}\right\}\right), \\
&= 2K_d \log_2 \left(1 + \left(\frac{N_t}{N_r + K_d}\right)^2 \lambda_d \Psi(\lambda_u)\right),
\end{aligned}
\tag{15}
$$

where the second equality is obtained by substituting (12) and (9) in (1). As a result, the expectation of the main signal is given by

$$
\mathbb{E}\left\{P_d\|\mathbf{h}_{k_d}\mathbf{w}_{k_d}\|^2\right\} = P_d\left(\frac{N_t}{N_r + K_d}\right)^2,
\tag{16}
$$

and the multiuser interference will vanish. Afterwards, it was proved in [10] that $\mathbb{E}\left\{\frac{1}{1 + \frac{1}{\lambda_u}\sum_{k_u=1}^{K_u} \|h_{k_d,k_u}\|^2}\right\} = \Psi(\lambda_u)$ defined in (14). ∎

Proposition 2 *A lower bound for the FD UL rate $\hat{C}_{UL|FD}$ is given by*

$$
\hat{C}_{UL|FD} = 2K_u \log_2 \left(\frac{P_u(N_r - 1)}{\sigma^2 + N_r(K_u - 1)P_u + \frac{N_r N_t}{(N_r + K_d)^2}CP_d}\right).
\tag{17}
$$

Proof: By applying the Jensen's inequality for the convex function $\log_2(1 + 1/x)$, we can get that

$$
\begin{aligned}
C_{UL|FD} &\geq \hat{C}_{UL|FD} \\
&= 2K_u \log_2 \left(1 + \right. \\
&\quad \left(\mathbb{E}\left\{\frac{\sigma^2 + MUI|_{UL|FD} + SI|_{UL|FD}}{P_u\|\mathbf{g}_{k_u}\|^2}\right\}\right)^{-1}\right), \\
&= 2K_u \log_2 \left(1 + (\Phi_1 \Phi_2)^{-1}\right), \\
&= 2K_u \log_2 \left(1 + \right. \\
&\quad \left.\frac{P_u D_{k_u}^{-\alpha}(N_r - 1)}{\sigma^2 + N_r(K_u - 1)P_u + \frac{N_r N_t}{(N_r + K_d)^2}CP_d}\right)
\end{aligned}
\tag{18}
$$

where, $MUI|_{UL|FD}$ is the MUI on the FD UL transmission, $SI|_{UL|FD}$ is the SI on the FD UL transmission, $\Phi_1 = \mathbb{E}\left\{1/P_u D_{k_u}^{-\alpha}\|\mathbf{g}_{k_u}\|^2\right\}$, and $\Phi_2 = \mathbb{E}\left\{\sigma^2 + MUI|_{UL|FD} + SI|_{UL|FD}\right\}$. The second equality is derived from the fact that $\mathbf{g}_{k_u}$ is independent of $\mathbf{g}_i$ $\forall i \neq k_u$, and $\mathbf{H}_a$ that influences $MUI|_{UL|FD}$ and $SI|_{UL|FD}$, respectively. The expected value of $1/\|\mathbf{g}_{k_u}\|^2$ can be calculated using the Wishart matrix property illustrated in (10). The expected value of the multiuser interference power on the UL transmission can be calculated as follows

$$
\begin{aligned}
\mathbb{E}\{MUI|_{UL|FD}\} &= \sum_{\substack{i=1 \\ i \neq k_u}}^{K_u} P_u D_i^{-\alpha}\mathbb{E}\left\{\|\mathbf{g}_i\|^2\right\}, \\
&= \sum_{\substack{i=1 \\ i \neq k_u}}^{K_u} P_u\mathbb{E}\left\{\text{tr}\left[\mathbf{g}_i^H \mathbf{g}_i\right]\right\} \\
&= N_r(K_u - 1)P_u,
\end{aligned}
\tag{19}
$$

where the last equality is obtained by applying the identity in (9). Finally, the value of $\mathbb{E}\left\{SI|_{UL|FD}\right\}$ can be derived as follows

$$
\begin{aligned}
\mathbb{E}\left\{SI|_{UL|FD}\right\} &= CP_d\mathbb{E}\left\{\|\mathbf{H}_a\mathbf{W}_a\|^2\right\}, \\
&= CP_d\mathbb{E}\left\{\text{tr}\left[\mathbf{W}_a^H\mathbf{H}_a^H\mathbf{H}_a\mathbf{W}_a\right]\right\}, \\
&= CP_d M_r\mathbb{E}\left\{\text{tr}\left[\mathbf{W}_a^H\mathbf{W}_a\right]\right\}, \\
&= \frac{N_r N_t}{(N_r + K_d)^2}CP_d,
\end{aligned}
\tag{20}
$$

where the last equality is obtained by applying both (11) and (12) ∎

Proposition 3 *An upper bound of the HD DL rate $\hat{C}_{DL|HD}$ and a lower bound for the HD UL rate $\hat{C}_{UL|HD}$ are given, respectively, by*

$$
\hat{C}_{DL|HD} = 2K_d \log_2 \left(1 + \left(\frac{N}{2K_d}\right)^2 \lambda_d\right),
\tag{21}
$$

$$
\hat{C}_{UL|HD} = 2K_u \log_2 \left(\frac{P_u(N - 1)}{\sigma^2 + N(2K_u - 1)P_u}\right).
\tag{22}
$$

Proof: The expressions in (21) and (22) can be proved following the same steps used in Propositions 1 and 2. ∎

From (13)-(20), it is noticed that FD DL transmission is still affected by the CCI. On the other hand, the FD UL transmission is still affected by the SI represented by the cancellation parameter C. Therefore, it is required to find the conditions under which the FD operation can outperform the HD operation. Since, in the practical communication network, it is always required to achieve more capacity in the DL transmission than the UL transmission, we start by defining the necessary conditions under which the DL FD capacity calculated in (13) is higher than the DL HD capacity calculated in (21).

Proposition 4 *In order to have $\hat{C}_{DL|FD} \geq \hat{C}_{DL|HD}$ the following conditions must be satisfied*

$$
\begin{aligned}
\tilde{N}_t &\geq \frac{N(N + K_d)}{N + 2K_d\sqrt{\Psi(\lambda_u)}}, \\
\tilde{N}_r &< N - \frac{N(N + K_d)}{N + 2K_d\sqrt{\Psi(\lambda_u)}} \\
\zeta &\leq 3\sigma^2,
\end{aligned}
\tag{23}
$$

where $\tilde{N}_t$ and $\tilde{N}_r$ are the number of transmission and receive antennas, respectively, that make $\hat{C}_{DL|FD} \geq \hat{C}_{DL|FD}$ and $\zeta = \mathbb{E}\{\sum_{k_u=1}^{K_u} P_u\|h_{k_d,k_u}\|^2\}$ is the expectation of the CCI on the FD DL transmission.

Proof: We begin our proof by calculating the number of transmission antennas $\tilde{N}_t$ that will make $\hat{C}_{DL|FD} \geq \hat{C}_{DL|FD}$. Therefore, we have

$$\left(\frac{\tilde{N}_t}{N_r + K_d}\right)^2 \lambda_d \Psi(\lambda_u) \geq \left(\frac{N}{2K_d}\right)^2 \lambda_d$$

$$\frac{\tilde{N}_t}{N_r + K_d} \geq \frac{1}{\sqrt{\Psi(\lambda_u)}} \left(\frac{N}{2K_d}\right). \quad (24)$$

After some mathematical manipulations, we can get the values of $\tilde{N}_t$ and $\tilde{N}_r$ defined in (24).

However, it must be noted that for (24) to be valid, the value of $\tilde{N}_t$ must be smaller than N. Hence, we get

$$\frac{N(N + K_d)}{N + 2K_d\sqrt{\Psi(\lambda_u)}} \leq N \longrightarrow \Psi(\lambda_u) \geq \frac{1}{4}. \quad (25)$$

Therefore, from the definition of $\Psi(\lambda_u) = \mathbb{E}\{1/1 + \frac{CCI|FD}{\sigma^2}\}$, where $CCI|FD$ is the CCI in the FD transmission, and Jensen's inequality for convex function, we can obtain the last constraint that $\zeta \leq 3\sigma^2$. ∎

Afterwards, after setting N_t and N_r to values satisfying $\tilde{N}_t$ and $\tilde{N}_r$, respectively, we will derive the SI cancellation threshold C^* that causes the FD UL capacity to be higher than the HD UL capacity

Proposition 5 *The SI cancellation threshold C^* that makes $\hat{C}_{UL|FD} \geq \hat{C}_{UL|HD}$ is given by*

$$C^* = \min(1, \hat{C}), \quad (26)$$

where

$$\hat{C} = (N_r + K_d)^2 \times \left(\frac{(N_r - 1)(\sigma^2 + M_{HD}) - (N - 1)(\sigma^2 + M_{FD})}{P_d N_r N_t (N - 1)}\right), \quad (27)$$

where $M_{HD} = \mathbb{E}\{MUI|_{UL|HD}\}$, $M_{FD} = \mathbb{E}\{MUI|_{UL|FD}\}$, are the expectations of the multiuser interference on the UL transmission in the case of FD and HD, respectively. It must be noted that C can not exceed the value of one. Therefore, we must always pick the minimum value between 1 and $\hat{C}$. [5]

Proof: The expression in (26) can be easily proved by from the expressions of $\hat{C}_{UL|FD}$ and $\hat{C}_{UL|HD}$. ∎

IV. NUMERICAL ANALYSIS

In this section, we validate the upper and lower bounds derived for the DL and UL capacities. Afterwards, we verify the obtained thresholds in Propositions 4 and 5. Fig. 2 validates the values of $\hat{C}_{DL|FD}$ and $\hat{C}_{UL|FD}$ derived in (13) and (17), respectively. Additionally, it shows the variation of both the DL and UL FD capacities with the UL transmission power P_u. It can be anticipated that increasing P_u increases the CCI interference on the DL transmission and then $C_{DL|FD}$ will decrease. On the other hand, increasing P_u will increase $C_{UL|FD}$. This behavior can be verified from Fig. 2. However, it can be seen that increasing P_u causes a slight increase in $C_{UL|FD}$. This response can be explained from (17) as increasing P_u increases both
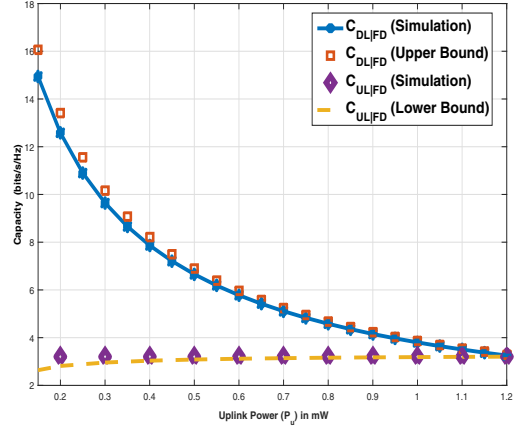

Fig. 2. Variation of Full Duplex Downlink and Uplink Capacities with Uplink Transmission Power
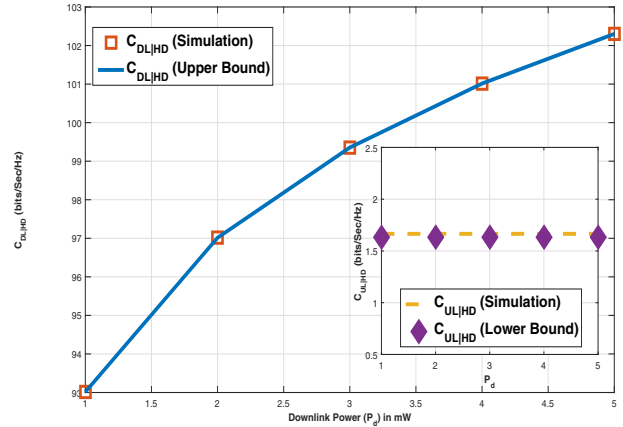

Fig. 3. Variation of Half Duplex Downlink and Uplink Capacities with Downlink Transmission Power

the UL signal power and the MUI power as well. Similarly, Fig. 3 validates the values of $\hat{C}_{DL|HD}$ and $\hat{C}_{UL|HD}$ derived in (21) and (22), respectively. Furthermore, it can be verified that increasing P_d increases the value of the HD DL capacity. On the other hand, it will have no impact on the value of the HD UL capacity.

Fig. 4 validates the value of $\tilde{N}_t$ and ζ derived in (23). It shows the variation $\hat{C}_{DL|FD}$ with increasing the number of transmission antennas N_t for different values of $\Psi(\lambda_u)$. It can be seen that, at $\Psi(\lambda_u) = 1$, which corresponds to the case in which there is no CCI, $\hat{C}_{DL|FD}$ equals $\hat{C}_{DL|HD}$ at $\tilde{N}_t = 253$ which matches the value calculated in (23). Similarly, in the case of $\Psi(\lambda_u) = 1/2$, the value of $\tilde{N}_t = 279$ matches the value calculated in (23)[6]. However, in the case of $\Psi(\lambda_u) = 1/6$, which corresponds to the case in which $\zeta > 3\sigma^2$, it can be noticed that $\hat{C}_{DL|FD}$ will be always smaller than $\hat{C}_{DL|HD}$. This result verifies the derived threshold for CCI expectation in (23).

Fig. 5 shows the variation of $\hat{C}_{UL|FD}$ with SI cancellation for

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[5]The results obtained in Preposition 5 indicate that the SI threshold is related to a weighted difference between the MUI experienced in HD and FD. This relation can be easily concluded because in order for FD to achieve better UL capacity, the SI must be canceled in a way that compensates for the difference between the MUI experienced in the two transmission modes.

[6]Increasing the value of CCI will degrade the DL SINR which will require increasing the transmission antennas in order to be able to compensate for that degradation.
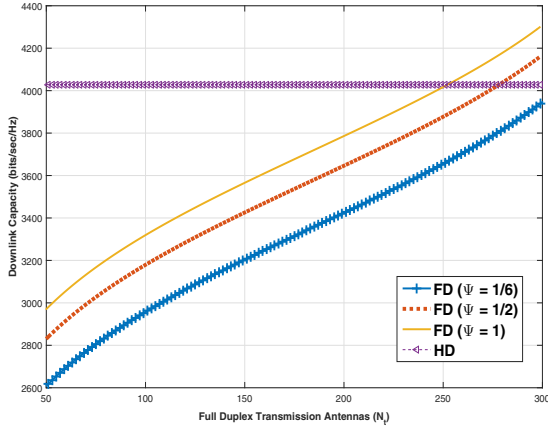
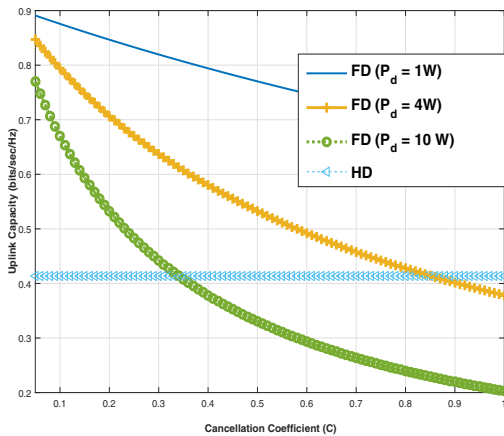Fig. 4. Validating the values of $\tilde{N}_t$ and ζ

Fig. 5. Validating the value of C^*

after setting the number of transmitting antennas, it is shown that the uplink full duplex rate will exceed that of half duplex when the self-interference cancellation is below a certain limit. Finally, we provide the numerical analysis to validate the derived capacity bounds and the required conditions for full duplex operation gains.

VI. Acknowledgment

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different values of P_d. The results shown validate the value of C^* derived in (26). In case of $P_d = 10W$, the value of $C^* = 0.34$ which matches the value calculated in (26). Additionally, decreasing P_d to $4W$ is expected to increase $\hat{C}_{UL|FD}$ and therefore the value of C^* will be larger as the quality of SI cancellation can be decreased for small values of P_d. This behavior can be verified from Fig. 5, as decreasing P_d from $10W$ to $4W$ increases the value of C^* from 0.34 to 0.85. However, further decrease in the value of P_d will make $\hat{C}_{UL|FD}$ always better than $\hat{C}_{UL|HD}$, and hence no cancellation will be required, i.e., $C^* = 1$.

V. Conclusion

In this paper, we consider a single network with a single massive MIMO base station that can operate either in full duplex or half duplex. We analyze the asymptotic behavior of the network when the number of BS antennas grows and investigate the necessary conditions under which the network operates in full duplex mode to achieve a higher capacity than the half duplex mode. It was shown that the full duplex downlink rate exceeds that of half duplex only if the expected value of the co-channel interference is below a certain value, and the number of transmitting antennas are set above a certain threshold. Furthermore,