# Full-Duplex Meets Multiuser MIMO: Comparisons and Analysis

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***Abstract*—Efficient utilization of wireless communication resources and increases in the capacity of the communication networks have become very crucial claims for next-generation cellular networks. Full-duplex (FD) is a very promising technique that allows for the efficient use of wireless communication resources, given that the self-interference level can be suppressed to an acceptable level. In this paper, we consider a simple FD communication system, consisting of one FD access point (AP) and two half-duplex (HD) mobile users, and investigate when it is more advantageous for the communication network to operate in FD or HD multiuser multiple-input multiple-output (MIMO) mode. Since FD transmission is degraded by self-interference while HD suffers from spatial correlation between MIMO antennas, which causes rate loss, we study the effect of both the self-interference cancelation parameter at the FD AP and the mutual distance between the HD users on FD performances and the effect of the spatial correlation coefficient on HD MIMO. Afterward, a switching criterion is proposed, which chooses the operation mode that maximizes the downlink channel capacity while maintaining the uplink channel capacity at a certain level. Subsequently, based on our study of the system's parameters that affect the performance of both FD and HD, theoretical thresholds for these parameters are derived. Finally, numerical analysis is presented, verifying the validity of the optimization problem solution and the derived thresholds.**

***Index Terms*—Full-duplex (FD), half-duplex (HD), resource allocation, transmission mode selection.**

## I. Introduction

NEXT-GENERATION cellular networks are required to utilize new techniques to provide increased capacity networks that accommodate the enormous growth in the number of wireless communication users and the required communication reliability. Therefore, efficient utilization of the valuable communication resources becomes inevitable. Unfortunately, most of the existing communication devices operate in half-duplex (HD), which dissipates the worthy resources by employing time-division duplexing or frequency-division duplexing. On the other hand, by using the same communication resources, communication networks can theoretically achieve double the HD capacity by utilizing full-duplex (FD). As mentioned in [1], enabling communication devices to send and receive data simultaneously at the same time slot and in the same channel has many advantages. First, in the physical layer, FD offers the potential to double the spectral efficiency, which is defined as the number of information bits reliably communicated per second per hertz. In addition, in the access layer, FD will allow for collision detection while transmitting in a contention-based network.

However, the main challenge that restrains FD operation is the increased self-interference caused by node transmission on node reception, particularly in a multiuser (MU) communication system. Accordingly, as the ability of a communication system increases to suppress the self-interference, the gain that can be achieved from FD will increase. Since the gain achieved by FD is controlled by self-interference cancelation, it is not always guaranteed that FD outperforms HD. In addition, in the case of MU-MIMO systems, HD may suffer from spatial correlation between antennas [2], which results from the limited spacing between antennas, and hence, the MIMO channels will be correlated. This correlation is expected to degrade the HD performance. Therefore, to be able to optimize the network performance and build a framework that optimizes the network's rate by choosing the transmission mode that offers the best performance, a clear investigation on the parameters that affect the performance of both FD and HD transmissions is needed. This investigation will give an insight into the conditions required for the communication system to operate either in FD or HD. Accordingly, an efficient switching algorithm between FD and HD can be established based on the derived conditions. The switching algorithm will be able to optimally utilize the available communication network resources.

In this paper, a complete study of the parameters that affect the performance of both FD and HD operation is presented. We consider a system with one access point (AP) such as a base station and two HD users. Accordingly, a switching criterion between FD and HD is proposed, which aims at choosing the transmission mode that maximizes the total downlink (DL) capacity while maintaining the uplink (UL) transmission's total

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capacity at a certain level. In addition, the switching criterion is formulated as a resource allocation problem, which is solved using the Karush–Kuhn–Tucker (KKT) conditions. Furthermore, theoretical expressions are derived for the system parameters' thresholds controlling the switching process between FD and HD. Finally, the network's operation regions are determined based on the derived thresholds. Our contributions in this paper can be summarized as follows:

1) presenting an investigation on the parameters affecting FD and HD performance;
2) proposing a switching criterion that chooses the best transmission mode for the communication network that maximizes the DL capacity and keeps the UL capacity at a certain level;
3) deriving theoretical threshold expressions for self-interference cancelation, mutual distance between users, and spatial correlation coefficient between antennas that dominate the switching between FD and HD, as well as deriving the network's operation regions based on the derived thresholds.

The rest of this paper is organized as follows. In Section II, a literature review on FD and HD resource allocation is presented. In Section III, the system model is presented. In Section IV, the switching criterion is formulated, and the analytical solution is presented. In Section V, numerical results are presented. Finally, this paper is concluded in Section VI.

## II. Related Work

Since reducing self-interference is considered the foremost challenge facing FD, it has been widely studied in the literature. In [3], three different self-interference cancelation algorithms have been presented: The first is based on antenna separation and digital cancelation; the second is based on antenna separation and analog cancelation; and the third is based on antenna separation with both analog and digital cancelation. In [4], a three-node FD network is studied in which the infrastructure node operates in FD with two HD nodes. A decode-and-cancel interference cancelation scheme is presented for improved interference cancelation. In [5], two models for loopback or self-interference cancelation have been proposed in FD wireless communication. In the first model, it is assumed that self-interference is precisely known, whereas in the second model, it is assumed that self-interference is random and unknown. In [6], a complete design of an FD system radio is presented. The proposed design, theoretically, has no bandwidth constraints, but it is restrained by practical limitations such as random signal attenuation and delay. Additionally, it can automatically change its self-interference cancelation according to the channel condition. In [7], new digital and analog self-interference cancelation schemes are presented along with a complete in-band FD radio that is capable of decreasing the self-interference level to noise floor.

Furthermore, the effect of FD on resource allocation problems has been newly studied in the literature. In [8], a study of the effect of FD on resource allocation for a small-cell network is presented. A resource allocation problem is formulated to set both the base station and the femtocell AP power to maximize the DL channel capacity while guaranteeing the UL channel quality of service (QoS). It was found that depending on the interference conditions, it can be decided when it is preferable to employ FD and when it is more advantageous to use HD. In [9], the optimal relay selection scheme that maximizes the effective signal-to-interference-and-noise ratio is proposed. Furthermore, the optimal power allocation and the optimal choice between FD and HD are calculated by minimizing the outage probability. In [10], the challenges in resource allocation in FD orthogonal frequency-division multiple access (OFDMA) networks are studied. In these types of problems, the transmitters and receivers have to be properly paired into separate transceiver units, an appropriate subcarrier should be allocated to each transceiver pair, and the base station must allocate a suitable power level to each transceiver pair. In [11], the optimum transmit power allocation for FD decode-and-forward (DF) relaying systems is studied. The optimization problem aims at minimizing the outage probability. In [12], a power allocation problem over the transmitting power of both the source and the relay in a Rayleigh fading FD-DF relay channel is presented. The optimization is subject to individual power constraints. In [13], an FD multiuser MIMO system is studied. An optimization problem is formulated to maximize the weighted sum data rate while keeping the maximum power constraints. Finally, it was shown that the proposed scheme can achieve higher rates than baseline HD schemes. In [14], the use of FD relaying in cognitive radio systems is studied. In addition, an optimal power allocation scheme based on minimizing the outage probability in cognitive FD relay networks is proposed. Afterward, the outage probability of the secondary user in the noise-limited and interference-limited environments is analyzed. In [15], a joint optimization problem for resource allocation and scheduling in the FD MIMO OFDMA relaying systems with amplify-and-forward (AF) and DF relaying protocols is presented. It was shown that the proposed distributed algorithm demonstrates the potential performance gains achievable with FD relaying protocols.

Moreover, studying the performance of systems combining FD and HD has been recently investigated in the literature. In [16], a $K$-link MIMO interference channel, where each link consists of two bidirectional FD nodes, is considered. The weighted sum-rate maximization problem subject to the sum-power constraint of the system or individual power constraints is proposed. It was shown via numerical results that FD achieves better rates than baseline HD schemes. In [17], the rate regions of HD and FD links using the OFDM technique are analyzed. This analysis takes into consideration the nonideality of practical transceivers. It is assumed that self-interference cancelation in an FD transceiver takes place in a three-step process. Finally, optimal power allocation schemes are proposed to maximize the rates of both the HD and FD OFDM links under different strategies. In [18], the spectral efficiency of FD small-cell wireless systems is investigated. Additionally, a joint beamformer design is proposed to maximize the spectral efficiency subject to certain power constraints. It was proven via numerical results that FD can outperform HD. In [19], the performance of a random-access time-slotted wireless network with a single AP and a mix of HD and FD stations is considered. In addition, generalized analytical formulations for the throughput for each station are presented. Finally, the effects of using FD on network behavior are discussed. In [20], a comparison between HD and FD relaying achievable rates is presented. The comparison is done after assuming a practical model for FD residual self-interference (RSI). In addition, the achievable rates and degrees of freedom of FD relaying are derived. Finally, it is proven that power scaling at the relay is required to maximize the degrees of freedom in FD relaying. In [21], the effect of jamming on the design of three-node two-hop cooperative AF communications with both HD and FD relaying is studied. In the case of HD relaying, it is assumed that the jammer can optimally allocate jamming power between listening and forwarding phases. Accordingly, the optimal jamming power allocation, with a total source and relay power constraint, is derived. However, in the case of FD relaying, it is proven that self-interference can bring substantial performance gain when it is appropriately controlled. In [22], capacity comparisons between FD and HD are presented when the total amount of analog radio hardware is limited. Under the radio hardware constraint, it must be determined when to use the radio to cancel self-interference and when to use it to have additional MIMO multiplexing gain. It is found that under certain scenarios, using radios for cancelation can be more advantageous, since the resulting FD system performs better than HD. In [23], the theoretical performance of an FD bidirectional communication system and an FD relay system is studied. Moreover, a gradient projection algorithm is developed, which makes use of both spatial and temporal degrees of freedom of the source covariance matrices of the MIMO links between transmitters and receivers, to optimize the power allocation vectors. It is concluded from numerical results that the FD mode is optimal when the nominal self-interference is low and that the HD mode is optimal when the nominal self-interference is high. In [24], the capacity tradeoff between FD and HD modes in a two-hop system with an AF relay is studied. In the beginning, closed-form expressions for the average end-to-end capacity in the relay link are derived. Afterward, maximum self-interference levels that still make FD achieve the same capacity as HD are evaluated. Finally, it was shown that FD achieves a better performance than that of HD in terms of system capacity. In [25], the problem of FD bidirectional MIMO communication is investigated. In addition, a transmission scheme based on maximizing the lower bound of the achievable rate is proposed, defining the thresholds between FD and HD regimes.

## III. System Model

In this paper, we consider a single channel network[1] with one AP equipped with two antennas. The cell has two active users $(m, n)$, which may be both transmitting in the UL mode or receiving in the DL mode, and in a special case, we will have one user transmitting in the UL and the other receiving in the DL. The AP is trying to choose the best transmission mode that will maximize the DL channel capacity while keeping a certain QoS for the UL transmission represented by the UL channel capacity. It must be taken into consideration that all channel state information for the UL transmissions, DL transmission, and channels between users must be available at the AP to be able to switch between FD and HD. Channel estimation is out of our work scope; different techniques of channel estimation are found in [26]–[29]. The system model is shown in Fig. 1. Based on the given assumptions, the difference between FD and HD operations will be illustrated.

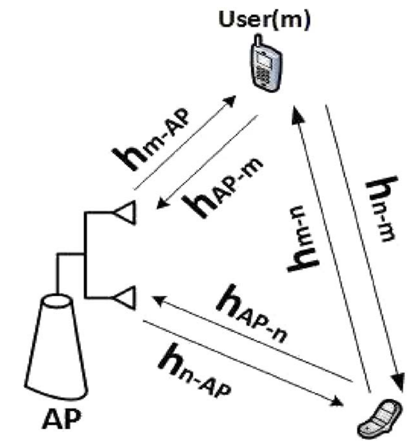



Fig. 1. System model.

### A. FD Transmission

In case of FD transmission, in the first time slot, the AP will be dedicated to serving the DL transmission of user $m$ along with the UL transmission of user $n$. However, in the second time slot, the AP will serve the DL transmission of user $n$ along with the UL transmission of user $m$. In addition, the AP is required to have a special FD radio [3], [30] that helps in self-interference cancelation. Accordingly, the DL received signal-to-interference-plus-noise ratio (SINR) of users $m$ and $n$ is given, respectively, by

$$\Gamma_{\mathrm{DL}_{m(n)}|\mathrm{FD}} = \frac{P_{\mathrm{AP}} D^{-\alpha}_{m(n)-\mathrm{AP}} \left|h_{m(n)-\mathrm{AP}}\right|^2}{\sigma^2 + \mathrm{P}\mathrm{D}^{-\alpha}_{m-n} \left|h_{m(n)-n(m)}\right|^2} \tag{1}$$

where $P_{\mathrm{AP}}$ is the AP transmission power; $D^{-\alpha}_{m-\mathrm{AP}}$ denotes the large-scale propagation loss between user $m$ and the AP, with distance $D_{m-\mathrm{AP}}$ and path-loss exponent $\alpha$; $h_{m-\mathrm{AP}}$ is the channel coefficient between user $m$ and the AP transmitting antenna[2]; $\sigma^2$ is the additive white Gaussian noise variance; and $D^{-\alpha}_{m-n}$ denotes the large-scale propagation loss between users $m$ and $n$. Finally, $\mathrm{PD}^{-\alpha}_{m-n}|h_{m(n)-n(m)}|^2$ denotes the interference introduced by the UL transmission. From (1), it is clear that the interference from the UL transmission on the DL transmission needs to be estimated; therefore, it is

[1]In our model, we assume a single-cell network by considering the other cells' interference as noise. Moreover, without loss of generality, we consider only one channel, since the multiple-channel case can be studied in a similar manner.

[2]Throughout this paper, $h_{x-y}$ denotes the channel coefficient between receiver $x$ and transmitter $y$.

sufficient to only estimate the SINR between the UL and the DL user. It must be noticed that SINR estimation is much simpler than channel estimation and can be done using simple power estimation techniques. After SINR estimation is done at the DL user, the estimation result is reported to the AP in a predetermined reporting period.[3] It must be mentioned that the notation $m(n)$ means that we can get the expression for the $n$th user from (1) by interchanging $m$ and $n$ in the equation. All channel coefficients are assumed to be independent and identically distributed zero-mean complex Gaussian random variables with unit variance, i.e., Rayleigh fading. On the other hand, the UL received SINR at the AP receiving antenna for users $m$ and $n$ is given, respectively, by

$$\Gamma_{\mathrm{UL}_{m(n)}|\mathrm{FD}} = \frac{\mathrm{PD}_{m(n)-\mathrm{AP}}^{-\alpha} \left|h_{\mathrm{AP}-m(n)}\right|^2}{\sigma^2 + P_{\mathrm{AP}}^{\lambda}/C} \tag{2}$$

where $P$ is the UL user transmission power; $D_{m-\mathrm{AP}}^{-\alpha}$ denotes the large-scale propagation loss, with distance $D_{m-\mathrm{AP}}$ and path-loss exponent $\alpha$; and $P_{\mathrm{AP}}^{\lambda}/C$ represents the RSI after using an FD radio at the AP with a cancelation parameter $C$, and $\lambda \in [0, 1]$ that reflect the quality of self-interference cancelation [3], [30], [32].[4] From (1) and (2), it can be noticed that the FD DL received SINR can be controlled by controlling the mutual distance between users. Moreover, the quality of the UL transmission depends on the FD radio used in the AP and, hence, on its self-interference cancelation parameter.

### B. HD Transmission

In the case of HD transmission, the AP will serve two simultaneous DL transmissions of users $m$ and $n$ in the first time slot. On the other hand, the AP will serve two simultaneous UL transmissions of users $m$ and $n$ in the second time slot. Therefore, the DL received SINR of users $m$ and $n$ is given, respectively, by

$$\Gamma_{\mathrm{DL}_{m(n)}|\mathrm{HD}} = \frac{P_{\mathrm{AP}} \left|\mathbf{h}_{m(n)-\mathrm{AP}} R^{\frac{1}{2}} \mathbf{w}_{\mathbf{m(n)-AP}}\right|^2 D_{m(n)-\mathrm{AP}}^{-\alpha}}{\sigma^2 + P_{\mathrm{AP}} \left|\mathbf{h}_{m(n)-\mathrm{AP}} R^{\frac{1}{2}} \mathbf{w}_{\mathbf{n(m)-AP}}\right|^2 D_{m(n)-\mathrm{AP}}^{-\alpha}} \tag{3}$$

where $\mathbf{w}_{\mathbf{m-AP}} \in \mathbb{C}^2$ is the precoding vector applied by the AP to user $m$ DL data, $R = \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix}$ is a $2 \times 2$ matrix that models the spatial correlation at the AP with spatial correlation coefficient $\beta$, $|\mathbf{h}_{m-\mathrm{AP}} R^{1/2} w_{n-\mathrm{AP}}|^2 D_{m-\mathrm{AP}}^{-\alpha}$ denotes the interference caused by the AP transmission to user $n$, and $w_{n-\mathrm{AP}}$ is the precoding vector applied by the AP to user $n$ data. The value of $\mathbf{w}_{\mathbf{m-AP}}$ is given by

$$w_{m-\mathrm{AP}} = \frac{R^{\frac{1}{2}} \mathbf{h}_{m-\mathrm{AP}}^{\dagger}}{\left\| R^{\frac{1}{2}} \mathbf{h}_{m-\mathrm{AP}}^{\dagger} \right\|_2} \tag{4}$$

where $\mathbf{h}_{m-\mathrm{AP}}^{\dagger}$ is the Hermitian transpose of $\mathbf{h}_{m-\mathrm{AP}}$ that denotes the channel vector between the AP and the DL user, and $\|.\|_2$ denotes the $\ell_2$-norm. It must be noticed that in the case of HD transmission, the DL user is receiving data from both antennas of the AP. This makes $\mathbf{h}_{m-\mathrm{AP}} \in \mathbb{C}^2$. In addition, it is clear that the $h_{m-\mathrm{AP}}$ channel coefficient defined in (1) represents an element in the channel vector $\mathbf{h}_{m-\mathrm{AP}}$. On the other hand, the UL received SINR of users $m$ and $n$ is given, respectively, by

$$\Gamma_{\mathrm{UL}_{m(n)}|\mathrm{HD}} = \frac{\mathrm{PD}_{m(n)-\mathrm{AP}}^{-\alpha} \left| R^{\frac{1}{2}} \mathbf{h}_{\mathrm{AP}-m(n)} \right|^2}{\sigma^2 + \left| \mathrm{PD}_{n(m)-\mathrm{AP}}^{-\alpha} R^{\frac{1}{2}} \mathbf{h}_{\mathrm{AP}-n(m)} \right|^2} \tag{5}$$

where $\mathbf{h}_{\mathrm{AP}-m}$ denotes the channel vector between user $m$ and the AP. Similarly, $\mathbf{h}_{\mathrm{AP}-m} \in \mathbb{C}^2$. In addition, $h_{\mathrm{AP}-m}$ defined in (2) represents an element in $\mathbf{h}_{\mathrm{AP}-m}$. $\mathrm{PD}_{n-\mathrm{AP}}^{-\alpha} |R^{1/2} \mathbf{h}_{\mathrm{AP}-n}|^2$ denotes the interference caused by the UL transmission of user $n$. From (3) and (5), it can be noticed that both the DL and UL received SINR, in the case of HD, depend on the spatial correlation between the antennas; hence, the value of the spatial correlation coefficient $\beta$ will influence the quality of HD operation.

To summarize, the relation between the HD and FD achieved capacities is mainly dependent on the self-interference cancelation, the mutual distance between users, and the spatial correlation between antennas, as follows.

- For the UL capacity, it is expected that HD will outperform FD for small values of self-interference cancelation $C$. However, it is expected that FD will offer higher UL capacity than HD for high values of spatial correlation coefficient $\beta$.
- For the DL capacity, it is anticipated that HD will outperform FD for a small mutual distance between the users $D_{m-n}$. On the other hand, FD will achieve higher DL capacity for large values of spatial correlation coefficient $\beta$.

## IV. Switching Criterion Between Full-Duplex and Half-Duplex

From the received SINR expressions given by (1)–(5), the total DL channel capacity per unit bandwidth in two successive time slots when operating in FD and HD transmissions is given, respectively, by

$$C_{\mathrm{DL}|\mathrm{FD}} = \log_2(1 + \Gamma_{\mathrm{DL}_m|\mathrm{FD}}) + \log_2(1 + \Gamma_{\mathrm{DL}_n|\mathrm{FD}}) \tag{6}$$

$$C_{\mathrm{DL}|\mathrm{HD}} = \log_2(1 + \Gamma_{\mathrm{DL}_m|\mathrm{HD}}) + \log_2(1 + \Gamma_{\mathrm{DL}_n|\mathrm{HD}}). \tag{7}$$

[3] Due to the centralized nature of the cellular network, in which the AP fully controls the user's procedures, the AP is capable of adjusting the channel estimation procedures with the existing network's users. Additionally, the problem of estimating the channel or SINR between the UL and DL users is the same as estimating the interuser channels in cooperative communication. A similar methodology to that proposed in [31] can be used to acquire the required channels.

[4] It must be pointed out that the transmission power of the AP must be very carefully chosen to avoid the receiver saturation problem. However, it must be mentioned that self-interference cancelation, the RSI model, and the receiver saturation problem are out of the scope of this work. The only focus of the proposed scheme is the resource allocation problem.

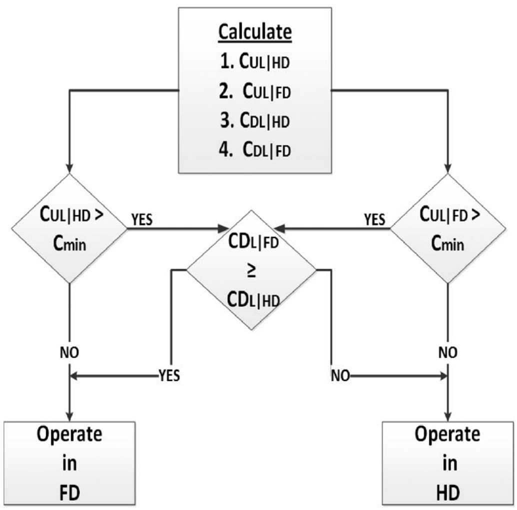



Fig. 2. Switching criterion.

In addition, the total UL channel capacity per unit bandwidth in two time slots when operating in FD and HD transmissions is given, respectively, by

$$C_{\mathrm{UL|FD}} = \log_2(1+\Gamma_{\mathrm{UL}_m|\mathrm{FD}}) + \log_2(1+\Gamma_{\mathrm{UL}_n|\mathrm{FD}}) \quad (8)$$

$$C_{\mathrm{UL|HD}} = \log_2(1+\Gamma_{\mathrm{UL}_m|\mathrm{HD}}) + \log_2(1+\Gamma_{\mathrm{UL}_n|\mathrm{HD}}). \quad (9)$$

We assume that the QoS of the UL transmission of each user is constrained by a certain minimum channel capacity $C_{\min}$. In other words, regardless of the operation mode, the UL transmission capacity for each user must be greater than $C_{\min}$. The total DL channel capacity and UL channel capacity per unit bandwidth for users $m$ and $n$ are given, respectively, by

$$C_{\mathrm{DL}} = T(C_{\mathrm{DL|FD}}) + (1-T)(C_{\mathrm{DL|HD}}) \quad (10)$$

$$C_{\mathrm{UL}} = T(C_{\mathrm{UL|FD}}) + (1-T)(C_{\mathrm{UL|HD}}) \quad (11)$$

where $T = 1$ denotes that the system will operate in the FD mode, whereas $T = 0$ denotes that the system will operate in the HD mode. The main idea of the switching criterion is to choose the transmission mode that will be able to do the following:

1) Provide the UL transmission with the minimum required channel capacity that is equal to $C_{\min}$.
2) Maximize the DL user capacity as given by (10).

Based on the previous assumptions, the switching criterion is shown in Fig. 2. In the beginning, both the DL and UL channel capacities for FD and HD are calculated, respectively, from (6)–(9). Afterward, the system will compare the achieved UL capacity with $C_{\min}$. If FD is not able to satisfy the UL channel capacity constraint, the system will operate in HD. On the contrary, when HD does not meet the UL channel capacity constraint, the system will operate in FD. On the other hand, if both FD and HD satisfy the minimum channel capacity for both users, the system will choose the transmission mode that will maximize the DL channel capacity given by (10). It is worth mentioning that if both FD and HD fail to satisfy the minimum channel capacity constraint for the users, the network will be considered in outage.

### A. Switching Criterion Problem Formulation

Here, we are going to provide the analysis for the proposed switching criterion. The proposed switching criterion can be formulated as the following optimization problem:

$$\begin{aligned} \max_{T^*} \quad & C_{\mathrm{DL}} \\ \text{subject to} \quad & C_{\mathrm{UL}} \geq C_{\min} \\ & T \in \{0, 1\}. \end{aligned} \quad (12)$$

It must be mentioned that the second binary constraint on $T$ makes the problem hard to solve. Therefore, to solve this problem, we are going to have two different ways. The first way is the exact solution. The exact solution is to enumerate all possible cases of the AP operation. In other words, we separately compare the system performance in the case of FD and HD operation, which correspond to the cases of $T = 0$ and $T = 1$, respectively. The other approach is to relax the second constraint on the $T$ parameter to be in the continuous domain [0, 1] instead of the binary domain {0, 1}.[5] After relaxing the $T$ coefficient to be in the [0, 1] interval, the problem will be solved using the KKT conditions. Afterward, the value of $T^*$ (the optimum $T$) will be remapped to the binary domain using a simple rounding rule as follows: If $T^* < 0.5$, it will be rounded to 0; otherwise, it will be rounded to 1. Accordingly, the relaxed optimization problem is given by

$$\begin{aligned} \max_{T^*} \quad & C_{\mathrm{DL}} \\ \text{subject to} \quad & C_{\mathrm{UL}} \geq C_{\min} \\ & 0 \leq T \leq 1. \end{aligned} \quad (13)$$

Accordingly, the solution of the optimization problem given by (13) using the KKT conditions will be as follows.

*Proposition 1:* The resource allocation problem given by (13) is solved in three different cases.

1) System Operates in HD Only
   In that case, $T^* = 0$. This occurs when FD transmission fails to satisfy the UL channel capacity constraint while HD does satisfy the constraint.
2) System Operates in FD Only
   In that case, $T^* = 1$. This occurs when HD transmission fails to satisfy the UL channel capacity constraint while FD does satisfy the constraint.
3) System Will Switch Between HD and FD
   This occurs when both FD and HD transmissions succeed in maintaining the UL channel capacity at $C_{\min}$.

[5]The "optimum" approach, which is simply an enumeration of all possible cases of the AP operation, can be easily applied in our model due to its simplicity. However, the enumeration method will be tedious if the number of APs increases. Therefore, we need a more generalized approach to solve the proposed allocation problem.

Therefore, the system will choose the mode that maximizes the DL channel capacity. In that case, $T^*$ is given by

$$T^* = \left| \frac{C_{\min} - C_{\mathrm{UL|HD}}}{C_{\mathrm{UL|FD}} - C_{\mathrm{UL|HD}}} \right|. \tag{14}$$

The proof of Proposition 1 is presented in Appendix A.

### *B. Self-Interference Cancelation, Mutual Distance Between Users, and Correlation Coefficient Thresholds for the Switching Criterion*

Since the performance of FD is greatly affected by the RSI and the mutual distance between the user transmitting in the UL and that receiving in the DL and the HD operation is influenced by the value of the spatial correlation between the channels, then it will be useful to derive the thresholds for the self-interference cancelation, the mutual distance between users, and the correlation coefficient that will cause the system to switch between FD and HD. In other words, the derived thresholds will define the regions in which the system will operate in FD or HD.

*1) Self-Interference Cancelation Threshold:* It can be noticed from (1) and (2) that the value of $C$ affects the value of the UL received SINR only. Therefore, FD operation will be allowable as long as the value of $C$ does not make the UL received SINR less than $C_{\min}$. In other words, the minimum allowable value for $C$ is that value that decreases the UL channel capacity for FD to be equal to $C_{\min}$. If the value of $C$ is less than $\hat{C}$, then the UL channel capacity of FD will be lower than $C_{\min}$, and hence, the system will operate directly in HD. This condition can be used to obtain threshold $\hat{C}$ as follows.

*Proposition 2:* The value of $\hat{C}$ at which $C_{\mathrm{UL|FD}} = C_{\min}$ is given by

$$\hat{C} = \frac{2P_{\mathrm{AP}}^{\lambda}(2^{C_{\min}} - 1)}{\sqrt{r_1^2 - 4(2^{C_{\min}} - 1)r_2} - r_1} \tag{15}$$

where

$$r_1 = \sigma^2(2^{C_{\min}+1} - 2) - \left(\mathrm{PD}_{m-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-m}|^2\right) - \left(\mathrm{PD}_{n-\mathrm{AP}}^{-\alpha}, |h_{\mathrm{AP}-n}|^2\right) \tag{16}$$

$$r_2 = \sigma^4(2^{C_{\min}}) - \left(\sigma^2 + \mathrm{PD}_{m-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-m}|^2\right)\left(\sigma^2 + \mathrm{PD}_{n-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-n}|^2\right). \tag{17}$$

The proof of Proposition 2 is presented in Appendix B.

*2) Mutual Distance Between Users Threshold:* It can be noticed from (1) and (2) that the value of the distance between users affects the value of the DL received SINR only. Hence, if it is assumed that both FD and HD satisfy the UL channel capacity constraint, the system will select the mode that will maximize the DL capacity. Therefore, $\hat{D}$ can be defined by the minimum allowable distance between users $m$ and $n$ such that $C_{\mathrm{DL|FD}}$ is equal to $C_{\mathrm{DL|HD}}$. If the distance between $m$ and $n$ becomes less than $\hat{D}$, then the interference introduced by the UL transmission will increase. Therefore, the DL received SINR given by (1) will decrease as well. Accordingly, $C_{\mathrm{DL|FD}}$ will be lower than $C_{\mathrm{DL|HD}}$. Therefore, the system will operate in HD. This condition can be used to obtain $\hat{D}$ as follows.

*Proposition 3:* The value of $\hat{D}$ at which $C_{\mathrm{DL|FD}} = C_{\mathrm{DL|HD}}$ is given by

$$\hat{D} = \sqrt[\alpha]{\frac{2(2^{C_{\mathrm{DL|HD}}} - 1)}{\sqrt{d_1^2 - 4d_2(2^{C_{\mathrm{DL|HD}}} - 1)} - d_1}} \tag{18}$$

where

$$d_1 = 2^{C_{\mathrm{DL|HD}}}(c_2 + c_4) - c_1 - c_2 - c_3 - c_4 \tag{19}$$

$$d_2 = 2^{C_{\mathrm{DL|HD}}}c_2c_4 - (c_1 + c_2)(c_3 + c_4) \tag{20}$$

$$c_{1(3)} = \frac{P_{\mathrm{AP}} D_{m(n)-\mathrm{AP}}^{-\alpha} \left|h_{m(n)-\mathrm{AP}}\right|^2}{P\left|h_{m(n)-n(m)}\right|^2} \tag{21}$$

$$c_{2(4)} = \frac{\sigma^2}{P\left|h_{m(n)-n(m)}\right|^2}. \tag{22}$$

The proof of Proposition 3 is presented in Appendix C.

*3) Spatial Correlation Coefficient Threshold:* Finding the spatial correlation coefficient threshold will be a little different. The first difference is that the value of $\beta \in [0, 1]$. In other words, the value of $\beta$ is constrained to be greater than zero and smaller than one. In addition, the value of $\beta$ influences both the UL and DL received SINR values of HD operation. Therefore, we will have two different thresholds for $\beta$ that control switching between HD and FD. The first threshold $\hat{\beta}_1$ will be a result of the necessity that the received UL SINR given by (5) is equal to $C_{\min}$. The second threshold $\hat{\beta}_2$ is the value of $\beta$ that makes the HD DL capacity given by (7) equal to the FD DL capacity given by (6).

Next, we are interested in finding the value of $\hat{\beta}_1$, which makes the achieved HD UL capacity equal to $C_{\min}$. If the actual value of $\beta$ exceeds $\hat{\beta}_1$, then the HD UL capacity will be less than $C_{\min}$. Therefore, the system will directly operate in FD. This condition can be used to obtain $\hat{\beta}_1$ as follows.

*Proposition 4:* The value of $\hat{\beta}_1$ at which $C_{\mathrm{UL|HD}} = C_{\min}$ is given by

$$\hat{\beta}_1 = \frac{\sqrt{b_2^2 - 4b_1b_3} - b_2}{2b_1}, \text{ for } C_{\min|\mathrm{HD}} \leq C_{\min} \leq C_{\max|\mathrm{HD}} \tag{23}$$

where $C_{\min|\mathrm{HD}}$ is the minimum possible value of HD UL capacity that corresponds to the case $\beta = 1$, and $C_{\max|\mathrm{HD}}$ is the maximum possible value of HD UL capacity that corresponds to the case $\beta = 0$. The constraint $C_{\min|\mathrm{HD}} \leq C_{\min} \leq C_{\max|\mathrm{HD}}$

TABLE I
SWITCHING THRESHOLDS

| | | |
|---|---|---|
| $\hat{C}$ | Self-Interference Cancellation Threshold | $\frac{2P_{AP}^{\lambda}(2^{C_{min}}-1)}{\sqrt{r_1^2-4(2^{C_{min}}-1)r_2}-r_1}$ |
| $\hat{D}$ | Mutual Distance between Users Threshold | $\sqrt[\alpha]{\frac{2(2^{C_{DL\|HD}}-1)}{\sqrt{d_1^2-4d_2(2^{C_{DL\|HD}}-1)}-d_1}}$ |
| $\hat{\beta}_1$ | Spatial Correlation Coefficient Threshold | $\frac{\sqrt{b_2^2-4b_1b_3}-b_2}{2b_1}$ |

is added to guarantee that the value of $\hat{\beta}_1$ is bounded in [0, 1]. In addition

$$b_1 = 2^{C_{\min}} H_2H_4 - (X_1H_2 + H_4)(X_3H_4 + H_2) \quad (24)$$

$$\begin{aligned} b_2 = {} & 2^{C_{\min}} (H_2(X_2 + H_3) + H_4(X_4 + H_1)) \\ & - (X_2 + H_3 + X_1H_1)(X_3H_4 + H_2) \\ & - (X_1H_2 + H_4)(X_4 + H_1 + X_3H_3) \end{aligned} \quad (25)$$

$$\begin{aligned} b_3 = {} & 2^{C_{\min}}(X_2 + H_3)(X_4 + H_1) \\ & - (X_2 + H_3 + X_1H_1)(X_4 + H_1 + X_3H_3) \end{aligned} \quad (26)$$

$$X_{1(3)} = \frac{D^{-\alpha}_{n(m)-\mathrm{AP}}}{D^{-\alpha}_{m(n)-\mathrm{AP}}}, \; X_{2(4)} = \frac{\sigma^2}{\mathrm{P}D^{-\alpha}_{m(n)-\mathrm{AP}}} \quad (27)$$

$$H_1 = h_1^2 + h_2^2 + h_3^2 + h_4^2, \; H_2 = 2(h_1h_3 + h_2h_4) \quad (28)$$

$$H_3 = h_5^2 + h_6^2 + h_7^2 + h_8^2, \; H_4 = 2(h_5h_7 + h_6h_8) \quad (29)$$

where it is assumed that $\mathbf{h_{AP-n}} = \begin{bmatrix} h1 + jh2 \\ h3 + jh4 \end{bmatrix}$, and $\mathbf{h_{AP-m}} = \begin{bmatrix} h5 + jh6 \\ h7 + jh8 \end{bmatrix}$.

The proof of Proposition 4 is presented in Appendix D.

The second threshold $\hat{\beta}_2$ is found by equating the HD DL capacity given by (7) to the FD DL capacity given by (6). If the exact value of $\beta$ exceeds $\hat{\beta}_2$, then the HD DL capacity will be smaller than the FD DL capacity. Therefore, the system will operate in FD. The value of $\hat{\beta}_2$ will be numerically computed.

Therefore, based on the derived thresholds, we can define the regions in which the system will suffer from outage $R_O$, operate in FD $R_{\mathrm{FD}}$, or operate in HD $R_{\mathrm{HD}}$. These regions can be defined, respectively, by

$$R_O = \{C < \hat{C}, \beta > \hat{\beta}_1\} \quad (30)$$

$$\begin{aligned} R_{\mathrm{FD}} = {} & \{C > \hat{C}, \beta > \hat{\beta}_1\} \\ & \bigcup \{C > \hat{C}, \beta < \hat{\beta}_1, D_{m-n} > \hat{D}, \beta > \hat{\beta}_2\} \end{aligned} \quad (31)$$

$$\begin{aligned} R_{\mathrm{HD}} = {} & \{C < \hat{C}, \beta < \hat{\beta}_1\} \\ & \bigcup \{C > \hat{C}, \beta < \hat{\beta}_1, D_{m-n} < \hat{D}, \beta < \hat{\beta}_2\} \end{aligned} \quad (32)$$

where the values of $\hat{C}$, $\hat{D}$, and $\hat{\beta}_1$ are summarized in Table I.

## V. NUMERICAL ANALYSIS

Here, we are going to validate our optimization problem solution by comparing the performance of the relaxed optimization problem solution presented in Proposition 1 and the exact solution. Furthermore, we are going to validate the derived threshold formulas by numerical results. In addition, we will show that our switching criterion is able to choose the transmission mode that offers the best performance represented by the maximum DL capacity. In addition, our results will study the effect of different system parameters on the overall system performance.

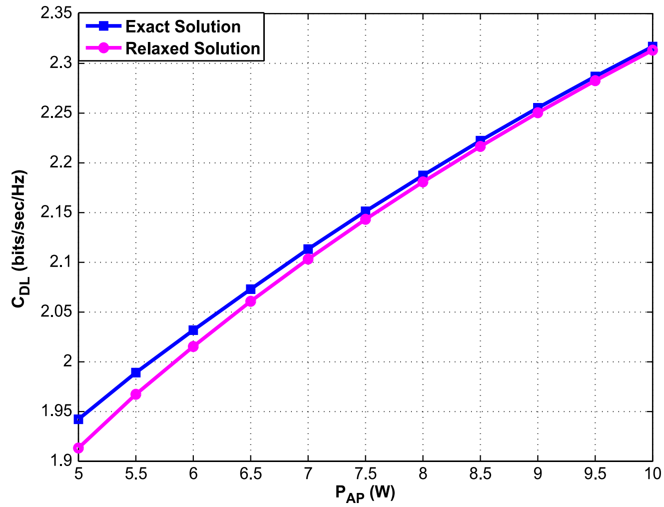


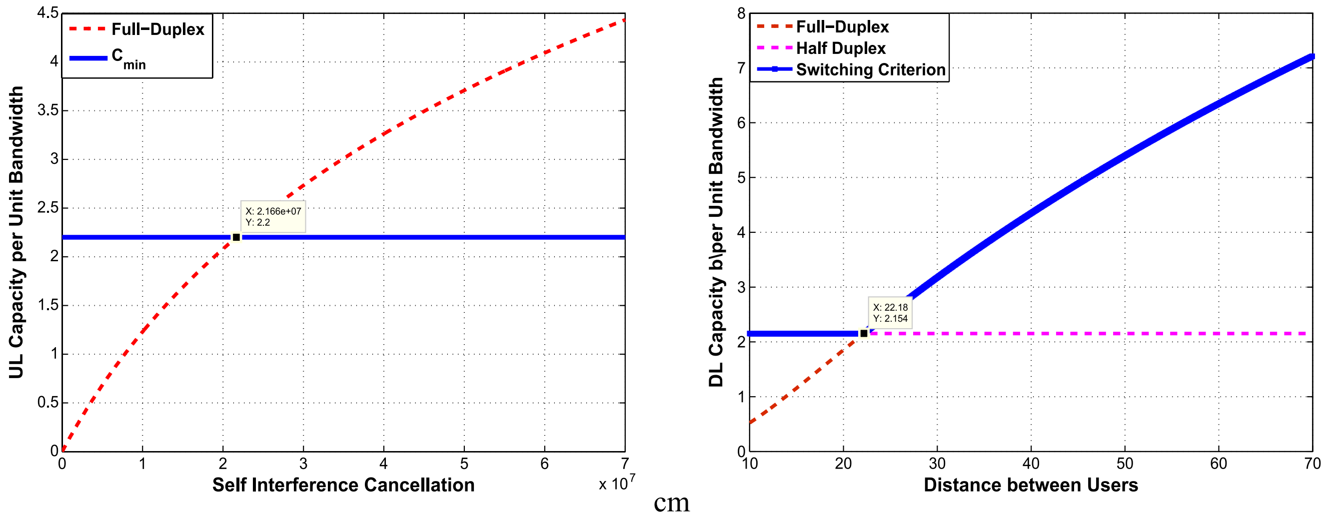

Fig. 3. Comparing between the relaxed and the exact solution achieved DL capacity.

### A. Validating the Performance of the Relaxed Optimization Problem

Here, we will validate our optimization problem solution proved in Proposition 1. Fig. 3 shows the variation of the achieved DL capacity with the AP transmission power. In addition, it compares between the achieved DL capacity in the case of the exact solution and the relaxed KKT solution given in Proposition 1. From the results shown in Fig. 3, it is shown that the biggest deviation between the relaxed and the exact solution is approximately equal to 1.5% at a value of $P_{\mathrm{AP}}$ approximately equal to 5 W. This indicates that the solution proposed using the KKT conditions gives a good approximation for the exact solution. In other words, relaxing the $T$ parameter to the continuous domain [0, 1] is a near-optimal approximation.

### B. Validating the Derived Thresholds for Self-Interference Cancelation, Mutual Distance Between Users, and Spatial Correlation Coefficient

Here, we are going to validate the derived thresholds in Propositions 2–4. Fig. 4(a) shows the variation of the total achieved UL capacity per unit bandwidth in the case of FD given by (8) with changing the self-interference cancelation parameter $C$. It is expected from (2) to have an increase in the received UL SINR by increasing $C$. This increase is attributable to the decrease in the self-interference level by increasing $C$. The increase in the UL received SINR will result in an increase in the value of the total UL received channel capacity given by (8). This behavior can be also verified from the results shown in Fig. 4(a). In addition, we can verify the value of $\hat{C}$ calculated




(a) (b)

Fig. 4. Verifying the validity of the theoretical expressions of (a) the self-interference cancelation parameter and (b) the mutual distance between users.

in (15). From the results shown in Fig. 4(a), it can be noticed that the FD achieved UL capacity will be equal to $C_{\min}$ at the value of self-interference cancelation $C$ approximately equal to $2.166 \times 10^7$. This value matches the $\hat{C}$ value calculated in (15).

Moreover, Fig. 4(b) shows the variation of the achieved DL capacity per unit bandwidth with the mutual distance between users. From (3), it is expected that the HD performance will not be affected by changing the mutual distance between users. However, from (1), it can be expected that the DL received SINR in the case of FD will increase with increasing the mutual distance between users. This increase is due to the decrease in the interference level introduced by the UL transmission on the DL transmission. Accordingly, the DL achieved capacity in the case of FD given by (6) will increase as well. It is obvious that the results obtained in Fig. 4(b) match the discussed performance behavior. Furthermore, our switching criterion effectiveness can be verified from the results shown in Fig. 4(b). It can be noticed that for small mutual distances between users, HD was able to achieve better DL capacity than FD. In that case, the proposed switching criterion chose to operate in HD. On the contrary, for large values of the mutual distance between users, FD transmission starts to outperform HD transmission. In that case, the proposed switching criterion chose to operate in FD. Finally, we can verify the value of $\hat{D}$ calculated in (18). From the results shown in Fig. 4(b), it can be noticed that switching between HD and FD occurs at $D_{m-n}$ equal to 22.18 m. In other words, the value of the achieved DL capacity in the case of FD given by (6) is equal to the value of the achieved DL capacity in the case of HD given by (7) at a value of $D_{m-n}$ equal to 22.18 m. This value matches the value of $\hat{D}$ calculated in (18).

Furthermore, Fig. 5 shows the variation of $C_{\mathrm{UL|HD}}$ given in (9) with the spatial correlation coefficient $\beta$. It is anticipated that increasing the spatial correlation coefficient will cause degradation in the performance of the HD transmission. Accordingly, it is expected to have a decrease in the value of $C_{\mathrm{UL|HD}}$ by increasing $\beta$. This behavior can be verified from

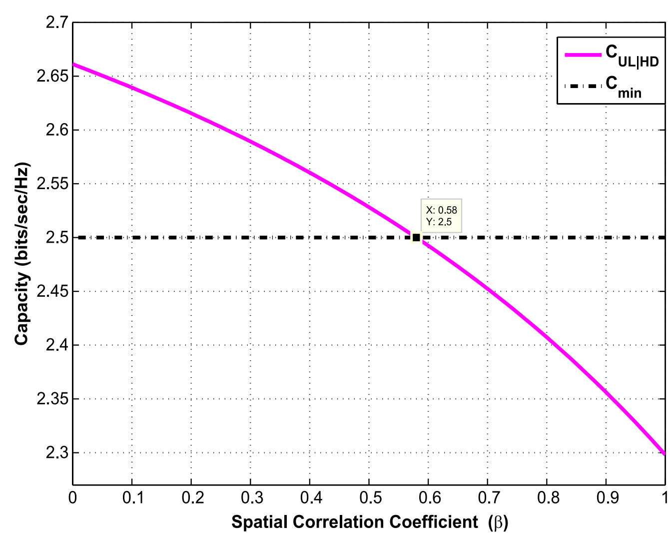



Fig. 5. Validating spatial correlation coefficient threshold.

the results shown in Fig. 5.[6] In addition, we can verify the value of $\hat{\beta}_1$ calculated in (23). It can be noticed that the value of $C_{\mathrm{UL|HD}}$ becomes equal to $C_{\min}$ at a value of $\beta = 0.58$. This value matches the value of $\hat{\beta}_1$ calculated in (23). In addition, it can be observed that the value of $C_{\max|\mathrm{HD}}$ is approximately equal to 2.66 bits/s/Hz, corresponding to $\beta = 0$. In addition, the value of $C_{\min|\mathrm{HD}}$ is approximately equal to 2.3 bits/s/Hz, corresponding to $\beta = 1$. Since $C_{\min|\mathrm{HD}} < C_{\min} < C_{\max|\mathrm{HD}}$, we are able to find a valid solution for $\hat{\beta} \in [0, 1]$. Otherwise, if $C_{\min}$ is not bounded between $C_{\min|\mathrm{HD}}$ and $C_{\max|\mathrm{HD}}$, there will be no valid solution for $\hat{\beta}_1 \in [0, 1]$. This result validates the added constraint on $C_{\min}$ in (23).

[6]It can be noticed that changing $\beta$ from 0 to 1 only changes the UL capacity from 2.65 to 2 bits/s/Hz, which is a very small variation. In Appendix E, according to the proposed model, the spatial correlation has a small effect on the UL capacity.

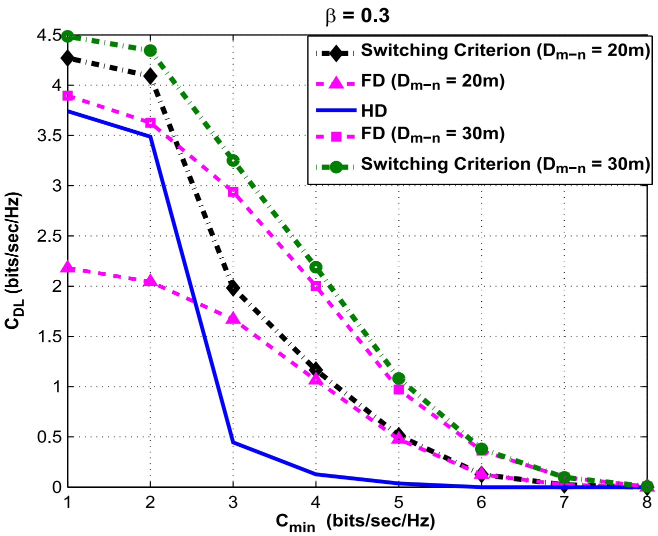



Fig. 6. Variation of $C_{\rm DL}$ with $C_{\min}$.

### *C. Studying the Effects of Different System Parameters on the Proposed System Performance*

Here, we are going to study the effect of the UL capacity constraint $C_{\min}$ on the achieved DL network capacity and the outage probability. In addition, the effects of the spatial correlation coefficient $\beta$ and the self-interference cancelation $C$ on the total achieved network DL capacity will be studied.

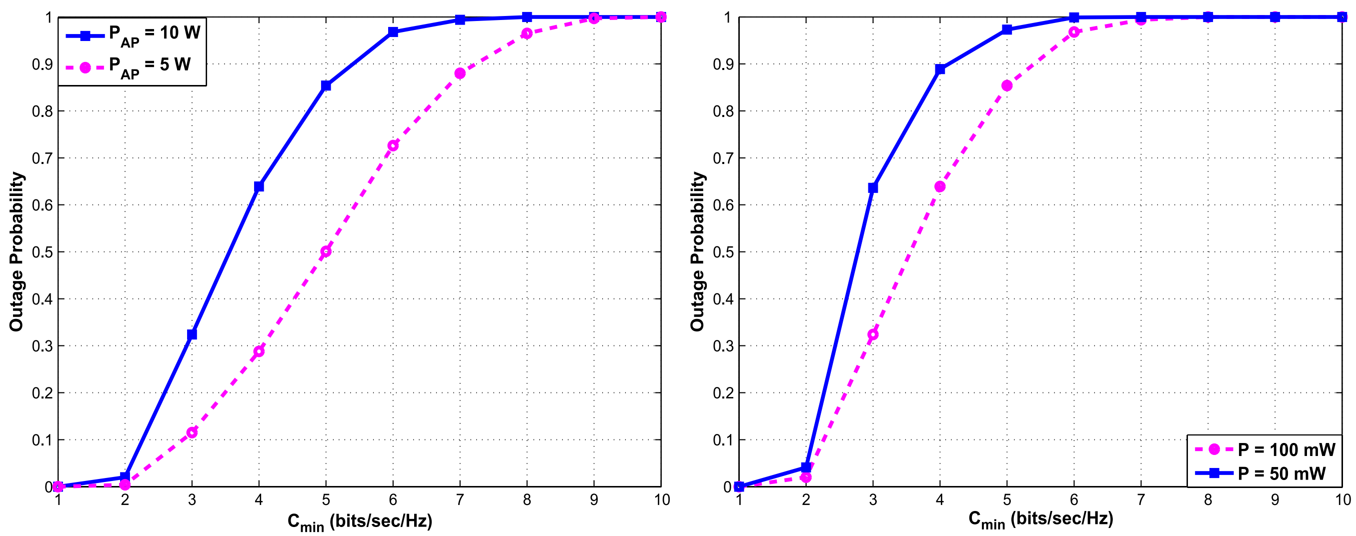
Fig. 6 shows the variation of the total DL capacity with $C_{\min}$. In the beginning, for small values of $C_{\min}$, both HD and FD will be able to fulfill the UL constraints, and the DL capacities for FD and HD are dependent on the values chosen in simulation for $D_{m-n}$ and $\beta$, respectively. As shown from the results, FD will always outperform HD for $D_{m-n} = 30$ m. However, when the distance decreases to 20 m, FD will outperform HD only for some values of $C_{\min}$. Afterward, as the value of $C_{\min}$ grows, the achieved DL capacity decreases. This can be simply explained by knowing that increasing $C_{\min}$ requires the system to allocate more resources for the UL transmission. However, if $C_{\min}$ continues to increase, the probability of fulfilling this QoS demand will decrease, and hence, the outage probability will increase. Additionally, in case of system outage, both the UL and DL capacity will be equal to zero. Therefore, on average, the achieved DL capacity will decrease. Accordingly, further increase in the $C_{\min}$ value will make the outage probability approach 1. In other words, the system will always fail to provide the UL transmission with its QoS demand. Accordingly, the DL capacity will be zero. In addition, the results shown in Fig. 6 verify the effectiveness of the proposed switching criterion in choosing the transmission mode that guarantees a better performance for the network. It can be observed that, in the case of $D_{m-n} = 20$ m and small values of $C_{\min}$, our switching criterion is able to achieve a gain of 15%. However, this gain decreases as $C_{\min}$ grows. The decrease in the achieved gain is attributable to the fact that increasing $C_{\min}$ will make both modes unable to fulfill this strenuous demand. Accordingly, the outage probability will increase. Similarly, in the case of $D_{m-n} = 30$ m, our switching criterion is able to choose the transmission mode that guarantees a better performance for the network.

The behavior of the network outage probability with $C_{\min}$ can be verified in Fig. 7(a); it shows the variation of the outage probability with $C_{\min}$, for different values of $P_{\rm AP}$. As shown, it can be seen that the outage probability increases by increasing $C_{\min}$. Furthermore, it can be noticed that by decreasing $P_{\rm AP}$ from 10 to 5 W, the outage probability decreases. It must be noticed from (2) and (5) that only $C_{\rm DL|FD}$ will increase by decreasing $P_{\rm AP}$ as a result of the decreased interference level; $C_{\rm DL|HD}$ is independent of $P_{\rm AP}$. However, based on the outage probability definition, decreasing $P_{\rm AP}$ will increase the chance of the FD mode to satisfy the UL constraint. Consequently, the overall outage probability will decrease. Additionally, Fig. 7(b) shows the variation of the outage probability with $C_{\min}$, for different values of the UL transmission power $P$. It can be noticed that increasing $P$ from 50 to 100 mW will result in a decrease in the system outage probability. Similarly, from (2) and (5), it is clear that increasing the value of $P$ will increase the value of the UL received SINR for both FD and HD, respectively. Therefore, the system ability to fulfill the $C_{\min}$ constraint increases. Hence, the outage probability decreases.

Moreover, the effect of $\beta$ on the average achieved DL capacity is shown in Fig. 8(a). It is expected that the FD achieved DL capacity calculated in (6) will be independent of $\beta$. However, it is expected that increasing the spatial correlation between the antennas will decrease the HD achieved DL capacity calculated in (7). This behavior can be verified by the results shown in Fig. 8(a). In addition, it can be observed that for $\beta < 0.8$, HD achieves better DL capacity than that of FD. However, for $\beta > 0.8$, FD starts to outperform HD. Therefore, it can be concluded that the value of $\hat{\beta_2}$, at which $C_{\rm DL|FD}$ calculated in (6) is equal to $C_{\rm DL|HD}$ calculated in (7), is equal to 0.8, which also corresponds to the switching point between HD and FD. In addition, it is clear that, on average, the proposed switching criterion is able to achieve a gain of about 24% for $\beta = 0$. However, this gain starts to decrease until reaching 20% for $\beta = 1$. Finally, the effect of varying the self-interference cancelation parameter on the system performance is shown in Fig. 8(b). As expected, the HD performance is independent of $C$. However, increasing $C$ will reduce the values of the self-interference introduced on the UL transmission in the case of FD. Accordingly, FD becomes more able to keep the $C_{\min}$ constraint. Accordingly, the achieved DL capacity will increase. In addition, our switching criterion efficacy can be verified from Fig. 8(b). It can be observed that the proposed switching criterion can achieve better DL capacity than those of FD and HD.

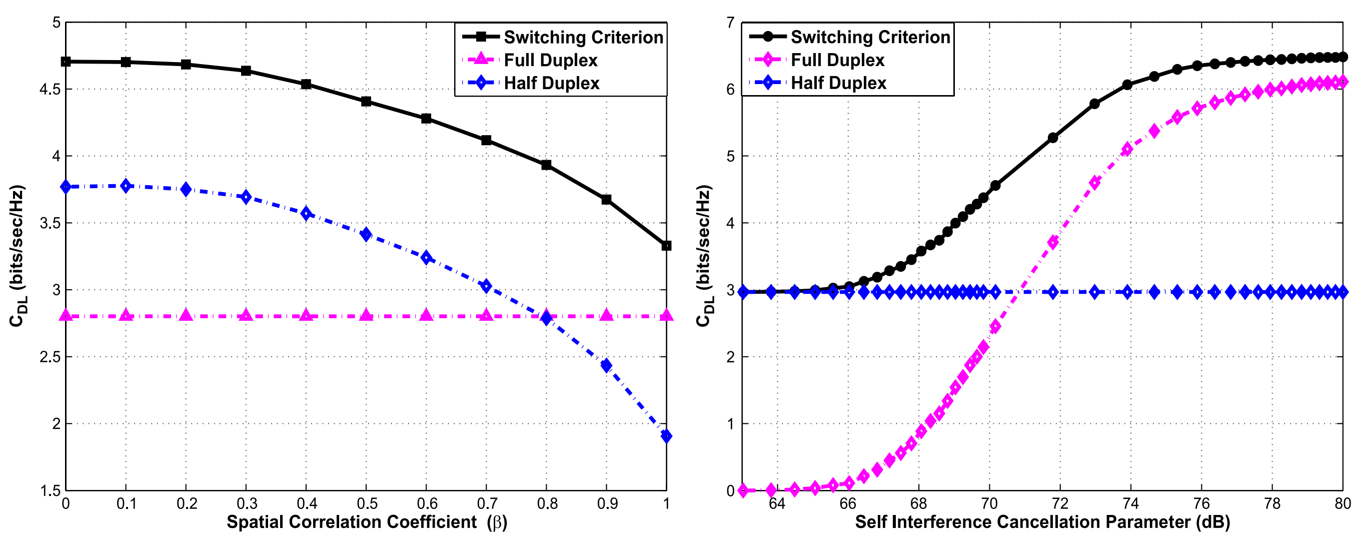
## VI. Conclusion

In this paper, a complete study of the parameters affecting the performances of FD and HD operations has been presented. In addition, we propose a switching criterion that chooses the transmission mode that maximizes the DL total channel capacity while keeping the UL channel capacity at a certain level. Additionally, the switching criterion was formulated as




(a) (b)

Fig. 7. Variation of outage probability with $C_{\min}$ with different values of (a) AP transmission power $P_{\mathrm{AP}}$ and (b) UL transmission power $P$.




(a) (b)

Fig. 8. Variation of $C_{Dl}$ with (a) spatial correlation coefficient $\beta$ and (b) self-interference cancelation parameter $C$.

a resource allocation problem and then solved using the KKT conditions. Furthermore, to identify the network's operation regions, we derived different thresholds for self-interference cancelation parameter, spatial correlation coefficient, and mutual distance between the users. This allowed for a clear definition of outage, FD, and HD operation regions. Moreover, with the aid of numerical results, we validated our optimization problem's solution and the derived thresholds. Furthermore, our system's performance is assessed and compared with both FD and HD performance for parameters from different systems. It was found that our system is able to achieve high performance gains for small values of UL channel capacity constraint and spatial correlation coefficient. In addition, it was revealed that increasing the AP transmission power increases the network outage probability due to the increased interference power introduced to the UL transmission in FD and HD mode. However, increasing the mobile user's transmission power will decrease the network outage probability. This decrease is due to the fact that increasing the value of mobile user transmission power will increase the value of the UL received SINR for both FD and HD. Finally, the effects of varying both the spatial correlation coefficient and the UL channel capacity constraint on the network performance are discussed.

## Appendix A
## Proof of Proposition 1

To solve the constrained optimization problem given by (12) after relaxing $T \in [0, 1]$, we are going to use the KKT conditions. Therefore, the Lagrangian function is given by

$$\begin{aligned} L(T, \lambda_1, \lambda_2, \lambda_3) &= -C_{\mathrm{DL}} + \lambda_1 \left(C_{\min} - T(C_{\mathrm{UL|FD}}) - (1-T)(C_{\mathrm{UL|HD}})\right) \\ &\quad + \lambda_2(-T) + \lambda_3(T-1). \end{aligned} \tag{33}$$

Therefore, the required KKT conditions that must be satisfied are as follows:

$$\frac{\partial L}{\partial T} = 0 \tag{34}$$
$$\lambda_1(C_{\min} - C_{\mathrm{UL}}) = 0 \tag{35}$$
$$\lambda_2(-T) = 0 \tag{36}$$
$$\lambda_3(T-1) = 0 \tag{37}$$
$$\lambda_1, \lambda_2, \lambda_3 \geq 0. \tag{38}$$

The Lagrangian derivative with respect to $T$ is given by

$$\frac{\partial L}{\partial T} = -C_{\mathrm{DL|FD}} + C_{\mathrm{DL|HD}} + \lambda_1(-C_{\mathrm{UL|FD}} + C_{\mathrm{UL|HD}}) - \lambda_2 + \lambda_3. \tag{39}$$

Therefore, the resource allocation problem can be solved in either one of the following three ways.

1) HD transmission only satisfies the UL capacity constraint. This corresponds to the case when $\lambda_1 = 0$, $\lambda_2 > 0$, and $\lambda_3 = 0$. Therefore, the optimization problem solution is given by

$$T^* = 0$$
$$\lambda_2^* = |C_{\mathrm{DL|HD}} - C_{\mathrm{DL|FD}}|. \tag{40}$$

2) FD transmission only satisfies the UL capacity constraint. This corresponds to the case when $\lambda_1 = 0$, $\lambda_2 = 0$, and $\lambda_3 > 0$. Therefore, the optimization problem solution is given by

$$T^* = 1$$
$$\lambda_3^* = |C_{\mathrm{DL|FD}} - C_{\mathrm{DL|HD}}|. \tag{41}$$

3) FD transmission and HD transmission both satisfy the UL capacity constraint. This corresponds to the case when $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_3 = 0$. Therefore, the optimization problem solution is given by

$$T^* = \left|\frac{C_{\min} - C_{\mathrm{UL|HD}}}{C_{\mathrm{UL|FD}} - C_{\mathrm{UL|HD}}}\right|$$
$$\lambda_1^* = \left|\frac{C_{\mathrm{DL|FD}} - C_{\mathrm{DL|HD}}}{C_{\mathrm{UL|HD}} - C_{\mathrm{UL|FD}}}\right|. \tag{42}$$

Therefore, the optimization problem solution will always assure that the UL channel capacity constraint is satisfied and guarantees the maximum achievable DL user channel capacity.

## Appendix B
## Proof of Proposition 2

$C^*$ can be found by equating the achieved UL channel capacity given in (8) by $C_{\min}$ as follows:

$$C_{\min} = \log_2\left(1 + \frac{\mathrm{P}D_{m-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-m}|^2}{\sigma^2 + R^*}\right) + \log_2\left(1 + \frac{\mathrm{P}D_{n-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-n}|^2}{\sigma^2 + R^*}\right) \tag{43}$$

where $R^* = P_{\mathrm{AP}}^{\lambda}/C^*$. The next step is to get rid of the $\log_2$ terms as follows:

$$2^{C_{\min}} = \left(1 + \frac{\mathrm{P}D_{m-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-m}|^2}{\sigma^2 + R^*}\right) \times \left(1 + \frac{\mathrm{P}D_{n-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-n}|^2}{\sigma^2 + R^*}\right). \tag{44}$$

Therefore

$$2^{C_{\min}}(\sigma^2 + R^*)^2 = \left(\sigma^2 + \mathrm{P}D_{m-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-m}|^2 + R^*\right) \times \left(\sigma^2 + \mathrm{P}D_{n-\mathrm{AP}}^{-\alpha}|h_{\mathrm{AP}-n}|^2 + R^*\right). \tag{45}$$

After some mathematical steps, it can be easily derived that

$$C^* = \frac{2P_{\mathrm{AP}}^{\lambda}(2^{C_{\min}} - 1)}{\sqrt{r_1^2 - 4(2^{C_{\min}} - 1)r_2} - r_1} \tag{46}$$

where $r_1$ and $r_2$ are given, respectively, by (16) and (17).

## Appendix C
## Proof of Proposition 3

$D^*$ can be found by equating $C_{\mathrm{DL|FD}}$ given in (6) by $C_{\mathrm{DL|HD}}$ given in (7) as follows:

$$C_{\mathrm{DL|HD}} = \log_2\left(1 + \frac{c_1}{c_2 + D^{*^{-\alpha}}}\right) + \log_2\left(1 + \frac{c_3}{c_2 + D^{*^{-\alpha}}}\right). \tag{47}$$

After removing the $\log_2$ terms, we obtain

$$\left(1 + \frac{c_1}{c_2 + D^{*^{-\alpha}}}\right)\left(1 + \frac{c_3}{c_2 + D^{*^{-\alpha}}}\right) = 2^{C_{\mathrm{DL|HD}}}. \tag{48}$$

Therefore

$$\left(\frac{c_1 + c_2 + D^{*^{-\alpha}}}{c_2 + D^{*^{-\alpha}}}\right)\left(\frac{c_2 + c_3 + D^{*^{-\alpha}}}{c_2 + D^{*^{-\alpha}}}\right) = 2^{C_{\mathrm{DL|HD}}}. \tag{49}$$

After some mathematical steps, it can be proven that

$$D^* = \sqrt[\alpha]{\frac{2(2^{C_{\mathrm{DL|HD}}} - 1)}{\sqrt{d_1^2 - 4d_2(2^{C_{\mathrm{DL|HD}}} - 1)} - d_1}} \tag{50}$$

where $d_1$, $d_2$, $c_1$, $c_2$, and $c_3$ are given in (19)–(22).

## Appendix D
## Proof of Proposition 4

$\hat{\beta}$ can be found by equating the achieved UL channel capacity given in (9) by $C_{\min}$ as follows:

$$= C_{\min} = \log_2\left(1 + \frac{X_1\left|R^{\frac{1}{2}}\mathbf{h_{AP-n}}\right|^2}{X_2 + \left|R^{\frac{1}{2}}\mathbf{h_{AP-m}}\right|^2}\right) + \log_2\left(1 + \frac{X_3\left|R^{\frac{1}{2}}\mathbf{h_{AP-m}}\right|^2}{X_4 + \left|R^{\frac{1}{2}}\mathbf{h_{AP-n}}\right|^2}\right). \tag{51}$$

After removing the $\log_2$ terms, we obtain

$$\left(1+\frac{X_1\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-n}}\right|^2}{X_2+\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-m}}\right|^2}\right)\left(1+\frac{X_3\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-m}}\right|^2}{X_4+\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-n}}\right|^2}\right)=2^{C_{\min}}. \tag{52}$$

It can be easily proved that

$$\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-n}}\right|^2 = H_1 + \hat{\beta}H_2$$
$$\left|R^{\frac{1}{2}}\mathbf{h}_{\mathbf{AP-m}}\right|^2 = H_3 + \hat{\beta}H_4. \tag{53}$$

Therefore, by substituting the values of $|R^{1/2}\mathbf{h}_{\mathbf{AP-n}}|^2$ and $|R^{1/2}\mathbf{h}_{\mathbf{AP-m}}|^2$ in (52), we obtain

$$\left(1+\frac{X_1(H_1+\hat{\beta}H_2)}{X_2+H_3+\hat{\beta}H_4}\right)\left(1+\frac{X_3(H_3+\hat{\beta}H_4)}{X_4+H_1+\hat{\beta}H_2}\right)=2^{C_{\min}}. \tag{54}$$

Finally, after some mathematical steps, we have

$$\hat{\beta} = \frac{\sqrt{b_2^2 - 4b_1b_3} - b_2}{2b_1} \tag{55}$$

where $b_1$, $b_2$, $b_3$, $X_1$, $X_2$, $X_3$, $X_4$, $H_1$, $H_2$, $H_3$, and $H_4$ are given in (24)–(29).

## Appendix E
## Clarifying the Effect of $\beta$ on the Uplink Capacity

Increasing the spatial correlation coefficient $\beta$ degrades the channel quality; therefore, it is expected that both the UL received power and interference power in (5) will decrease. In addition, from the UL capacity calculation in (9), it can be noticed that

$$\begin{aligned}
C_{\mathrm{UL|HD}} &= \log_2(1+\Gamma_{\mathrm{UL}_m|\mathrm{HD}}) + \log_2(1+\Gamma_{\mathrm{UL}_n|\mathrm{HD}}) \\
&= \log_2\left(1+\frac{\mathrm{PD}_{m-\mathrm{AP}}^{-\alpha}\left|R^{\frac{1}{2}}\mathbf{h}_{\mathrm{AP}-m}\right|^2}{\sigma^2+\left|\mathrm{PD}_{n-\mathrm{AP}}^{-\alpha}R^{\frac{1}{2}}\mathbf{h}_{\mathrm{AP}-n}\right|^2}\right) \\
&\quad + \log_2\left(1+\frac{\mathrm{PD}_{n-\mathrm{AP}}^{-\alpha}\left|R^{\frac{1}{2}}\mathbf{h}_{\mathrm{AP}-n}\right|^2}{\sigma^2+\left|\mathrm{PD}_{m-\mathrm{AP}}^{-\alpha}R^{\frac{1}{2}}\mathbf{h}_{\mathrm{AP}-m}\right|^2}\right) \\
&= \log_2\left(1+\frac{G_1}{\sigma^2+G_2}\right) + \log_2\left(1+\frac{G_2}{\sigma^2+G_1}\right) \\
&= \log_2\left(\left(1+\frac{G_1}{\sigma^2+G_2}\right)\times\left(1+\frac{G_2}{\sigma^2+G_2}\right)\right)
\end{aligned} \tag{56}$$

where $G_1 = \mathrm{PD}_{m-\mathrm{AP}}^{-\alpha}|R^{1/2}\mathbf{h}_{\mathrm{AP}-m}|^2$, and $G_2 = |\mathrm{PD}_{n-\mathrm{AP}}^{-\alpha} R^{1/2}\mathbf{h}_{\mathrm{AP}-n}|^2$. Therefore, if $\Gamma_{\mathrm{UL}_m|\mathrm{HD}}$ and $\Gamma_{\mathrm{UL}_n|\mathrm{HD}}$ are relatively greater than 1, additionally, if $G_1$ and $G_2$ are relatively greater than $\sigma^2$, then approximately, decreasing $G_1$ and $G_2$ by increasing $\beta$ will have a small effect on the UL capacity, as the decrease in the received power is compensated by a decrease in the interference power. In other words, the range of variation of the UL capacity corresponding to changing $\beta$ from 0 to 1 will be very small. This explains the results previously obtained in Fig. 5.

## References


[1] A. Sabharwal *et al.*, "In-band full-duplex wireless: Challenges and opportunities," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1637–1652, Jun. 2014.

[2] A. Tulino, A. Lozano, and S. Verdu, "Impact of antenna correlation on the capacity of multiantenna channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 7, pp. 2491–2509, Jul. 2005.

[3] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *Conf. Rec. 44th ASILOMAR Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2010, pp. 1558–1562.

[4] J. Bai and A. Sabharwal, "Decode-and-cancel for interference cancellation in a three-node full-duplex network," in *Conf. Rec. 46th ASILOMAR Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2012, pp. 1285–1289.

[5] A. Thangaraj, R. Ganti, and S. Bhashyam, "Self-interference cancellation models for full-duplex wireless communications," in *Proc. Int. Conf. SPCOM*, Bangalore, India, Jul. 2012, pp. 1–5.

[6] M. Jainy *et al.*, "Practical, real-time, full duplex wireless," in *Proc. ACM MobiCom*, Las Vegas, NV, USA, Sep. 2011, pp. 301–312.

[7] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in *Proc. ACM SIGCOMM*, Hong Kong, Aug. 2013, pp. 375–386.

[8] R. Sultan, L. Song, and Z. Han, "Impact of full duplex on resource allocation for small cell networks," in *Proc. IEEE GlobalSIP Conf.*, Atlanta, GA, USA, Dec. 2014, pp. 1257–1261.

[9] H. Cui, M. Ma, L. Song, and B. Jiao, "Relay selection for two-way full duplex relay networks with amplify-and-forward protocol," *IEEE Trans. Commun.*, vol. 13, no. 7, pp. 3768–3777, Jul. 2014.

[10] B. Di, S. Bayat, L. Song, and Y. Li, "Radio resource allocation for full-duplex OFDMA networks using matching theory," in *Proc. IEEE INFOCOM WKSHPS*, Toronto, ON, Canada, May 2009, pp. 197–198.

[11] Y. Bo, L. Yang, X. Cheng, and R. Cao, "Transmit power optimization for full duplex decode-and-forward relaying," in *Proc. IEEE GLOBECOM*, Atlanta, GA, USA, Dec. 2013, pp. 3347–3352.

[12] M. J. Emadi, A. G. Davoodi, and M. R. Aref, "Analytical power allocation for a full-duplex decode-and-forward relay channel," *IET Commun.*, vol. 7, no. 13, pp. 1338–1347, Sep. 2013.

[13] S. Li, R. D. Murch, and V. K. Lau, "Linear transceiver design for full-duplex multi-user MIMO system," in *Proc. IEEE ICC*, Jun. 2014, pp. 4921–4926.

[14] H. Kim, S. Lim, H. Wang, and D. Hong, "Optimal power allocation and outage analysis for cognitive full duplex relay systems," *IEEE Trans. Commun.*, vol. 11, no. 10, pp. 3768–3777, Oct. 2012.

[15] D. Ng, E. Lo, and R. Schober, "Dynamic resource allocation in MIMO-OFDMA systems with full-duplex and hybrid relaying," *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1291–1304, Apr. 2012.

[16] A. C. Cirik, R. Wang, Y. Hua, and M. Latva-aho, "Weighted sum-rate maximization for full-duplex MIMO interference channels," *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 801–815, Mar. 2015.

[17] W. Li, J. Lilleberg, and K. Rikkinen, "On rate region analysis of half- and full-duplex OFDM communication links," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1688–1698, Jun. 2014.

[18] D. Nguyen, L.-N. Tran, P. Pirinen, and M. Latva-aho, "On the spectral efficiency of full-duplex small cell wireless systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 4896–4910, Sep. 2014.

[19] V. Aggarwal and N. Shankaranarayanan, "Performance of a random-access wireless network with a mix of full- and half-duplex stations," *IEEE Commun. Lett.*, vol. 17, no. 11, pp. 2200–2203, Oct. 2013.

[20] N. Shende, O. Gurbuz, and E. Erkip, "Half-duplex or full-duplex relaying: A capacity analysis under self-interference," in *Proc. 47th Annu. CISS*, Baltimore, MD, USA, Mar. 2013, pp. 1–6.

[21] G. Zheng, E. Jorswieck, and B. Ottersten, "Cooperative communications against jamming with half-duplex and full-duplex relaying," in *Proc. IEEE 77th VTC Spring*, Dresden, Germany, Jun. 2013, pp. 1–5.

[22] V. Aggarwal, M. M. Duarte, A. Sabharwal, and N. Shankaranarayanan, "Full- or half-duplex? A capacity analysis with bounded radio resources," in *Proc. IEEE ITW*, Lausanne, Switzerland, Sep. 2012, pp. 207–211.

[23] A. C. Cirik, Y. Rong, and Y. Hua, "Achievable rates of full-duplex MIMO radios in fast fading channels with imperfect channel estimation," *IEEE Trans. Signal Process.*, vol. 62, no. 15, pp. 3874–3886, Aug. 2014.

[24] T. Riihonen, S. Werner, and R. Wichman, "Comparison of full-duplex and half-duplex modes with a fixed amplify-and-forward relay," in *Proc. IEEE WCNC*, Budapest, Hungary, Apr. 2009, pp. 207–211.

[25] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex bidirectional MIMO: Achievable rates under limited dynamic range," *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3702–3713, Jul. 2012.

[26] Y. El Hajj Shehadeh and S. Sezginer, "Fast varying channel estimation in downlink LTE systems," in *Proc. IEEE 21st Int. Symp. PIMRC*, Istanbul, Turkey, Sep. 2010, pp. 608–613.

[27] H. Sahlin and A. Persson, "Aspects of MIMO channel estimation for LTE uplink," in *Proc. IEEE VTC Fall*, Sep. 2011, pp. 1–5.

[28] X. Xia, H. Zhao, and C. Zhang, "Improved SRS design and channel estimation for LTE-advanced uplink," in *Proc. IEEE 5th Int. Symp. MAPE EMC Technol. Wireless Commun.*, Oct. 2013, pp. 84–90.

[29] Q. Li *et al.*, "On channel estimation for multi-user MIMO in LTE-A uplink," in *Proc. IEEE 79th VTC Spring*, May 2014, pp. 1–5.

[30] M. Duarte, C. Dick, and A. Sabharwal, "Experiment-driven characterization of full-duplex wireless systems," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4296–4307, Nov. 2012.

[31] M. K. Awad, V. Mahinthan, J. W. Mark, and X. Shen, "Impact of imperfect channel state information on cooperative communications," in *Proc. IEEE PacRim Conf. Commun., Comput. Signal Process.*, Victoria, BC, Canada, Aug. 2011, pp. 232–237.

[32] K. Akcapinar and O. Gurbuz, "Full-duplex bidirectional communication under self-interference," in *Proc. 13th Int. ConTEL*, Jul. 2015, pp. 1–7.

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