Joint Resource Management with Distributed Uplink Power Control in Full-Duplex OFDMA Networks

Radwa Sultan[†], *IEEE Member*, Lingyang Song^{*}, *IEEE Senior Member*, Karim G. Seddik[‡], *IEEE Senior Member*, and Zhu Han^{** §}, *IEEE Fellow*

[†] Electrical and Computer Engineering Department, Manhattan College, NY, USA

* School of Electrical Engineering and Computer Science, Peking University, Beijing, China

[‡] Electronics and Communications Engineering Department, American University in Cairo, New Cairo, Egypt

** Electrical and Computer Engineering Department, University of Houston, TX, USA

[§] Department of Computer Science and Engineering, Kyung Hee University, Seoul, South Korea rsultan02@manhattan.edu, lingyang.song@pku.edu.cn, kseddik@aucegypt.edu, zhan2@uh.edu

Abstract-Resource allocation problems in full-duplex orthogonal frequency division multiple access (FD-OFDMA) networks are challenging due to their combinatorial, non-convex nature. In this paper, user pairing, subcarrier and power allocation in a single cell FD-OFDMA network are considered. A joint optimization problem is formulated to maximize the network's sum rate while satisfying downlink (DL) and uplink (UL) transmission power' constraints. Due to the sheer complexity of the proposed formulation, mainly due to its combinatorial nature, an efficient, iterative two-step solution algorithm for the joint problem is proposed. In the first step, based on defining the DL user equipment (UE) signal to noise ratio (SNR) threshold, which is the least SNR that can be detected by the DL-UE, an algorithm is proposed for user pairing and subcarrier assignment. In the second step, the power allocation problem for the assigned users' pairs is formulated and solved using the Alternating Direction Method of Multipliers (ADMM) in the high signalto-interference-noise ratio regime. Finally, numerical results are presented to validate the performance of the proposed algorithms. We show that the performance of our proposed computationallyefficient two-step algorithm is very close to the sum rate upper bound derived from solving the dual problem.

Keywords. ADMM, Full Duplex, OFDMA, Power Allocation, Resource Allocation, SNR Threshold, Subcarrier Allocation.

I. INTRODUCTION

Modern wireless communication networks are continuously required to offer a significant increase in the network capacity in order to be able to support the enormous growth in the number of wireless communication users. Accordingly, efficient resource allocation algorithms have become a crucial need. However, most of the existing communication networks waste the available resources by utilizing half-duplex (HD) communication. Theoretically speaking, enabling the network nodes to simultaneously transmit and receive data at the same time slot and same channel, i.e. utilizing full-duplex (FD) communication, can double the aggregate network throughput [1].

A. Literature Review

Although FD communication was considered unfeasible, because of the high self-interference (SI) from the node transmission on the node reception, the recent evolution in

SI cancellation techniques [2]-[5] reinvigorates the attention to FD communication and nominates the FD communication as a technique that is able to supply the needed high rates [6]. In [7], it was shown that both user diversity gain and FD communication gain can be achieved, and the performance of FD communication highly depends on the strength of the residual SI. Additionally, studying the recent development and future directions of resource allocation in different FD systems attracts recent research work [8]-[10] to explore the new network resources in different domains, including power, space, frequency, and device dimensions. It is found that FD can outperform HD in both interference-unaware and interference-aware scenarios [11]. Additionally, in [12], the authors derived necessary conditions for the FD mode achieves a better energy efficiency (EE)-spectral efficiency (SE) tradeoff than the HD mode. Moreover, it was proved that it is possible for the industry to design the optimal EE-oriented resource allocation strategy while guaranteeing a given required SE. Accordingly, to fully utilize the available FD resources while taking into consideration the new challenges that will arise from deploying FD, efficient and novel resource allocation schemes are strongly needed [13], [14].

In addition, deploying FD in multiple access networks like orthogonal division multiple access (OFDMA) recently gains a lot of attention. However, different from HD-OFDMA networks that require subcarrier (SC) allocation, in FD-OFDMA each subcarrier serves simultaneous transmissions in the uplink (UL) and the downlink (DL) modes. Therefore, efficient pairing between users transmitting in the UL and users receiving in the DL into independent transceivers is required to decrease the co-channel interference (CCI) introduced from the UL transmission on the DL reception¹. This additional optimization requirement increases the complexity of the resource allocation in FD-OFDMA networks due to the combinatorial nature of the subcarrier assignment and users' pairing. In [15], a joint subcarrier scheduling and power allocation problem to maximize the sum rate under both perfect and imperfect SI cancellation scenarios is proposed.

¹Throughout the paper, users who are transmitting in the UL are denoted by UL users and users who are receiving in the DL are denoted by DL users. For the perfect cancellation scenario, subcarrier scheduling and power allocation are optimized by applying the Lagrange duality method. For the imperfect cancellation, an iterative algorithm based on the projected gradient method is proposed. In [16], the joint problem of subchannel assignment and power allocation in FD-OFDMA network considering the inter-node interference is investigated with both full and limited channel state information (CSI) knowledge. In the case of limited CSI, a low-complexity inter-node interference estimation is presented. In [17], the joint optimization problem of transmission mode selection, subcarrier assignment, relay selection, subcarrier pairing as well as power allocation is investigated for OFDMA networks. The binary assignment problem is transformed into a maximum weighted bipartite matching problem which can be solved by the classical Hungarian method.

The aforementioned work did not consider the user pairing optimization in maximizing the FD-OFDMA sum rate. However, user pairing is considered in [18], [19], in which the joint optimization problem of subcarrier assignment, UL-DL user pairing, and power allocation is solved by the dual method in which it is decomposed into a primal problem and a dual problem. The concave-convex procedure is used to transform the primal problem into a tractable form through sequential convex approximations while the sub-gradient method is utilized to solve the dual problem. In [20], the effects of different system parameters on the FD-OFDMA network performance are studied. Moreover, a joint resource allocation problem which aims at maximizing the network sum rate by considering mode selection, user pairing, subcarrier allocation and power control is proposed and solved by relaxing the subcarrier assignment variables to the continuous domain. In [21], the FD-OFDMA allocation problem is discussed and solved using the matching theory. In [22], a joint algorithm that aims at maximizing the utility sum of users while fully exploiting the capacity benefit of FD communication is proposed. The key idea of the proposed algorithm is the assignment of a transmission mode, users and transmit power levels jointly for a frequency resource block, which is a group of contiguous subcarriers, based on the awareness of residual SI. Moreover, since maximizing the network's sum rate will affect the fairness among the netwrk's users, some work discussed the fairness problem in FD-OFDMA networks. Fairness can be achieved through multiple approaches like maximizing the max-min fairness rate, by imposing the minimum rate constraint for each UL and DL user, or guaranteeing at least on subcarrier for each UL and DL user. Maximizing the system max- min fairness rate in an FD multi-user OFDMA system is addressed in [23], where the uplink/downlink transmission direction assignment, user paring, and power allocation problem are jointly optimized to maximize the system max-min fairness rate. To solve the joint NP hard problem, the authors proposed efficient methods based on simple relaxation and greedy rounding techniques. In [24], the authors proposed a queue-aware, fair scheduling and power allocation problem for FD-OFMDA networks. The proposed problem aims at maximizing the user equipments (UE) signal-to-interference noise ratio (SINR) values, while at the same time enforcing fairness among the UEs. Solving

such a problem requires information on the UE radio conditions, their queue statuses, as well as an innate definition of fairness. Accordingly, the authors define a UE pair priority and formulate the problem with the objective of maximizing the sum of these priorities. It is shown that the proposed approach improves fairness among the user equipment at no cost in the system's performance.

B. Contributions

To maximize the network DL and UL sum rate while satisfying transmission power constraints, we consider the joint users' pairing, subcarrier allocation, and power allocation problem. Our main contributions in this work are as follows

- We propose a computationally efficient, polynomialcomplexity joint user pairing and subcarrier allocation algorithm which is based on defining the least detectable received SNR by the UE, i.e., the signal-to-noise (SNR) threshold [25], [26]. The existence of the SNR threshold is a practical specification of the DL-UE that will help in defining a set of candidate DL, UL, and subcarriers that have a very low CCI level. It is shown that the complexity of the proposed algorithm is lower than the algorithm presented in [21].
- 2) Different from the water filling power allocation [27] presented in [21], we analyze and solve the power allocation problem for the proposed FD-OFDMA network. The power allocation problem's approximation for the high signal-to-interference noise ratio (SINR) is proved to be convex, and hence, it is formulated and solved distributively using the Alternating Direction Method of Multipliers (ADMM) [28], [29]². Additionally, the proposed power allocation problem is different from the power allocation problem in the FD bidirectional channel presented in [31] as the power allocation in the case of FD-OFDMA is done over all subcarriers which results in different objective function and constraints.
- 3) From numerical results, it is shown that the convergence of the proposed power allocation algorithm is faster than the centralized interior-point based algorithm. Additionally, we show that our proposed algorithm can achieve a performance that is very close to the optimal solution. Additionally, these results validate the effectiveness of the proposed low-complexity algorithm in achieving a good performance that is close to the upper bound obtained by the iterative approximation approach considered in [19];

however, it should be stated that proposed sum rate maximization may not achieve fairness among the network's users, however, fairness is out of our work scope and will be

²The optimal power allocation in FD-OFDMA networks is different from the traditional water-filling in HD-OFDMA [30] as in HD-OFDMA, both the UL and DL transmissions are independent, in which water-filling is proved to be optimal. The main idea of water-filling is to allocate more power for the channel with better signal-to-interference noise ratio (SINR) either in the UL or the DL transmission. However, in the case of FD, the presence of residual SI from the DL transmission on the UL transmission, and the presence of the CCI from the UL transmission on the DL transmission will make both UL and DL transmissions dependent. Hence, the optimal power allocation will be different.



Fig. 1: System model with user pairing and subcarrier allocation.

considered in our future work. As mentioned before, fairness in FD-OFDMA networks is discussed in [23], [24].

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the joint user pairing, subcarrier, and power allocation problem, along with the proposed solution algorithm are presented. In Section IV, numerical analysis is presented to validate the performance of the proposed solution algorithm. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

In this paper, we consider a single cell, time division duplex (TDD) network with a FD-AP operating in OFDMA with S subcarriers. In order to realize FD feasibility, the AP is equipped with a special FD radio which provides linear cancellation, non-linear cancellation, and analog cancellation to cover up for the self-interference, nonlinear harmonic components, quantization and transmitter noise [2]-[4]. All subcarriers are assumed to be perfectly orthogonal, i.e., there is no inter-subcarrier interference. There are N HD single antenna users, and therefore, in a given time slot, the AP connects with N/2 UL users and N/2 DL users. In every time slot, the AP assigns a given subcarrier s to simultaneously serve the n^{th} UL user transmission along with the m^{th} DL user transmission. In that case, the received UL signal at the AP will be affected by the SI from the AP DL transmission. Additionally, the received signal from the AP at the m^{th} DL user will suffer from the CCI from the UL transmission that shares the same subcarrier. Therefore, in order to improve the network spectral efficiency, it is needed to decrease the interference on both the UL and DL transmission. The SI on the UL transmission is controlled by the FD radio implemented at the AP [2]-[4]. On the other hand, the CCI on the DL transmission can be controlled by proper pairing between the DL and UL users that share the same subcarriers. In other words, the DL-UL pair that is chosen to share a given subcarrier should guarantee a small CCI. The system model with user pairing and subcarrier allocation is shown in Fig. 1; as shown, pairing is most probable between UL and DL users which are far away from each other to limit the interference from the UL transmission on the DL transmission.

Additionally, in order to maximize the network sum rate, it is needed to optimize the power allocation among different subcarriers for both the DL and UL transmissions. Based on the above assumptions, if we consider that the m^{th} DL user transmission is paired with the n^{th} UL user transmission on the s^{th} subcarrier, then the received SINR for the m^{th} DL user is given by

$$\Gamma_{m|DL_s} = \frac{P_s D_{m-AP}^{-\alpha} |h_{m-AP}^s|^2}{\sigma^2 + CCI_{m-n}^s},$$
(1)

where, P_s is the s^{th} subcarrier DL AP transmission power, $D_{m-AP}^{-\alpha}$ denotes the large scale propagation fading between the m^{th} user and the AP with distance D_{m-AP} and path loss exponent α , h_{m-AP}^s is the channel coefficient between the m^{th} user and the AP transmission antenna on the s^{th} subcarrier³, where all the channel coefficients are assumed to be an i.i.d. zero mean complex Gaussian random variables with unit variance, i.e., Rayleigh fading, σ^2 is the additive white Gaussian noise (AWGN) variance, and CCI_{m-n}^s denotes the co-channel interference on the m^{th} DL user from the n^{th} UL user transmission; the value of CCI_{m-n}^s is given by

$$CCI_{m-n}^{s} = P_{ns}D_{m-n}^{-\alpha}|h_{m-n}^{s}|^{2},$$
(2)

where P_{ns} is the n^{th} UL transmission power in a given subcarrier s. As clear from (2), the value of the CCI is decreased by choosing a DL-UL pair with large mutual distance D_{m-n} . Furthermore, the received SINR from the n^{th} UL user at the AP receiving antenna is given by

$$\Gamma_{n|UL_{s}} = \frac{P_{ns} D_{n-AP}^{-\alpha} |h_{AP-n}^{s}|^{2}}{\sigma^{2} + P_{s}/C},$$
(3)

where P_s/C represents the residual SI (RSI) after using a FD radio with a cancellation parameter C > 1, which is available at the AP [2], [32]. Accordingly, the value of the RSI is controlled by the implemented FD radio. From the received DL SINR and the received UL SINR, calculated in (1) and (3), respectively, the network DL and UL sum rates per unit time and unit bandwidth (*bits/sec/Hz*) are given, respectively, by

$$R_{T|DL} = \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} \sum_{s=1}^{S} a(m, n, s) \log_2(1 + \Gamma_{m|DL_s}),$$

$$R_{T|UL} = \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} \sum_{s=1}^{S} a(m, n, s) \log_2(1 + \Gamma_{n|UL_s}),$$
(4)

where the first summation sums over all DL users, the second summation sums over all UL users, and the last summation

 $^{^3 {\}rm Throughout}$ this paper, h^s_{x-y} denotes the channel coefficient between the transmitter y and the receiver x on the s subcarrier.

Algorithm 1: Proposed Joint User Pairing and Subcarrier Allocation Algorithm

Data: UL channels CSI, DL channels CSI, CCI information between each UL-DL user-pair, P_{max} , $P_{ul|max}$, α , SNR_{th} **Result**: Find $a(m, n, s) \forall m \in \{1..N/2\}, n \in \{1..N/2\},$ and $s \in \{1..S\}$

Initially:

 $\Phi_i = \emptyset,$

1. Form the initial candidate pair set Φ_i given in (6) 2. while $\Phi_i \neq \emptyset$ do

 $\forall s^* \in \{1..S\}$ $if \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} a(m, n, s^*) = 1$ then $a. There is no conflict for the given subcarrier s^*,$ $b. <math>\Phi_f = \Phi_f \cup a(m, n, s^*),$ $c. Update the candidate pairs set <math>\Phi_i$ by removing the assigned subcarrier s^* else

a. There is a conflict between more than one DL-UL pairs on the given subcarrier s^* , or there are no UL-DL pair on s^* with $CCI_{m-n}^s/\sigma^2 < SN\mathcal{R}_{th}$ b. Choose trio (m, n, s^*) that achieves highest sum rate on s^*

sums over all subcarriers. The coefficient $a(m, n, s) \in \{0, 1\}$ is a binary variable that indicates the user-pairing and the subcarrier assignment. a(m, n, s) = 1 means that the m^{th} DL user is paired with the n^{th} UL user and the (m-n) pair is served by the s^{th} subcarrier; otherwise, the value of a(m, n, s)will be equal to zero. Finally, the network sum rate per unit time and unit bandwidth (bits/sec/Hz) is the sum of the DL and UL rates calculated in (4), and is given by

$$R_T = R_{T|DL} + R_{T|UL}.$$
(5)

III. JOINT USER PAIRING, SUBCARRIER AND POWER Allocation in Full-Duplex OFDMA Networks

In this section, we optimize the user pairing, power allocation, and subcarrier allocation to maximize the UL and DL network sum throughput given in (5), while satisfying the transmission power constraints imposed on the AP and the UEs. When formulating the joint allocation problem, it should be noted that each of the DL and UL users is allowed to receive and send data, respectively, on multiple subcarriers. However, each subcarrier is allowed to be assigned only once to a single transceiver pair. Therefore, the joint resource allocation problem is given by

$$\max_{\mathbf{A},\mathbf{P}_{s},\mathbf{P}_{ns}} R_{T}$$
s.t.
$$\sum_{s=1}^{S} P_{s} \leq P_{max},$$

$$\sum_{s=1}^{S} P_{ns} \leq P_{ul|max}, \forall n \in \{1 \cdots N/2\},$$

$$\sum_{n=1}^{N/2} \sum_{m=1}^{N/2} a(n,m,s^{*}) = 1 \forall s^{*} \in \{1 \cdots S\},$$
(P1)

where A is a vector that contains all the a(m, n, s) variables for all combinations of UL users, DL users, and subcarriers. $\mathbf{P_s} = [P_1, P_2, \dots P_S]^T$ is a vector that includes the DL transmission powers on each subcarrier and P_{ns} = $[P_{11}, P_{21}, \dots, P_{N/2 1}, P_{12}, P_{22}, \dots, P_{N/2S}]^T$ is a vector that includes all the UL users transmission powers on different SCs. The first constraint in (P1) guarantees that the AP total transmission power will not exceed the maximum allowable transmission power P_{max} . Similarly, the second constraint is to limit the UL transmission power to its maximum value denoted by $P_{ul|max}$. Finally, the last constraint guarantees that each subcarrier is assigned to a single pair. It must be noticed that the formulation proposed in (P1) is a hard problem due to its combinatorial nature as a result of the presence of the binary variables a(m, n, s). Therefore, obtaining the optimal solution using brute-force exhaustive search will be very challenging especially for a large number of users and subcarriers. Accordingly, a suboptimal solution algorithm is proposed. First, we propose a solution algorithm for joint user pairing and subcarrier allocation to find the values of a(m, n, s) variables. Second, we derive a solution for the power allocation problem. Finally, we explain the iterative algorithm for solving the joint resource allocation problem in **(P1)**.

A. Joint User Pairing and Subcarrier Allocation Solution Algorithm

In the beginning, to pair the DL and UL users into independent transceivers, we must reassure that the main purpose of the pairing process is to decrease the CCI. Therefore, it is not required to estimate the channel between each DL-UL pair, it is sufficient to estimate the SNR or the received powers between each DL-UL pair and choose the ones with the least interference ⁴. Therefore, during the UL channels' estimation ⁵, the DL users can overhear the UL users' pilot transmission to the AP and report the UL interference levels estimation results to the AP. However, if the received SNR from specific UL transmissions on some particular subcarriers is below

⁴Due to the centralized nature of the cellular network, in which the AP fully control the users procedures, the AP is capable of adjusting the channel estimation procedures with the existing network's users.

⁵The UL channels can be estimated by having the UL users periodically inserting reference pilot signals in the transmitted data. The transmission patterns of these reference signals are adjusted such that in a given time-frequency resource, a single UL user will be sending its reference signal. For more information on channel estimation techniques for OFDM networks, please refer to [33], [34].

	$Subcarrier_1$			$Subcarrier_2$			$Subcarrier_3$			$Subcarrier_4$						
	r	t	w	v	r	t	w	v	r	t	w	v	r	t	w	v
a						\checkmark										
b															\checkmark	
c									\checkmark	\checkmark			\checkmark			
d				\checkmark						\checkmark						\checkmark

TABLE I: Reporting received power between different UL-DL pairs

the DL-UE SNR threshold, the DL user will not report any received powers from these UL transmissions ⁶. Accordingly, it is favorable to pair the DL user with these UL transmissions. Obviously, by following this pairing procedure, the CCI can be significantly reduced, and hence, the sum rate is expected to increase. The proposed pairing and subcarrier assignment algorithm is described in Algorithm 1. At the beginning, after having all the required UL channels, DL channels and UL-DL SNRs available at the AP, the AP constructs an initial candidates' set Φ_i by including all (m, n, s) combinations that were not reported to the AP in the training phase. In other words, in the training phase, the n^{th} UL transmission was not detected by the m^{th} DL user on the s^{th} subcarrier because the received SNR at the DL user is smaller than the SNR threshold detectable at the DL UE denoted by SNR_{th} . Accordingly, Φ_i is defined as,

$$\Phi_{i} = \{(m, n, s), m, \in \{1 \cdots N/2\}, n \in \{1 \cdots N/2\}, \\ s \in \{1 \cdots S\} | CCI_{m-n}^{s} / \sigma^{2} < SN\mathcal{R}_{th}\},$$
(6)

where CCI_{m-n}^s/σ^2 is the received SNR from the n^{th} UL transmission at the m^{th} DL user on the s^{th} subcarrier; the value of CCI_{m-n}^s is given in (2). Since each subcarrier is allowed to be assigned once to a single DL-UL pair, therefore, the next step is to check for conflicts between different DL-UL pairs on different SCs. If there is no conflict on a given SC, i.e., there is only one DL-UL pair that satisfies the pairing condition indicated in (6) on that specific subcarrier, this (m, n, s) trio will be included in the final candidates set Φ_f . On the other side, if a conflict exists on a given SC, the algorithm will pick the DL-UL pair that achieves the highest sum rate on that SC, and includes the picked pair along with that SC in the final set Φ_f . Therefore, for a DL user to be paired with an UL user on a certain subcarrier, the following conditions must be satisfied:

- 1) the received SNR from the UL transmission is below the DL user SNR threshold,
- 2) when a conflict occurs between multiple pairs, the pair with highest sum rate is chosen.

It should be noticed that the complexity of joint user pairing and subcarrier allocation described in Algorithm 1 is in the order of $\mathcal{O}(SN^2)$. When comparing the exhaustive search solution complexity with the complexity of the proposed algorithm, it can be readily seen that the exhaustive search complexity grows exponentially with the number of users and subcarriers, i.e. $\mathcal{O}((N)^{2S})$. Additionally, the transmittersubchannel-receiver threesided matching algorithm, proposed in [21], is in the order of $\mathcal{O}(N^3S^2)$, while as mentioned above, the proposed algorithm complexity is polynomial in the number of users and subcarriers, in other words, it has lower complexity than the exhaustive search and the algorithm proposed in [21].

For a better understanding of the joint user pairing and subcarrier allocation, we provide an illustrative example. Consider a system with 4 DL users, 4 UL users, and 4 subcarriers. As mentioned before, in the UL channels estimation, the DL users will overhear the UL pilot transmission, and report the received SINR to the AP. Accordingly, after the training phase, the AP will have a report for the received power between each DL and UL users pair, as shown in Table I, where $\{a, b, c, d\}$ is the set of DL users, and $\{r, t, w, v\}$ is the set of UL users. The check marks indicate that the received SNR at the DL user from a given UL on a given SC is below the SNR threshold. The first step is to define the set Φ_i , in which the AP determines the candidate pairs whose received SNRs between the UL and DL users are less than SNR_{th} . Accordingly, Φ_i is given by

$$\Phi_i = \{ (d, v, 1), (a, t, 2), (c, r, 3), (c, t, 3), (d, t, 3), (b, w, 4), (d, v, 4), (c, r, 4) \}.$$
(7)

From the candidate pairing chances obtained in Φ_i , it can be noticed that there are no conflicting pairs on both the first and the second subcarriers. However, for subcarrier 3 and subcarrier 4, the choice among conflicting pairs will be based on maximizing the sum rate. Accordingly, after calculating the sum rates for (c, r, 3), (c, t, 3), (d, t, 3) and (b, w, 4), (d, v, 4), (c, r, 4) and choosing the ones with highest sum rates, the final pairing set will be given by

$$\Phi_f = \{ (d, v, 1), (a, t, 2), (c, r, 3), (b, w, 4) \}.$$
(8)

The next step is to allocate power to the transceiver pairs on the given subcarriers specified by Φ_f .

B. Power Allocation in FD OFDMA

The next step after user pairing and subcarrier allocation is to optimize the power allocation among the existing pairs. The solution of the power allocation problem in FD OFDMA will be different from the traditional water-filling in HD networks due to the correlation between the UL and DL transmissions.

⁶We should note that, during the pilot transmission, it is assumed that the power is uniformly distributed among the subcarriers. Pilot transmission power adjustment is out of our work scope. Some pilot transmission power optimization techniques can be found in [35], [36].

Algorithm 2: Power Allocation Algorithm for OFDMA FD network

- **Data:** all CSI information, P_{max} , $P_{ul|max}$, α , $\mathbf{P_s^0}$, $\mathbf{P_{ns}^0}$, $\mathbf{Z_s^0}$ and $\mathbf{Z_{ns}^0}$, λ
- **Result**: Find \mathbf{P}_{s} and \mathbf{P}_{ns} maximizing R_{T} while keeping total transmission power constraints P_{max} for the AP and $P_{ul|max}$ for the UL transmissions.

Intially:

$$\begin{split} \mathbf{P}_{s} &= \mathbf{P}_{s}^{0}, \, \mathbf{P}_{ns} = \mathbf{P}_{ns}^{0}, \\ \mathbf{Z}_{s} &= \mathbf{Z}_{s}^{0}, \, \, \mathbf{Z}_{ns} = \mathbf{Z}_{ns}^{0} \\ k &= 1; \\ 1. \, P_{s}^{k+1} &:= \mathbf{prox}_{\lambda f(P_{s})}(P_{s}^{k} - Z_{s}^{k} - U_{s}^{k}), \\ 2. \, Z_{s}^{k+1} &:= \mathbf{prox}_{\lambda g(P_{ns})}(P_{ns}^{k} - Z_{ns}^{k} - u^{k}), \\ 3. \, P_{ns}^{k+1} &:= \mathbf{prox}_{\lambda g(P_{ns})}(P_{ns}^{k} - Z_{ns}^{k} - u^{k}), \\ 4. \, Z_{ns}^{k+1} &:= \mathbf{I}_{\mathcal{C}_{\mathcal{U}\mathcal{L}}}(P_{ns}^{k+1} + U_{ns}^{k}), \\ 5. \, U_{s}^{k+1} &:= U_{s}^{k} + P_{s}^{k+1} - Z_{ns}^{k+1}, \\ 6. \, U_{ns}^{k+1} &:= U_{ns}^{k} + P_{ns}^{k+1} - Z_{ns}^{k+1}, \\ 7. \, \text{if } P_{s} &= Z_{s} \, \forall \, s \in \{0,S\} \text{ and } P_{ns} = Z_{ns} \, \forall \, s \in \{0,S\} \text{ and } n \in \{1,N/2\} \text{ then} \\ P_{s} &= P_{ns}^{k+1} \\ P_{ns} &= P_{ns}^{k+1} \, \forall \, s \in \{0,S\} \text{ and } n \in \{1,N/2\} \text{ end} \\ \text{else} \\ k = k + 1 \\ \text{Return to step. 1} \end{split}$$

The power allocation problem for the proposed model is derived from the initial problem formulated in (P1) after considering only the active subcarriers which are assigned to the formed users' pairs in the user pairing and subcarrier allocation step. For instance, in the illustrative example, after obtaining the final pairing set is given in (8), the sum rate R_T^{ex} is given by

$$R_T^{ex} = \log_2(1 + \Gamma_{d|DL_1}) + \log_2(1 + \Gamma_{v|UL_1}) + \log_2(1 + \Gamma_{a|DL_2}) + \log_2(1 + \Gamma_{t|UL_2}) + \log_2(1 + \Gamma_{c|DL_3}) + \log_2(1 + \Gamma_{r|UL_3}) + \log_2(1 + \Gamma_{b|DL_4}) + \log_2(1 + \Gamma_{w|UL_4}).$$
(9)

Therefore, the power allocation problem is given by

$$\max_{\mathbf{P}_{s}, \mathbf{P}_{ns}} R_{T}$$
s.t.
$$\sum_{s=1}^{S} P_{s} \leq P_{max},$$

$$\sum_{s=1}^{S} P_{ns} \leq P_{ul|max}, \forall n \in \{1 \cdots N/2\}.$$
(P2)

In solving the power allocation problem, we will consider the high SINR case because the main objective of the user pairing and subcarrier allocation step is to guarantee the least possible interference as well as the highest possible rate on each subcarrier. Therefore, after the pairing and subcarrier assignment step, it is reasonable to consider the high SINR case ⁷. In that case, the proposed power allocation problem can be readily proved to be approximately convex, which can be solved using ADMM. First, we start by proving the convexity of the power allocation problem at high SINR approximation. Afterwards, we present the steps of the ADMM solution algorithm.

Proposition 1. In high SINR case, the power allocation problem (**P2**) is approximately a convex optimization problem. It is rewritten as

$$\min_{\mathbf{P_s, P_{ns}}} F(\mathbf{P_s}) + \mathbf{G}(\mathbf{P_{ns}})$$
s.t.
$$\sum_{s=1}^{S} P_s \le P_{max},$$

$$\sum_{s=1}^{S} P_{ns} \le P_{ul|max}, \forall \ n \in \{1 \cdots N/2\},$$
(P3)

where

$$F(\mathbf{P_s}) = -\sum_{s=1}^{S} \log_2 \left(\frac{P_s D_{m-AP}^{-\alpha} |h_{m-AP}^s|^2}{\sigma^2 + P_s / C} \right),$$

$$G(\mathbf{P_{ns}}) = -\sum_{s=1}^{S} \log_2 \left(\frac{P_{ns} D_{n-AP}^{-\alpha} |h_{AP-n}^s|^2}{\sigma^2 + P_{ns} D_{m-n}^{-\alpha} |h_{m-n}^s|^2} \right)$$
(10)

It should be noted that the objective function and the constraints functions, in (**P3**), are fully separable in the DL transmission powers P_s and the UL transmission powers P_{ns} . Since the constraints are all affine constraints, then the convexity of the approximate high SINR problem (**P3**) can be proved by proving the convexity of $F(\mathbf{P_s})$ with respect to P_s and the convexity of $G(\mathbf{P_{ns}})$ with respect to P_{ns} . The proof of Proposition 1 is presented in Appendix A.

There are many algorithms that are used to solve these types of convex problems. In this paper, we are going to adopt the ADMM algorithm, which is a method for solving generic convex constrained problems and only uses the proximal operator of the objective function and projection onto the constraint set [28], [29], [37]. The convergence of the ADMM algorithm is discussed in [29]. Additionally, it is shown that ADMM achieves a linear convergence rate [38], [39]. The main advantage of using ADMM is its ability to solve separable optimization problems in a distributed fashion. The first step towards solving (P3) using ADMM is to rewrite the constrained optimization problem as a sum of the objective function and an indicator function of the convex set of the constraints. The solution of the power allocation problem in (P3) can be described as follows.

Proposition 2. The ADMM solution algorithm for the problem

⁷It should be reassured that, in the UL transmission, the SI is cancelled by the used FD radio in the AP, which covers up for the self-interference to the receiver noise floor. Accordingly, the high SINR assumption is also valid for the UL transmission.

Algorithm 3: Joint User Pairing, Subcarrier and Power Allocation Algorithm for OFDMA FD network

- **Data**: all CSI information, P_{max} , $P_{ul|max}$, α , $\mathbf{P_s^0}$, $\mathbf{P_{ns}^0}$, $\mathbf{Z_s^0}$ and $\mathbf{Z_{ns}^0}$, λ , β , SNR_{th}
- **Result**: Find $a(m, n, s) \forall m \in \{1..N/2\}, n \in \{1..N/2\},$ and $s \in \{1..S\}, \mathbf{P_s}$ and $\mathbf{P_{ns}}$ maximizing R_T while keeping total transmission power constraints P_{max} for the AP and $P_{ul|max}$ for the UL transmissions.

Intially: Assume DL and UL power are distributed equally among active users and subcarriers.

 $\Phi_i = \emptyset, \, \Phi_f = \emptyset,$

iteration t = 1,

Step 1: Joint User Pairing and Subcarrier Allocation

a. Form the initial candidate pair set Φ_i given in (6),

b. Apply Steps described in Algorithm. 1 to find A(t),

Step 2: Power Allocation

a. From $\Phi_f(t)$, formulate power allocation problem in **P3**,

b. Run the power allocation scheme described in Algorithm. 2. to find $P_s(t)$ and $P_{ns}(t)$,

Step 3: Check convergence

a. if $\mathbf{A}(t) = \mathbf{A}(t-1) \& R_T(t) = R_T(t-1)$ then $\mathbf{A} = \mathbf{A}(t)$ $\mathbf{P_s} = \mathbf{P_s}(t)$ $\mathbf{P_{ns}} = \mathbf{P_{ns}}(t)$ else t = t+1Return to Step 1 with the calculated transmission

powers from Step 2 to form $\Phi_i(t+1)$

(P3) is given by

$$P_{s}^{k+1} := prox_{\lambda f(P_{s})}(P_{s}^{k} - Z_{s}^{k} - U^{k}),$$

$$Z_{s}^{k+1} := \Pi_{\mathcal{C}_{\mathcal{D}\mathcal{L}}}(P_{s}^{k+1} + U_{s}^{k}),$$

$$P_{ns}^{k+1} := prox_{\lambda g(P_{ns})}(P_{ns}^{k} - Z_{ns}^{k} - U_{s}^{k}),$$

$$Z_{ns}^{k+1} := \Pi_{\mathcal{C}_{\mathcal{U}\mathcal{L}}}(P_{ns}^{k+1} + U_{ns}^{k}),$$

$$U_{s}^{k+1} := U_{s}^{k} + P_{s}^{k+1} - Z_{s}^{k+1},$$

$$U_{ns}^{k+1} := U_{ns}^{k} + P_{ns}^{k+1} - Z_{ns}^{k+1},$$
(11)

where

$$f(P_s) = -\log_2\left(\frac{P_s D_{m-AP}^{-\alpha} |h_{m-AP}^s|^2}{\sigma^2 + P_s/C}\right),$$

$$g(P_{ns}) = -\log_2\left(\frac{P_{ns} D_{n-AP}^{-\alpha} |h_{AP-n}^s|^2}{\sigma^2 + P_{ns} D_{m-n}^{-\alpha} |h_{m-n}^s|^2}\right).$$
(12)

The variables Z_s and $Z_{ns} \forall s \in \{1 \cdots S\}$ and $n \in \{1 \cdots N/2\}$ are equivalent to the variables P_s and $P_{ns} \forall s \in \{1 \cdots S\}$ and $n \in \{1 \cdots N/2\}$, respectively. **prox**_{$\lambda f(P_s)$}(ν) and **prox**_{$\lambda g(P_{ns})$}(ν) are the proximal operators on $f(P_s)$

and $g(P_{ns})$, respectively, with a scaling factor of λ . Finally, $\Pi_{C_{D\mathcal{L}}}(\nu)$ and $\Pi_{C_{\mathcal{UL}}}(\nu)$ are the Euclidean projections onto the constraints set $C_{D\mathcal{L}}$ and $C_{\mathcal{UL}}$, which are defined, respectively, by

$$C_{D\mathcal{L}} = \{z_1, z_2, ..., z_S | \sum_{s=1}^{S} P_s \le P_{max} \},$$

$$C_{\mathcal{UL}} = \{z_{11}, z_{12}, z_{13} ..., z_{S*N/2} |$$

$$\sum_{s=1}^{S} P_{ns} \le P_{ul|max} \ \forall n \in \{1 \cdots N/2\} \}.$$
(13)

Proof: Since the optimization problem given in (**P3**) is fully separable in the DL and UL transmission powers, then the first step is to write (**P3**) as two distinct convex optimization problems. The first problem is in terms of the DL transmission powers (**P3.1**) and the second problem is in terms of the UL transmission powers (**P3.2**). The first optimization problem (**P3.1**) is given by

$$\min_{\mathbf{P}_{s}} \sum_{s=1}^{S} f(P_{s})$$
s.t.
$$\sum_{s=1}^{S} P_{s} \leq P_{max}.$$
(P3.1)

where $f(P_s)$ is defined in (12). Afterwards, (P3.1) is reformulated into the canonical form defined in [28], as follows

$$\min_{\mathbf{P}_{\mathbf{s}}} \sum_{s=1}^{S} f(P_s) + \Psi(Z_s), \qquad (\mathbf{P3.1.1})$$

where $\Psi(Z_s) = \mathcal{I}_{\mathcal{C}_{D\mathcal{L}}}(z_1, z_2 \cdots z_S)^8$, $\mathcal{I}_{\mathcal{C}_{D\mathcal{L}}}$ is an indicator function which is defined as

$$\mathcal{I}_{\mathcal{C}_{\mathcal{DL}}}(x) = \begin{cases} 0, & x \in \mathcal{C}_{\mathcal{DL}}, \\ +\infty, & x \notin \mathcal{C}_{\mathcal{DL}}, \end{cases}$$
(14)

where the set C_{DL} is defined in (13). Similarly, the second problem is in terms of the UL transmission powers (**P3.2**), and is given by

$$\min_{\mathbf{P}_{ns}} \sum_{s=1}^{S} g(P_{ns})$$
s.t.
$$\sum_{s=1}^{S} P_{ns} \leq P_{ul|max}, \forall \ n \in \{1 \cdots N/2\}.$$
(P3.2)

where $g(P_{ns})$ is defined in (12). Afterwards, (P3.2) is reformulated into the canonical form as follows

$$\min_{\mathbf{P}_{ns}} \sum_{s=1}^{S} g(P_{ns}) + \Theta(Z_{ns}), \qquad (\mathbf{P3.2.2})$$

where $\Theta(Z_{ns}) = \mathcal{I}_{C_{UL}}(z_{11}, z_{12} \cdots z_{SN/2}), \mathcal{I}_{C_{UL}}$ is an indicator function which is defined as

⁸In the canonical form definition, precisely speaking, in $\Psi(Z_s)$ definition, a change of variables from P_s to z_s is required $\forall s = \{0, 1, \dots S\}$, where $Z_s = [z_1, z_2, \dots z_S]^T$ [28], [29].



Fig. 2: Variation of R_T versus C: a. N = 16, S = 8, b. N = 24and S = 16, 32, 48.

$$\mathcal{I}_{\mathcal{C}_{\mathcal{U}\mathcal{L}}}(x) = \begin{cases} 0, & x \in \mathcal{C}_{\mathcal{U}\mathcal{L}}, \\ +\infty, & x \notin \mathcal{C}_{\mathcal{U}\mathcal{L}}. \end{cases}$$
(15)

Finally, the problems in (**P3.1.1**) and (**P3.2.2**) can be solved using the algorithm given in (11), where the proximal operator and the Euclidean projection are given, respectively, by

$$\mathbf{prox}_{\lambda f}(\nu) = \underset{\mathbf{x}}{\operatorname{argmin}} f(x) + (1/(2\lambda)) \|x - \nu\|_{2}^{2},$$

$$\Pi_{\mathcal{C}}(\nu) = \underset{x \in \mathcal{C}}{\operatorname{argmin}} \|x - \nu\|_{2}.$$

$$(16)$$

The complete solution algorithm for the proposed allocation problem is given in Algorithm 2. In the beginning, it is assumed that all UL channel state information (CSI), DL CSI, and mutual SINR information between UL and DL users are available at the AP. Next, the parameters α , P_{max} , and $P_{ul|max}$ are determined. Afterwards, initial vectors P_s^0 , P_{ns}^0 , Z_s^0 and \mathbf{Z}_{ns}^{0} are determined. Then, in steps 1 through 6, the algorithm continues on updating the values of P_s , Z_s , P_{ns} , and Z_{ns} . It must be noticed that the dual variables U_s and U_{ns} are updated to measure the deviation of P_s from Z_s and P_{ns} from Z_{ns} , respectively. The algorithm will stop iterating when $\mathbf{P_s}$ converges to $\mathbf{Z_s}$ and $\mathbf{P_{ns}}$ converges to $\mathbf{Z_{ns}}.$ It should be emphasized that steps 3 and 4, which are used to update P_{ns} and \mathbf{Z}_{ns} , are performed in parallel at each UL user. However, the remaining steps of Algorithm 2, which include updating $\mathbf{P}_{s}, \mathbf{Z}_{s}, U_{s}, \text{ and } U_{ns}$ and checking the algorithm convergence, are performed at the AP.

Finally, the complete solution algorithm for solving the joint user pairing and subcarrier allocation, formulated in **P1**, is given in Algorithm 3. Initially, to solve the user-pairing and subcarrier allocation, we set $\Phi_i = \emptyset$, and the number of iteration t = 1. The first step, after assuming equal power allocation over the UL and DL transmissions, is to apply the joint user pairing and subcarrier algorithm explained in



Fig. 3: Variation of R_T versus grid size: a. N = 16, and S = 8, b. N = 24 and S = 16, 32, 48.

Algorithm 1. Using the user pairing and subcarrier allocation results, the power allocation problem is formulated in (**P2**). Assuming the high SINR case, the problem's approximation in the case of the high SINR case (**P3**) is formulated and solved using Algorithm 2. Finally, if the users' pairing, subcarrier allocation, and sum capacity remain unchanged, the algorithm will stop and the final solution for the joint problem is obtained. Otherwise, the algorithm will rerun the joint user pairing and subcarrier allocation described in Algorithm 1 with the new power allocation and then the power allocation scheme described in Algorithm 2.

IV. NUMERICAL ANALYSIS

In this section, first, we evaluate the FD power allocation performance. Second, we study the effects of different system parameters on the power allocation problem as well as the joint allocation problem. We are considering a square grid with the access point in its center. All the users are uniformly distributed inside the grid. It is assumed that the SNR threshold SNRth = 20dB, the SI cancellation parameter C = 70dB, the maximum DL transmission power $P_{max} = 2W$, and the maximum UL transmission power for each UL user Pul|max = 1mW. The proposed system is simulated using Monte Carlo simulations on MATLAB. Unless stated otherwise, the simulation parameters are given in Table II.

TABLE II: Simulation Parameters

N	16 users	S	16 SCs
\mathcal{SNR}_{th}	$-20 \ dB$	α	2.7
P_{max}	2 W	$P_{ul max}$	1 mW
σ^2	$-110 \ dBm$	<i>C</i> [3]	10^{7}



Fig. 4: Validation of the performance of the proposed joint allocation algorithm with different grid sizes. Parameters used to generate this figure: $\{N = 6, S = 3, P_{max} = 2W, 6W, P_{ul|max} = 1mW\}$

A. Power Allocation Performance Evaluation

In this section, the behavior of the power allocation problem proposed in Section III-B is evaluated. First, we validate our solution algorithm by comparing its performance with the interior point algorithm [40], implemented in *Matlab Optimization Toolbox*. Second, we compare the required run time needed by both algorithms to solve the power allocation problem in (**P3**). Finally, we study the effect of the SI cancellation parameter (C) and the mutual distance between users on the network sum throughput.

1) Validating Proposed ADMM Algorithm for Power Allocation Problem: Fig. 2 shows the variation of the sum throughput R_T with the SI cancellation parameter C, for different number of users and subcarriers. Additionally, it compares the proposed ADMM solution and the interior-point algorithm. It is expected that increasing C will cause an increase in the sum throughput as a result of decreasing the RSI level. Furthermore, increasing the number of subcarriers is expected to offer more transmission channels, and as a consequence, the sum throughput will increase as well. This behavior can be validated from the results shown in Fig. 2, as the enhancement of the sum capacity is noticed by increasing the value of C from 70dB to 110dB. Furthermore, increasing the number of subcarriers causes an increase in the network sum rate. Additionally, it is clear that the proposed distributed ADMM solution matches the interior point solution for different network conditions. Furthermore, Table III shows the difference between the required run time for both the proposed ADMM algorithm and the interior-point algorithm for different numbers of users and subcarriers⁹. Even though both ADMM and interior-point algorithm have linear convergence rates [38]-[40], the per-iteration complexity of



Fig. 5: Validation of the performance of the proposed joint allocation algorithm versus different SI cancellation parameter. Parameters used to generate this figure: $\{N = 6, S = 3, P_{max} = 2W, 6W, P_{ul|max} = 1mW\}$

the interior-point algorithm is expected to be much higher than that of ADMM because in the case of the interior-point algorithm, the solution of the power allocation problem in centralized. On the other hand, in the case of the ADMM, the UL power allocation is performed in parallel, which decreases the complexity of the solution algorithm. This can be validated from the results shown in Table III as it can be noticed that the needed run time for the proposed ADMM algorithm to solve the power allocation problem in (P3) is less than that required by the interior-point algorithm implemented in Matlab Optimization Toolbox. Additionally, it is observed that the difference between the required run time between the two algorithms increases with the number of available subcarriers. For instance, the ADMM reduced the required run time by about 21% for S = 16 and N = 24, and this reduction in run time increased to about 77% for S = 64 and N = 24. Based on the results shown in Fig. 2 and Table III, it is quite clear that the proposed ADMM algorithm requires less run time than that taken by the interior-point algorithm, to solve the power allocation problem in (P3) with almost similar performance.

Fig. 3 shows the variation of the sum throughput R_T with the grid size, for different numbers of subcarriers. First of all, since the users are uniformly distributed along the considered square grid, then as the grid size becomes bigger, the average distance between the AP and the users becomes larger. Hence, both the UL and DL received powers decrease. Therefore, it is expected that the larger the grid size the smaller the achieved capacity. The decrease in R_T with respect to the grid size can be verified from the results shown in Fig. 3. Additionally, the results in Fig. 3 show the proposed ADMM solution and the interior-point solution result in similar performance.

⁹The comparison between the ADMM and interior-point algorithms is performed under the same optimality tolerance with the same CPU specifications.

Subcarriers	bcarriers Users ADMM		Interior-Point Algorithm	Time Difference percentage			
16	24	3.48s	4.45 <i>s</i>	21%			
32	24	4.35s	17.72 <i>s</i>	75%			
64	24	8.78s	38.89 <i>s</i>	77%			

TABLE III: Comparison between run time for the proposed ADMM algorithm and the Interior-Point Algorithm

B. Joint User Pairing, Subcarrier Allocation and Power Allocation Performance Evaluation

In this subsection, we study and evaluate the performance of the joint allocation problem. First, the proposed solution algorithm performance is validated by comparing it with the optimal solution which is obtained through exhaustive search that enumerates all possible user pairings and subcarrier allocations, and chooses the combination that results in the maximum sum rate. The total number of combinations for exhaustive search equals $(N/2)^{2*S}$ ¹⁰. It is quite clear that, for large networks, obtaining the optimal solution is very complicated as a result of the large number of combinatorial possibilities of users' pairings, and subcarrier allocation. Afterwards, we explore how varying different network parameters can affect the achieved sum capacity.

1) Validating Proposed User Pairing and Subcarrier Allocation Algorithm: Fig. 4 compares the performance of the proposed joint allocation algorithm with the exhaustive search solution for a 3-SC, 6-user network, with a varying grid size from 200m to 600m. In that case, the exhaustive search solution runs over 729 different combinations of DL users, UL users, and SC allocations. From the results in Fig. 4, it can be seen that the difference between the proposed algorithm and the exhaustive search solution is almost 1%, which indicates that our proposed approach is very close to being optimal. Also, when the value of P_{max} is increased from 2W to 6W, the proposed algorithm sum rate increased and achieved a performance which is very close to the optimal. Additionally, when calculating the run time for both algorithms, it is found that the proposed algorithm takes much less time to find the solution, as compared to the exhaustive search based approach¹¹. Additionally, the proposed algorithm and the exhaustive search solution are compared when varying the self-interference cancellation parameter in Fig. 5. We can observe that the proposed algorithm approaches the optimal solution for different values of the self-interference cancellation parameter. Which again emphasizes the merits of our proposed algorithm as compared to the "optimal" exhaustive search solution in terms of performance and complexity.

For further validation, the two solution methods are compared for a 4-SC, 4-user network which requires the enumeration of 256 different combinations of DL users, UL users, and SC allocations. In Fig. 6, the two solution methods are compared while changing the grid size, and similar to the results shown in Fig. 4, the difference between the optimal capacity and the capacity achieved by the proposed algorithm is very small. Additionally, in Fig. 7, the upper-bound obtained by solving the dual problem in [19], is simulated for comparison. The three methods are compared for different values of SI cancellation parameter. From the results in Fig. 7, we can observe that the performance of the optimal solution is very close to the upper bound obtained in [19], which means that the duality gap is very small, which matches the results obtained in [41] where it is proved that solving the dual problem of the non-convex multi-carrier spectrum sharing problems gives a solution with a negligible duality gap. Furthermore, it is observed that the proposed algorithm can achieve a performance close to the upper bound, found through solving the dual problem and that is very close to the optimal, exhaustive search based solution.

The next step is to validate the proposed algorithm performance in larger networks. In large networks, the implementation of the exhaustive search solution is impractical. Therefore, we will compare the proposed algorithm performance with the dual upper bound solution presented in [19]. In Fig. 8, the proposed algorithm and the upper bound solution are compared for a network with 16 SC and 16 users and a network with 48 SCs, which is the same as the number of SCs used in certain amendments of the IEEE 802.11 standard of Wi-Fi networks for example, and 20 users. In both cases, we can observe that the gap between the proposed algorithm and the upper bound is very small which validates the effectiveness of the proposed algorithm in achieving a "close to optimal" performance.

Next, we compare the performance of our proposed algorithm against the random pairing FD network and the HD network. In the random pairing FD network, UL users and DL users are randomly paired and allocated to available subcarriers. In the HD network, we assume a square grid where the HD AP is centered in the middle. In the first time slot, the AP serves N DL users and each subcarrier is allocated to the DL transmission with the highest SNR. Similarly, in the second time slot, the AP serves N UL user and each subcarrier is allocated to the UL transmission with the highest SNR. In Fig. 9, the three systems are compared while varying the grid size. From the results shown in Fig. 9, it can be noticed that the proposed scheme greatly outperforms both the random pairing and the HD systems. It can be noticed that, for 200mgrid, the proposed algorithm sum rate is 1.5 times that of

¹⁰The implementation of the exhaustive search is done in this work only for comparison purpose.

¹¹Note that the complexity of the exhaustive search algorithm is exponential in the number of users and the number of subcarriers; therefore, for larger networks, with large number of users and large number, the exhaustive search based approach becomes impractical to implement.



Fig. 6: Validation of the performance of the proposed joint allocation algorithm versus different grid sizes. Parameters used to generate this figure: $\{N = 4, S = 4, P_{max} = 1W, 2W, P_{ul|max} = 1mW\}$

the random pairing sum rate, and 1.91 times that of the HD system sum rate. Furthermore, as the grid size increases to 2Km, the proposed algorithm still guarantees a better rate than that offered by both the random pairing and HD schemes ¹².

Moreover, Fig. 10 shows the variation of the sum rate with the SI cancellation parameter C. It is obvious that changing the SI cancellation parameter will not affect the HD system performance. However, increasing C is expected to increase the sum rate for both the proposed and random pairing schemes, as a result of reduced RSI. From the results displayed in Fig. 10, the decrease in the sum rate with SI cancellation coefficient is verified, for both the proposed and random pairing scheme. Additionally, the advantage of the proposed scheme over the random pairing scheme can be justified, as over different values of C, the proposed scheme achieves a better rate than random pairing and the HD scheme. Similarly, from the results shown in Fig. 10, it can be noticed that further decrease in C may cause HD to outperform the FD performance. In that case, mode selection will be essential to switch the network to HD operation to maximize the network spectral efficiency. Mode selection is addressed in the work presented in [20] for a FD-OFDMA network with multiples APs.

V. CONCLUSION

In this paper, a single cell OFDMA network is considered, wherein a joint user pairing, subcarrier, and power allocation algorithm that aims at maximizing the DL and UL sum rate is proposed. A two-step solution algorithm is presented; the



Fig. 7: Validation of the performance of the proposed joint allocation algorithm versus different SI cancellation parameter. Parameters used to generate this figure: $\{N = 4, S = 4, P_{max} = 2W, P_{ul|max} = 1mW\}$

first step is the joint user pairing and subcarrier allocation by considering the pairs whose co-channel interference is below the downlink user equipment SNR threshold. The second step is to consider the power allocation in high signal-tointerference-noise ratio. Using the alternating direction method of multipliers the power allocation problem is solved as two separate problems in the uplink and downlink transmissions. Through numerical simulations, we show that the distributed nature of the proposed power allocation solution enables it to converge to the optimal solution faster than the "centralized" interior point based algorithm. We also show that our proposed algorithm achieves a performance that is very close to the sum rate upper bound, obtained from solving the dual problem. This, in turn, proves that our proposed approach achieves a performance that is very close to the optimal, "impractical" exhaustive search based solution.

In the future work, fairness will be considered in the joint optimization problem. The fairness issues while maximizing the networks' sum rate can be addressed by maximizing the min-max fairness rate, by adding a minimum rate constraint for each user, or by guaranteeing at least one subcarrier for each user in the network.

APPENDIX A PROOF OF PROPOSITION 1

In case of high SINR, and after assigning the available subcarrier to the DL-UL user pairs, the sum DL and UL rates in (4) are, respectively, modified to

$$\overline{R}_{T|DL} = \sum_{s=1}^{S} \log_2(\Gamma_{m|DL_s}),$$

$$\overline{R}_{T|UL} = \sum_{s=1}^{S} \log_2(\Gamma_{n|UL_s}).$$
(17)

¹²As the grid size increases, the DL power increases to increase the coverge for larger grid size. Accordingly, the SI levels increase and the gap between the FD and the HD performance decreases.



Fig. 8: Validation of the performance of the proposed joint allocation algorithm versus different SI cancellation parameter. Parameters used to generate this figure: $\{N = 16, S = 16, and N = 20, S = 48 P_{max} = 2W, P_{ul|max} = 1mW\}$

Accordingly, the sum rate approximation, in high SINR, is given by,

$$\overline{R}_{T} = \sum_{s=1}^{S} \log_{2}(\Gamma_{m|DL_{s}}) + \log_{2}(\Gamma_{n|UL_{s}}),$$

$$= \sum_{s=1}^{S} \log_{2}\left(\frac{P_{s}D_{m-AP}^{-\alpha}|h_{m-AP}^{s}|^{2}}{\sigma^{2} + P_{s}/C}\right) + \log_{2}\left(\frac{P_{ns}D_{n-AP}^{-\alpha}|h_{AP-n}^{s}|^{2}}{\sigma^{2} + P_{ns}D_{m-n}^{-\alpha}|h_{m-n}^{s}|^{2}}\right),$$

$$= F(\mathbf{P_{s}}) + G(\mathbf{P_{ns}}).$$
(18)

Accordingly, the power allocation problem is reformulated, as given in (P3). Now the next step, to prove the convexity of the power allocation problem approximation, is to prove the convexity of $F(\mathbf{P_s})$ with respect to $\mathbf{P_s}$, and $G(\mathbf{P_{ns}})$ with respect to $\mathbf{P_{ns}}$. In other words, it is required to prove that both functions have a positive semi-definite Hessian matrix, i.e., $\nabla^2 F(\mathbf{P_s}) \ge 0$ and $\nabla^2 G(\mathbf{P_{ns}}) \ge 0$. Starting with $F(\mathbf{P_s})$, the first derivative of $F(\mathbf{P_s})$ with respect to $\mathbf{P_s}$ is given by

$$\nabla F(\mathbf{P_s}) = \left[\frac{\partial F(\mathbf{P_s})}{\partial P_1}, \cdots, \frac{\partial F(\mathbf{P_s})}{\partial P_S}\right]^T,$$

$$= \left[\frac{\partial f_1(P_1)}{\partial P_1}, \cdots, \frac{\partial f_S(P_S)}{\partial P_S}\right]^T,$$
(19)

where $f_s(P_s) \forall s \in \{1, \dots, S\}$ is given in (12). The second equality in (19) is due to the fact that $F(\mathbf{P_s})$ is fully separable over $P_s \forall s \in \{1, \dots, S\}$. The value of $\partial f_s / \partial P_s \forall s \in \{1, \dots, S\}$ is given by

$$\frac{\partial f_s}{\partial P_s} = \frac{-\sigma^2}{P_s} \frac{1}{\sigma^2 + P_s/C}.$$
(20)



Fig. 9: Variation of R_T versus grid size for HD, random allocation FD and proposed scheme. Parameters used to generate this figure: { N = 24, S = 16, $P_{max} = 2W$, $P_{ul|max} = 1mW$, C = 80dB }

The next step is to calculate the value of $\nabla^2 F(\mathbf{P_s})$. Since, $F(\mathbf{P_s})$ is fully separable over $P_s \forall s \in \{1 \cdots S\}$, therefore $\nabla^2 F(\mathbf{P_s}) = \operatorname{diag} \left[\frac{\partial^2 f_1}{\partial 2s_1}, \cdots, \frac{\partial^2 f_S}{\partial 2S}\right]$ is a diagonal matrix whose diagonal entries are given by

$$\nabla^2 F(\mathbf{P_s})|_{(s,s)} = \left(\frac{\sigma^2}{P_s} \times \frac{1/C}{(\sigma^2 + P_s/C)^2}\right) + \left(\frac{1}{\sigma^2 + P_s/C} \times \frac{\sigma^2}{P_s^2}\right)$$
(21)

where $\nabla^2 F(\mathbf{P_s})|_{(s,s)}$ is the (s,s) element in the Hessian matrix of $F(\mathbf{P_s})$. From (21), it can be noticed that all the diagonal elements $\nabla^2 F(\mathbf{P_s})$ are positive and hence, $\nabla^2 F(\mathbf{P_s})$ is a positive semi-definite matrix. Therefore, $F(\mathbf{P_s})$ is a convex function with respect to $\mathbf{P_s}$. Using the same procedures, the convexity of $G(\mathbf{P_{ns}})$ with respect to $\mathbf{P_{ns}}$ can be proved. Finally, Since both $F(\mathbf{P_s})$ and $G(\mathbf{P_{ns}})$ are convex function, and all the constraints in (**P3**) are linear, therefore, the power allocation problem approximation in high SINR regime is a convex optimization problem.

ACKNOWLEDGMENT

This work is partially supported by US MURI AFOSR MURI 18RT0073, NSF EARS-1839818, CNS1717454, CNS-1731424, CNS-1702850, and CNS-1646607.

REFERENCES

- A. Sabharwal, P. Schniter, D. Guo, D. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: Challenges and opportunities," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 9, pp. 1637 1652, Jun. 2014.
- [2] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in Proceedings of the ACM SIGCOMM, Hong Kong, China, Aug. 2013.
- [3] M. Jainy, J. I. Choiy, T. M. Kim, D. Bharadia, S. Seth, K. Srinivasan, P. Levis, S. Katti, and P. Sinha, "Practical, real-time, full duplex wireless," in the 17th Annual International Conference on Mobile Computing and Networking (MobiCom), Las Vegas, NV, Sep. 2011.



Fig. 10: Variation of R_T versus SI cancellation parameter for HD, random allocation FD and proposed scheme. Parameters used to generate this figure: {N = 24, S = 16, $P_{max} = 2W$, $P_{ul|max} = 1mW$ }

- [4] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems* and Computers (ASILOMAR), Pacific Grove, CA, Nov. 2010.
- [5] A. Masmoudi and T. Le-Ngoc, "Self-interference cancellation limits in full-duplex communication systems," in *IEEE Global Communications Conference (GLOBECOM)*, Washington, DC, Dec. 2016.
- [6] L. Song, R. Wichman, Y. Li, and Z. Han, Full-Duplex Communications and Networks. Cambridge University Press, UK, 2017.
- [7] D. Wen and G. Yu, "Time-division cellular networks with full-duplex base stations," *IEEE Communications Letters*, vol. 20, no. 2, pp. 392– 395, Feb. 2015.
- [8] N. V. Shende, Ö. Gürbüz, and E. Erkip, "Half-duplex or full-duplex communications: Degrees of freedom analysis under self-interference," *IEEE Transactions on Wireless Communications*, vol. 17, no. 2, pp. 1081–1093, Feb. 2018.
- [9] L. Song, Y. Li, and Z. Han, "Resource allocation in full-duplex communications for future wireless networks," *IEEE Wireless Communications*, vol. 22, no. 4, pp. 88–96, Aug. 2015.
- [10] R. Sultan, L. Song, and Z. Han, "Impact of full duplex on resource allocation for small cell networks," in *IEEE Signal and Information Processing (GlobalSIP), Global Conference on*, Atlanta, GA, Dec. 2014, pp. 1257–1261.
- [11] N. H. Mahmood, G. Berardinelli, F. M. Tavares, and P. Mogensen, "On the potential of full duplex communication in 5G small cell networks," in *IEEE Vehicular Technology Conference (VTC Spring)*, 81st, Glasgow, UK, May. 2015.
- [12] D. Wen, G. Yu, R. Li, Y. Chen, and G. Y. Li, "Results on energy-and spectral-efficiency tradeoff in cellular networks with full-duplex enabled base stations," *IEEE Transactions on Wireless Communications*, vol. 16, no. 3, pp. 1494–1507, Mar. 2017.
- [13] R. Sultan, L. Song, K. Seddik, and Z. Han, "Full-duplex meets multiuser MIMO: Comparisons and analysis," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 1, pp. 455 – 467, Jan. 2017.
- [14] Z. Zhang, K. Long, A. V. Vasilakos, and L. Hanzo, "Full-duplex wireless communications: Challenges, solutions, and future research directions," *Proceedings of IEEE*, vol. 104, no. 7, pp. 1369–1409, Jul. 2016.
- [15] H. Kim, H. Lee, M. Ahn, H.-B. Kong, and I. Lee, "Joint subcarrier and power allocation methods in full duplex wireless powered communication networks for OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4745–4753, Jul. 2016.
- [16] C. Nam, C. Joo, and S. Bahk, "Resource allocation in full-duplex OFDMA networks: Approachs for full and limited CSIs," *Journal of Communications and Networks*, vol. 18, no. 6, pp. 913 – 925, Dec. 2016.

- [17] Y. Jiang, F. C. M. Lau, I. W.-H. Ho, and Y. Gong, "Resource allocation for multi-user OFDMA hybrid full-/half-duplex relaying systems with direct links," *Vehicular Technology, IEEE Transactions on*, vol. 65, no. 8, pp. 6101–6118, Aug. 2016.
- [18] S. Xiao, X. Zhou, Y. Yuan-Wu, G. Y. Li, and W. Guo, "Robust resource allocation in full-duplex-enabled ofdma femtocell networks," *IEEE Transactions on Wireless Communications*, vol. 16, no. 10, pp. 6382–6394, Oct. 2017.
- [19] S. Xiao, S. Guo, X. Zhou, D. Feng, Y. Yuan-Wu, G. Y. Li, and W. Guo, "Joint uplink and downlink resource allocation in full-duplex OFDMA networks," in *IEEE International Conference on Communications (ICC)*, Kuala Lumpur, Malaysia, May 2016.
- [20] R. Sultan, L. Song, K. G. Seddik, Y. Li, and Z. Han, "Mode selection, user pairing, subcarrier allocation and power control in fullduplex OFDMA hetnets," in *Communications Workshops (ICC), IEEE International Conference on*, London, UK, Jun. 2015.
- [21] B. Di, S. Bayat, L. Song, Y. Li, and Z. Han, "Joint user pairing, subchannel, and power allocation in full-duplex multi-user ofdma networks," *IEEE Transactions on Wireless Communications*, vol. 15, no. 12, pp. 8260–8272, Dec. 2016.
- [22] J.-H. Yun, "Intra and inter-cell resource management in full-duplex heterogeneous cellular networks," *IEEE Transactions on Mobile Computing*, vol. 15, no. 2, pp. 392–405, Feb. 2016.
- [23] X. Zhang, T.-H. Chang, Y.-F. Liu, C. Shen, and G. Zhu, "Max-min fairness user scheduling and power allocation in full-duplex ofdma systems," *IEEE Transactions on Wireless Communications*, vol. 18, no. 6, pp. 3078 – 3092, Jun. 2019.
- [24] H. Fawaz, S. Lahoud, M. El Helou, and J. Saad, "Queue-aware priority based scheduling and power allocation in full-duplex OFDMA cellular networks," in 25th International Conference on Telecommunications (ICT), St. Malo, France, Jun. 2018, pp. 15–20.
- [25] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 4–17, Feb. 2008.
- [26] —, "Fundamental limits on detection in low SNR under noise uncertainty," in 2005 International Conference on Wireless Networks, Communications and Mobile Computing, vol. 1, Maui, HI, Jun. 2005, pp. 464–469.
- [27] T. M. Cover and J. A. Thomas, *Elements of information theory*. John Wiley and Sons, 1st ed., 1991.
- [28] N. Parikh and S. Boyd, "Proximal algorithms," Foundations and Trends in optimization, vol. 1, no. 3, pp. 123–231, Nov. 2013.
- [29] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, Jan. 2011.
- [30] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on selected areas in communications*, vol. 21, no. 2, pp. 171–178, Feb. 2003.
- [31] W. Cheng, X. Zhang, and H. Zhang, "Optimal dynamic power control for full-duplex bidirectional-channel based wireless networks," in *IN-FOCOM*, Turin, Italy, Jul. 2013, pp. 3120–3128.
- [32] M. Duarte, C. Dick, and A. Sabharwal, "Experiment-driven characterization of full-duplex wireless systems," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 12, pp. 4296–4307, Dec. 2012.
- [33] P. Krishna, Y. A. Kumar, and K. K. Rao, "Pilot based LMMSE channel estimation for multi-user MIMO-OFDM systems with power delay profile," in *IEEE Asia Pacific Conference on Circuits and Systems* (APCCAS), Ishigaki, Japan, Nov. 2014, pp. 487–490.
- [34] M. K. Ozdemir and H. Arslan, "Channel estimation for wireless ofdm systems," *IEEE Communications Surveys & Tutorials*, vol. 9, no. 2, pp. 18–48, Jul. 2007.
- [35] S. Ohno, S. Hosokawa, and K. A. D. Teo, "Pilot power optimization for channel estimation in ofdm system," in *IEEE Asia-Pacific Conference* on Communications (APCC), Tokyo, Japan, Oct. 2008.
- [36] M. Karami, A. Olfat, and N. C. Beaulieu, "Pilot symbol parameter optimization based on imperfect channel state prediction for ofdm systems," *IEEE Transactions on Communications*, vol. 61, no. 6, pp. 2557–2567, Mar. 2013.
- [37] Z. Han, M. Hong, and D. Wang, Signal Processing and Networking for Big Data Applications. Cambridge University Press, Apr. 2017.
- [38] W. Shi, Q. Ling, K. Yuan, G. Wu, and W. Yin, "On the linear convergence of the ADMM in decentralized consensus optimization," *IEEE Transactions on Signal Processing*, vol. 62, no. 7, pp. 1750–1761, Apr. 2014.

- [39] W. Deng and W. Yin, "On the global and linear convergence of the generalized alternating direction method of multipliers," *Journal of Scientific Computing*, vol. 66, no. 3, pp. 889–916, May. 2016.
- [40] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.
- [41] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Transactions on Communications*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.



Radwa Sultan (S'05-M'17) received her B.S.(with highest honors) and the M.S.degrees in electrical engineering from Alexandria University, Alexandria, Egypt, in 2009 and 2013, respectively, and her PhD degree in Electrical and Computer Engineering Department at the University of Houston, Texas, in 2017. Currently, she is an assistant professor in the Electrical and Computer Engineering Department in Manhattan College, New York. Dr. Sultan has served on the technical program committees of several

IEEE journals and conferences in the areas of wireless communication. Her primary research interests are full-duplex communication, massive MIMO, signal processing, wireless resource allocation, and big data analysis in wireless communication.



Karim G. Seddik (S'04–M'08–SM'14) is a professor in the Electronics and Communications Engineering Department at the American University in Cairo (AUC), Egypt. Before joining AUC, he was an assistant professor in the Electrical Engineering Department at Alexandria University, Egypt. Dr. Seddik received the B.S. (with highest honors) and M.S. degrees in electrical engineering from Alexandria University, Alexandria, Egypt, in 2001 and 2004, respectively. He received his Ph.D. degree at the

Electrical and Computer Engineering Department, University of Maryland, College Park in 2008. His research interests include applications of machine learning in communication networks, cognitive radio communications, and layered channel coding.

Dr. Seddik has served on the technical program committees of numerous IEEE conferences in the areas of wireless networks and mobile computing. Dr. Seddik is a recipient of the State Encouragement Award in 2016. Dr. Seddik is a recipient of the Certificate of Honor from the Egyptian President for being ranked first among all departments in the College of Engineering, Alexandria University in 2002, the Graduate School Fellowship from the University of Maryland in 2004 and 2005 and the Future Faculty Program Fellowship from the University of Maryland in 2007.



Lingyang Song (S'03-M'06-SM'12-F'19) received his PhD from the University of York, UK, in 2007, where he received the K. M. Stott Prize for excellent research. He worked as a research fellow at the University of Oslo, Norway until rejoining Philips Research UK in March 2008. In May 2009, he joined the School of Electronics Engineering and Computer Science, Peking University, and is now a Boya Distinguished Professor. His main research interests include wireless communications, mobile

computing, and machine learning.

Dr. Song has co-authored of 4 text books, including "Wireless Device-to-Device Communications and Networks," and "Full-Duplex Communications and Networks," by Cambridge University Press, UK. He has co-edited two books: "Orthogonal Frequency Division Multiple Access (OFDMA)-Fundamentals and Applications," and "Evolved Network Planning and Optimization for UMTS and LTE (IEEE ComSoc Best Readings), " by Auerbach Publications, CRC Press, USA.

Dr. Song is the co-author of 10 best paper awards and 1 best demo award, including IEEE Leonard G. Abraham Prize in 2016 (publication in the IEEE Journal on Selected Areas in Communications in the previous 3 calendar years), best paper awards from IEEE Communication Society Flagship Conference: IEEE ICC 2014, IEEE ICC 2015, IEEE Globecom 2014, and the best demo award in the ACM Mobihoc 2015. He received National Science Fund for Distinguished Young Scholars in 2017, First Prize in Nature Science Award of Ministry of Education of China in 2017, National Science Fund for Excellent Young Scholars in 2013, and IEEE Communications Society Asia Pacific (AP) Young Researcher Award in 2012.

Dr. Song has served as a Distinguished Lecturer of IEEE Communications Society (2015-2018), an Area Editor of IEEE Transactions on Vehicular Technology (2019-), an Editor of IEEE Transactions on Communications (2019-), Editor of China Communications (2015-), and Editor of Transactions on Wireless Communications (2013-2018). He also serves as a Section Editor for Springer Handbook of Cognitive Radio (2016-). He served as the TPC co-chairs for ICUFN 2011/2012 and IEEE ICCC 2019. He served as symposium co-chairs for IEEE ICC 2014/2016, IEEE VTC 2016 spring, and IEEE Globecom 2016. He has served as Vice Chair (2016-) of IEEE Communications Society Cognitive Network Technical Committee, and Vice Chair (2016-) of IEEE Communications Society Asia Pacific Board Technical Affairs Committee.



Zhu Han (S'01-M'04-SM'09-F'14) received the B.S. degree in electronic engineering from Tsinghua University, in 1997, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, in 1999 and 2003, respectively.

From 2000 to 2002, he was an R&D Engineer of JDSU, Germantown, Maryland. From 2003 to 2006, he was a Research Associate at the University of Maryland. From 2006 to 2008, he was an assistant

professor at Boise State University, Idaho. Currently, he is a John and Rebecca Moores Professor in the Electrical and Computer Engineering Department as well as in the Computer Science Department at the University of Houston, Texas. He is also a Chair professor in National Chiao Tung University, ROC. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. Dr. Han received an NSF Career Award in 2010, the Fred W. Ellersick Prize of the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the Journal on Advances in Signal Processing in 2015, IEEE Leonard G. Abraham Prize in the field of Communications Systems (best paper award in IEEE JSAC) in 2016, and several best paper awards in IEEE conferences. Currently, Dr. Han is an IEEE communications Society Distinguished Lecturer from 2015-2018, and ACM distinguished Member since 2019. Dr. Han is 1% highly cited researcher since 2017 according to Web of Science.