On the Degrees of Freedom of IRS-Assisted Non-coherent MIMO Communications

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***Abstract*—In this paper, we consider the problem of intelligent reflecting surfaces (IRS) assisted non-coherent MIMO communications. The transmitter employs non-coherent MIMO codes and we allow the IRS to send its own information by switching the phases of some/all of its reflecting elements. We prove that the phase-modulated information from the IRS is completely transparent to the non-coherent MIMO transmitter and receiver, with no need for any performance provisioning or even knowing that the IRS is transmitting data. Moreover, we characterize the achievable degrees of freedom of the proposed transmission scheme gained by allowing the IRS to send extra data. We show that although non-coherent communications do not require channel estimates at the receiver, IRSs can still be used to enhance performance as more energy can be geared towards the receiver, although no co-phasing is done in our case. Moreover, the IRS can increase the achievable degrees of freedom of the system without requiring any extra power at the IRS.**

***Keywords*—Degrees of Freedom (DoF), Intelligent Reflecting Surface (IRS), Non-coherent MIMO Communications.**

I. Introduction

Wireless channel fading has always been one of the main challenges facing achieving the promised gains of future wireless systems. Over the years, researchers have been dealing with wireless channel fading as an inevitable, uncontrollable impairment and focused on alleviating its effect by devising better transmission and reception schemes. Intelligent reflecting surfaces (IRSs) present a paradigm shift aimed at controlling the communication channel. An IRS consists of a large number of small reflecting, normally passive, elements whose phases can be controlled to achieve some target performance. The interested reader is referred to [1]–[3] and references therein for a holistic overview of this emerging topic.

On the other hand, non-coherent MIMO communication is envisioned to be one of the eminent candidate technologies for future wireless communication systems [4]. Non-coherent communication alleviates the need for the transmission of pilot signals and can simplify the receiver design requirements as no channel estimation or phase estimation/compensation blocks are needed. However, these gains come at the expense of some loss of the degrees of freedom (DoF) [5] (the reader is referred to [6] for a comprehensive review of the concept of DoF). For some recent advances on the use and design of non-coherent MIMO codes, the reader is referred to [7]–[10] and references therein.

In this paper, we characterize the achievable gains by merging IRS with non-coherent MIMO transmission schemes. At a first glance, it might seem that the IRS does not provide any performance gains to a non-coherent MIMO communication system as the main advantage of IRS is to achieve some sort of channel co-phasing to enhance the receiver signal-to-noise ratio (SNR). However, in the context of non-coherent MIMO communications, the IRS can provide a communication path if the channel between the transmitter and the receiver is blocked or increase the power of the received signal by reflecting more power towards the receiver enhancing its SNR. Moreover, in our system, we allow the IRS to transmit its own phase-modulated data on top of the underlying non-coherent MIMO code. We show that these extra IRS data symbols can be transmitted completely *transparent* to the transmitter and they do not require any extra power provisioning from the transmitter node or the IRS. We characterize the extra achievable degrees of freedom (DoF) by allowing the IRS to transmit its own data. Finally, we show that allowing the IRS to simultaneously transmit the same phase-modulated symbol from multiple IRS elements can result in symbol error rate (SER) performance gains.

***Notations*:** The notations $\mathbf{A}^T$ and $\mathbf{A}^{\mathcal{H}}$ are used to denote the transpose and the Hermitian (conjugate) transpose for the matrix $\mathbf{A}$, respectively. For a matrix $\mathbf{A}$, the notation $\mathbf{a}_{i,:}$ denotes the i-th row of $\mathbf{A}$, $\mathbf{a}_{:,j}$ denotes the j-th column of $\mathbf{A}$, and a_{ij} denotes the element in i-th row and the j-th column of $\mathbf{A}$. For a vector $\mathbf{a}$, the notation a_k denotes the k-th element of $\mathbf{a}$. The symbol $\odot$ denotes the Hadamard (element-wise) product where $\mathbf{C} = \mathbf{A} \odot \mathbf{B}$ means that $c_{ij} = a_{ij} b_{ij}\ \forall i, j$.

II. Preliminaries and System Model

In this section, we provide a quick overview of the Grassmann manifold and our system model. For the non-coherent MIMO communication system, the capacity at high SNRs has been shown to be achievable using Grassmannian codes [5]; each codeword represents a subspace that is not distorted by the channel at high SNRs [5].

A. The Grassmann Manifold

Consider the set of all $T \times M$ unitary matrices for $T \geq M$. This set defines the Stiefel manifold of matrices of size $T \times M$ $\mathbb{S}_{T,M}$. Define an equivalence relation where two points $\mathbf{P}$ and $\mathbf{Q}$ on the Stiefel manifold are equivalent if their T-dimensional column vectors span the same subspace. In other words, $\mathbf{P} \equiv \mathbf{Q}$ if they are related by right multiplication of a unitary matrix $\mathbf{\Omega}$ such that

$$\mathbf{P} = \mathbf{Q}\mathbf{\Omega}, \qquad \mathbf{\Omega} \in \mathbb{U}_M, \tag{1}$$

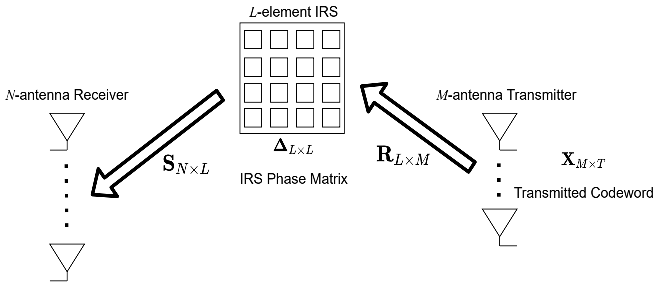

Fig. 1: System Model

where $\mathbb{U}_M$ is the unitary group consisting of all $M\times M$ unitary matrices. The Grassmann manifold $\mathbb{G}_{T,M}$ is defined as the quotient manifold of the Stiefel manifold $\mathbb{S}_{T,M}$ with respect to the equivalence relation in (1). Hence, every point on the Grassmann manifold defines a unique subspace of dimension M. The Grassmann manifold $\mathbb{G}_{T,M}$ has a complex dimension of $M(T-M)$ [5].

B. System Model

In this work, we focus on the use of IRS in the context of non-coherent communications. We assume a single M-antenna transmitter as shown in Fig. 1. The transmitter communicates to a destination node that has N receive antennas. We assume that the direct channel between the transmitter and the receiver is blocked and the transmitter can reach the receiver only through the IRS; this assumption is a practical assumption as IRSs are normally used to provide coverage for blocked/uncovered areas. We assume, without loss of generality, that $N \geq M$.[1] We assume that an L-element IRS is inserted in the system as shown in Fig. 1. We assume that L is very large compared to M and N given the fact that an IRS is expected to have a large number of small reflecting elements. The received signal at the non-coherent MIMO receiver can be expressed as

$$\mathbf{Y}_{N\times T} = \mathbf{S}_{N\times L}\boldsymbol{\Delta}_{L\times L}\mathbf{R}_{L\times M}\mathbf{X}_{M\times T} + \mathbf{N}_{N\times T}, \qquad (2)$$

where $\mathbf{X}$ ($\mathbf{X}^T \in \mathbb{G}_{T,M}$) is the transmitted Grassmannian codeword of size $M\times T$ that represents a subspace of the T-dimensional space, T is the channel coherence time and is assumed to be such that $T \geq 2M$, $\mathbf{R}$ is the $L\times M$ channel matrix between the transmitter and the IRS assumed to be a complex Gaussian matrix with i.i.d. entries with zero mean and unit variance, $\boldsymbol{\Delta}$ is an $L\times L$ diagonal matrix whose i-th diagonal element is given by $e^{j\theta_i}$, $i = 1,\cdots,L$, which represents the phase introduced by the i-th element of the IRS while all off-diagonal elements of $\boldsymbol{\Delta}$ are zeros, $\mathbf{S}$ is the $N\times L$ channel matrix between the IRS and the receiver assumed to be a complex Gaussian matrix with i.i.d. entries with zero mean and unit variance[2], and $\mathbf{N}$ is the noise matrix of size $N\times T$ whose elements are modeled to be i.i.d. complex Gaussian variables with zero means and variance N_0.

In a conventional system, the phases introduced at the *passive* IRS elements are adjusted to *control/co-phase* the channel at the receiver to introduce favorable channel conditions. However, in our model, where we assume a non-coherent communication system, the presence of the IRS can result in the following two benefits; first, the IRS can direct more power towards the receiver, which would enhance the receiver SNR, and in some cases, the IRS can provide a means for the transmitter to reach the receiver if the direct link is blocked as what we assume in our model. Second, the IRS phases can be used to transmit *extra* information from the IRS that is totally transparent to the non-coherent receiver as will be explained later. It should be noted that the IRS data can be intended for the non-coherent MIMO code receiver, to transmit for example control data, or for some other node in the network. It should also be noted that allowing phase modulation at the IRS elements increases the exploited degrees of freedom in the system. Next, we characterize the extra achieved degrees of freedom by allowing phase modulation of the IRS elements.

III. Degrees of Freedom Analysis

In this section, we provide the degrees of freedom (DoF) analysis of the presented transmission model in (2). First, note that for point-to-point system with M transmit antennas and N receive antennas communicating over T time slots, and under our assumption of having $N \geq M$, the degrees of freedom, $d_{\text{point-to-point, coherent}}$, is given by [5]

$$d_{\text{point-to-point, coherent}} = M \times T. \qquad (3)$$

These degrees of freedom are achievable under a coherent communication model where the receiver has access to channel state information (CSI). However, for a non-coherent point-to-point communication where the receiver has no CSI, the degrees of freedom, $d_{\text{point-to-point, non-coherent}}$, reduces to [5]

$$d_{\text{point-to-point, non-coherent}} = M \times (T-M). \qquad (4)$$

The last expression reflects the fact that at high SNRs, the optimum non-coherent MIMO codes are Grassmannian codes, where each codeword represents a subspace of dimension M. These subspaces are not affected by the channel and the receiver can distinguish between different received codewords by decoding the span of the received matrix. The degrees of freedom, $M\times(T-M)$, in this case, reflect the dimensionality of the Grassmannian manifold as explained above.

Going back to our model in (2), we have two sources of data transmission: the transmitted code matrix $\mathbf{X}$ and the phases transmitted from the IRS. Next, we calculate the achievable degrees of freedom, d, from our proposed transmission model. First, let us focus on the transmitted code matrix $\mathbf{X}$ and assume that the IRS does not apply any phase modulations, i.e., $\boldsymbol{\Delta} = \mathbf{I}_L$, where $\mathbf{I}_L$ is the identity matrix of size $L\times L$. Our model in (2) can be rewritten as

$$\mathbf{Y}_{N\times T} = \mathbf{S}_{N\times L}\mathbf{R}_{L\times M}\mathbf{X}_{M\times T} + \mathbf{N}_{N\times T}. \qquad (5)$$

[1]For the case of $N < M$, it has been shown that the transmitter cannot benefit, in terms of DoF, from the antennas beyond N; the transmitter can limit its number of active antennas to N, and that will result in the maximal achievable DoF [11].

[2]For simplicity of presentation, we have assumed that all channel gains are of equal variance. However, the more general model should take into consideration the effect of distances and path loss exponents in the considered cascade channel model and its effect of the overall system performance [12].

In this model, the effective channel between the transmitter and the receiver is $\mathbf{SR}$ while, in (2), the effective channel is given by $\mathbf{S\Delta R}$. However, given our model for the channel $\mathbf{R}$ between the transmitter and the IRS, it can be easily seen that $\mathbf{R} =_D \mathbf{\Delta R}$, where $=_D$ denotes equality in distribution, i.e., the variables on the two sides of the equality follow the same probability distribution; the phase of each element of the $\mathbf{R}$ matrix has a uniform distribution of their phases, so introducing any phase shift at the IRS elements, whether deterministic or random, will not change the distributions of the phases of these elements. Hence, the code matrix $\mathbf{X}$ will encounter the same effective channel distribution whether the IRS transmitted data through phase variations or not. Therefore, the transmission of data from the IRS by controlling the phases θ_i's, $i = 1, \cdots, L$ is completely transparent to the transmitted MIMO code $\mathbf{X}$. Finally, it should be noted that the effective $N \times M$ channel matrix $\mathbf{S\Delta R}$ is full rank (almost surely) for any $\mathbf{\Delta}$. Hence, the achievable DoF from the transmission of the code matrix $\mathbf{X}$ is $M(T-M)$ under our system model assumptions.

Next, we aim at calculating the extra DoF achievable by controlling the phases of the IRS elements. To carry out this calculation, we assume that the receiver has successfully decoded $\mathbf{X}$ and calculate the extra achieved DoF. The received signal model in (2) can now be written as

$$\begin{aligned} \mathbf{Y} &= \mathbf{S\Delta R X} + \mathbf{N} \\ &= \mathbf{S\Delta Z} + \mathbf{N}, \end{aligned} \tag{6}$$

where $\mathbf{Z} = \mathbf{RX}$ is an $L \times T$ matrix. Given that the matrix $\mathbf{X}$ has a rank of M by construction, it can be easily seen that the matrix $\mathbf{Z}$ has a rank of M as well. The last equation can be written as

$$\mathbf{Y} = \left[e^{j\theta_1}\mathbf{s}_{:,1}\ e^{j\theta_2}\mathbf{s}_{:,2} \cdots e^{j\theta_L}\mathbf{s}_{:,L}\right]\mathbf{Z} + \mathbf{N}. \tag{7}$$

Define the vectorized version of the matrix $\mathbf{Y}$, $\mathbf{y}_v$, of size $NT \times 1$, as

$$\mathbf{y}_v = \begin{bmatrix} \mathbf{y}_{:,1} \\ \mathbf{y}_{:,2} \\ \vdots \\ \mathbf{y}_{:,T} \end{bmatrix}. \tag{8}$$

Equation (7), and after some manipulation, can be rewritten as (ignoring the noise terms as the DoF are calculated as SNR tends to infinity)

$$\mathbf{y}_v = \underbrace{\begin{bmatrix} z_{11}\mathbf{s}_{:,1} & z_{21}\mathbf{s}_{:,2} & \cdots & z_{L1}\mathbf{s}_{:,L} \\ z_{12}\mathbf{s}_{:,1} & z_{22}\mathbf{s}_{:,2} & \cdots & z_{L2}\mathbf{s}_{:,L} \\ \vdots & \vdots & \cdots & \vdots \\ z_{1T}\mathbf{s}_{:,1} & z_{2T}\mathbf{s}_{:,2} & \cdots & z_{LT}\mathbf{s}_{:,L} \end{bmatrix}_{NT\times L}}_{\mathbf{H}_{\text{eff}}} \begin{bmatrix} e^{j\theta_1} \\ e^{j\theta_2} \\ \vdots \\ e^{j\theta_L} \end{bmatrix}_{L\times 1}. \tag{9}$$

The degrees of freedom in the last expression are limited by the rank of the effective channel matrix, $\mathbf{H}_{\text{eff}}$.

Theorem III.1. *The achievable degrees of freedom of the IRS-assisted non-coherent MIMO system is given by*

$$d = M\left(T + \frac{N}{2} - M\right)$$

Proof. The effective channel can be written as the Hadamard product of two matrices as

$$\mathbf{H}_{\text{eff}} = \underbrace{\begin{bmatrix} \mathbf{s}_{:,1} & \mathbf{s}_{:,2} & \cdots & \mathbf{s}_{:,L} \\ \mathbf{s}_{:,1} & \mathbf{s}_{:,2} & \cdots & \mathbf{s}_{:,L} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{s}_{:,1} & \mathbf{s}_{:,2} & \cdots & \mathbf{s}_{:,L} \end{bmatrix}}_{\mathbf{S}_{e_{NT\times L}}} \odot \underbrace{\begin{bmatrix} z_{11} & z_{21} & \cdots & z_{L1} \\ \vdots & \vdots & \cdots & \vdots \\ z_{11} & z_{21} & \cdots & z_{L1} \\ z_{12} & z_{22} & \cdots & z_{L2} \\ \vdots & \vdots & \cdots & \vdots \\ z_{12} & z_{22} & \cdots & z_{L2} \\ \vdots & \vdots & \cdots & \vdots \\ z_{1T} & z_{2T} & \cdots & z_{LT} \\ \vdots & \vdots & \cdots & \vdots \\ z_{1T} & z_{2T} & \cdots & z_{LT} \end{bmatrix}}_{\mathbf{Z}_{e_{NT\times L}}}. \tag{10}$$

Hence, we have $\mathbf{H}_{\text{eff}} = \mathbf{S}_e \odot \mathbf{Z}_e$. The following inequality holds for the rank of the Hadamard multiplication of two matrices [13]

$$\text{rank}(\mathbf{H}_{\text{eff}}) \leq \text{rank}(\mathbf{S}_e)\ \text{rank}(\mathbf{Z}_e). \tag{11}$$

It can be readily proved that $\text{rank}(\mathbf{S}) = N$ and $\text{rank}(\mathbf{Z}) = M$ almost surely. It follows that $\text{rank}(\mathbf{S}_e) = \text{rank}(\mathbf{S}) = N$ and $\text{rank}(\mathbf{Z}_e) = \text{rank}(\mathbf{Z}) = M$ due to the structure of $\mathbf{S}_e$ and $\mathbf{Z}_e$ given at the top of the next page. Hence, the $\text{rank}(\mathbf{H}_{\text{eff}})$ is upper-bounded by MN. It remains to see if the $\text{rank}(\mathbf{H}_{\text{eff}})$ hits its upper-bound. As given above, $\mathbf{H}_{\text{eff}}$ can be written as

$$\mathbf{H}_{\text{eff}} = \begin{bmatrix} z_{11}\mathbf{s}_{:,1} & z_{21}\mathbf{s}_{:,2} & \cdots & z_{L1}\mathbf{s}_{:,L} \\ z_{12}\mathbf{s}_{:,1} & z_{22}\mathbf{s}_{:,2} & \cdots & z_{L2}\mathbf{s}_{:,L} \\ \vdots & \vdots & \cdots & \vdots \\ z_{1T}\mathbf{s}_{:,1} & z_{2T}\mathbf{s}_{:,2} & \cdots & z_{LT}\mathbf{s}_{:,L} \end{bmatrix}_{NT\times L}. \tag{12}$$

Then we divide $\mathbf{H}_{\text{eff}}$ into T sub-matrices each consisting of N consecutive rows; the first sub-matrix is composed of the first N rows and so on. It should be noted that each column in the formed sub-matrices is a scaled version of the corresponding column from the $\mathbf{S}$ matrix. The columns' scaling factors in any sub-matrix correspond to a column in the $\mathbf{Z}$ matrix. Selecting any independent set of M columns from $\mathbf{Z}$ (which is always guaranteed to exist as $\mathbf{Z}$ has a rank of M), we can easily see that the sub-matrices corresponding to this set are linearly independent and can be used to represent any other sub-matrix in $\mathbf{H}_{\text{eff}}$. It remains to prove that the rows across these M sub-matrices are linearly independent[3]. Now, we aim at proving that the MN rows in the linearly independent sub-matrices are also linearly independent. For the sake of contradiction, assume that some row, $\mathbf{h}_l$, in one of the linearly independent sub-matrices, can be written as a linear combination of the rows of the set of linearly independent sub-matrices. It can be readily seen that this linear combination will correspond to a linear combination of the corresponding M columns of the $\mathbf{Z}$ matrix (transposed in a row form). This, in turn, means that

[3]It should be noted that if two sub-matrices are linearly independent this does not mean that the rows in these two sub-matrices are also linearly independent. For example, if the two sub-matrices share the same set of rows except for one then the sub-matrices are clearly linearly independent; however, the rows across the two sub-matrices are clearly linearly dependent because of the repeated rows across the two sub-matrices

these columns of the matrix $\mathbf{Z}$ are linearly dependent, which is a contradiction given that these columns of the matrix $\mathbf{Z}$ were chosen to be linearly independent. Hence, the rank of the matrix $\mathbf{H}_{\text{eff}}$ is proved to be MN.

Given that the rank of the $\mathbf{H}_{\text{eff}}$ matrix is MN, the degrees of freedom that can be generally achieved by the insertion of the IRS is MN. However, given that we have restricted the IRS effect to merely phase rotations to guarantee that the IRS data transmission is transparent to the system, the extra achieved degrees of freedom are halved to be $\frac{1}{2}MN$. The loss of half of the dimension is because no amplitude scaling is allowed at the IRS elements. Hence, the overall DoF, d, achieved by the transmitter node and the IRS transmission is given by

$$\begin{aligned} d &= M(T-M) + \frac{1}{2}MN \\ &= M\left(T + \frac{N}{2} - M\right). \end{aligned} \tag{13}$$

□

IV. Simulation Results

In this section, we present some simulation results to validate the theoretical analysis presented above.

In the first scenario, we investigate the effect of applying phase modulation at the IRS on the non-coherent MIMO receiver. Fig. 2 shows the performance for $M = 2$, $N = 2$, and $T = 2M = 4$. The constellation size is set to be 256 points (which is used in all simulations) designed on the Grassmannian manifold using the direct design approach [11], [14]. The number of the elements of the IRS is set to be $L = 8$, $L = 20$, and $L = 40$; four elements from the IRS, where $MN = 4$, are used to transmit IRS data using QPSK modulated symbols and the remaining elements do not apply any specific phase shifts, i.e., they just reflect the impinged transmitter signal. The non-coherent receiver applies the maximum likelihood (ML) detector given by [15]

$$\mathbf{X}_{\text{ML}} = \arg\max_{\mathbf{X}} \ \operatorname{Tr}\left(\mathbf{Y}\mathbf{X}^{\mathcal{H}}\mathbf{X}\mathbf{Y}^{\mathcal{H}}\right), \tag{14}$$

where $\operatorname{Tr}(.)$ denotes the trace of a matrix[4].

From Fig. 2, it is clear that the transmission of phase-modulated symbols from some of the IRS has no effect on the SER performance of the non-coherent receiver compared to the system where no phase shifts are applied at the IRS elements. This is attributed to the fact that introducing phase shifts at the IRS elements does not change the distribution of the effective channel between the transmitter and the receiver as explained above. Moreover, it can be seen, as expected, that increasing the number of elements at the IRS enhances the SER performance at the non-coherent receiver; although no channel control/co-phasing is done in this case, nevertheless, increasing the number of elements at the IRS will increase the average received power at the receiver; this, in turn, enhances the overall non-coherent system error performance.

In the second scenario, we investigate the SER performance of the IRS transmitted phase-modulated data. It should be

[4]The ML detector tries to find the closest subspace in the constellation set to the subspace of the received matrix $\mathbf{Y}$.

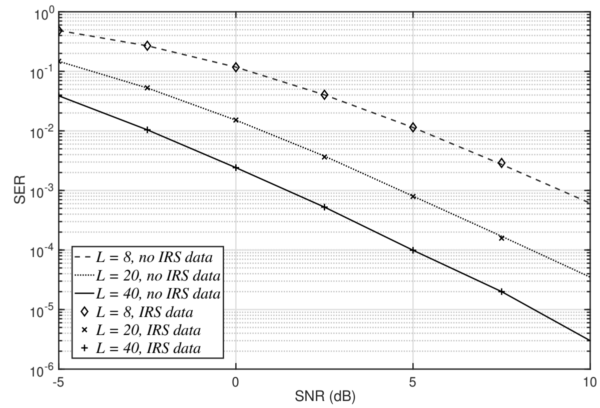

Fig. 2: SER of the schemes with IRS data transmissions and no IRS data transmission for $L = 8$, $L = 20$, and $L = 40$ with $M = N = 2$ (in the legends, no IRS data means that the IRS merely reflects the transmitter impinged energy; IRS data means that some IRS elements transmit extra data symbols by controlling their phases)

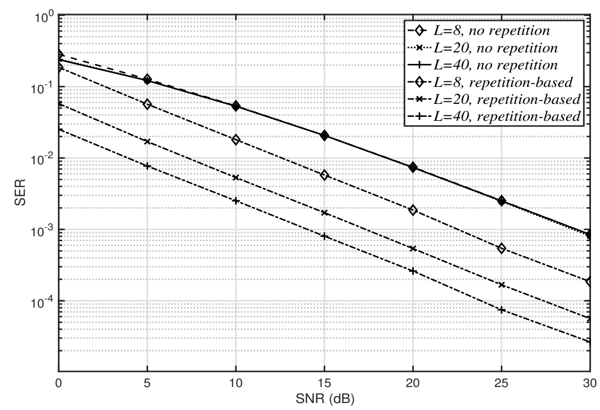

Fig. 3: SER of the QPSK modulated IRS data for $L = 8$, $L = 20$, and $L = 40$ with $M = N = 2$; the results are shown for the no-repetition and repetition-based schemes

noted that the receiver in this case has to estimate the channel state information. There are many options to implement the receiver decoder in this case. One option is to do joint decoding of the non-coherent code matrix $\mathbf{X}$ and the phases of the IRS elements transmitting data using ML decoding through, e.g., exhaustive search. Another option is to have a two-stage decoder [14], [16], in which the non-coherent code matrix $\mathbf{X}$ is decoded first and then the data from the IRS elements are decoded using some MIMO spatial multiplexing decoder, e.g., the Zero-Forcing (ZF) or Minimum Mean-Square Error (MMSE) detectors. In Fig. 3, we resort to the second approach of having a two-step detector. We first decode the non-coherent MIMO codeword $\mathbf{X}$ and use it to construct $\mathbf{H}_{\text{eff}}$ in (12) for decoding the IRS modulated phases. We use the ZF detector to decode the IRS phases.

In Fig. 3, we consider two different approaches for transmitting IRS QPSK modulated phases, namely, no-repetition and repetition-based approaches. In the no-repetition approach, the phase of only one IRS element is used to transmit a specific phase-modulated symbol. In the repetition-based approach, the same modulated phase is transmitted from several IRS elements. For example, in the case of $M = N = 2$ and $L = 40$, the IRS can transmit up to 4 different phase-modulated symbols, and each can be simultaneously transmitted from 10 different IRS elements. The repetition-based approach benefits from the increased transmission power as we can think of each of the IRS elements as a separate antenna that is, on average, transmitting the same power[5]. Therefore, allowing more IRS elements to apply (transmit) the same phase is in some sense equivalent to having more antennas transmitting more energy. The gains of the repetition-based approach are clear from Fig. 3. It can be seen that for the no repetition transmission scheme, all the SER curves coincide on top of each other as the number of IRS elements has no effect because the phases are transmitted from only 4 IRS elements in our case irrespective of the actual number of IRS elements. However, the repetition-based approach can always benefit from increasing the number of elements of the IRS resulting in significant performance gains that are proportional to the number of reflecting elements.

V. Conclusions

In this paper, we characterize some of the gains achievable by the use of IRS in the context of non-coherent communications. The benefit is two-fold; first, although the non-coherent receiver does not require CSI, the insertion of IRS can still result in performance gains by directing more energy towards the receiver increasing its SNR. Moreover, we characterize the degrees of freedom gained by allowing the IRS to transmit phase-modulated symbols. We proved that this will not affect the transmission of the non-coherent data and no provisions are required at the non-coherent transmitter or receiver (in terms of power provisions or even having to know that the IRS is transmitting data). The IRS can still be passive and transmit its data reflecting the transmitted signal. Finally, we have shown that allowing the IRS to repeat its phase-modulated symbols, by having multiple IRS elements simultaneously transmit the same symbol, results in significant performance gains.

[5]Note that the IRS is not actually transmitting any extra energy, i.e., it still acts as a passive surface, but it benefits from the reflected energy of the transmitter node. This allows for *almost free* transmissions of the extra IRS data, which is one of the advantages of our proposed system.

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