On the Stability of Random Multiple Access with Feedback Exploitation and Queue Priority

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Abstract— In this paper, we study the stability of two interacting queues under random multiple access in which the queues leverage the feedback information. We derive the stability region under random multiple access where one of the two queues exploits the feedback information and backs off under negative acknowledgement (NACK) and the other, higher priority, queue will access the channel with probability one. We characterize the stability region of this feedback-based random access protocol and prove that this derived stability region encloses the stability region of the conventional random access (RA) scheme that does not exploit the feedback information.

I. INTRODUCTION

The stability of interacting queues has been extensively considered in literature. Several works have considered the characterization of the stability region of interacting queues under random access protocols. The stability region is characterized for the case $M = 2$ and $M = 3$ interacting queues as well as the case of $M > 3$ with symmetric arrivals. The stability region for the general case of $M > 3$ with asymmetric arrivals is still an open problem and only inner achievable bounds are known.

Recently, many papers have considered the problem of interacting queues in different contexts. For example, [1] considers the problem of interacting queues in a TDMA system where a relay is used to help the source nodes in forwarding their lost packets. In [2], the stability of interacting queues under a random access protocol in the context of *Cognitive Radio Network* was derived. In [3], the stability region of two interacting queues under random access protocol where the two queues harvest energy was characterized. Other works can be found in [4], [5], where derivations of the stability regions in the context of different cognitive radio networks were considered.

In this paper, we derive the stability region of a two-queue random access (RA) protocol with priorities. The queues will apply the conventional RA protocol but in the case of packet loss due to collision the two queues will exploit the feedback information to provide some level of coordination. We set a priority to one of the two queues as follows. In the case of a negative acknowledgement, the queue with the higher priority will attempt transmission in the following time slot with probability one and the other queue will back off to allow for collision-free transmission of the higher priority queue. This priority may arise in cognitive radio scenarios where the primary user has a higher priority than the secondary user, or in the case of multiple users with different quality of service
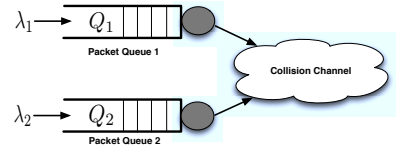

Fig. 1: The system model.

(QoS) constraints. We derive the expression for the boundary of the stability region and prove that the RA with priority scheme encloses the stability region of the conventional RA scheme. To the best of our knowledge, the problem of characterizing the stability region of the random access protocol with feedback leveraging has not been considered before.

II. SYSTEM MODEL

The system model is shown in Fig. 1. We consider the case of two interacting packet queues, namely Q_1 and Q_2. Q_1 and Q_2 have infinite buffers for storing fixed length packets. The channel is slotted in time and any slot duration equals one packet transmission time. The arrival processes at the two queues, Q_1 and Q_2, are modeled as Bernoulli arrival processes with means λ_1 and λ_2, respectively [3]. Under our system model assumptions, the average arrival rates are λ_1 and λ_2 packets per time slot, and are bounded as $0 < \lambda_i < 1$, $i = 1, 2$[1]. We can assume that the packets arrive at the start of the time slot.

The channel is modeled as a collision channel, where packet loss results only in the case of simultaneous transmissions from the two queues. If only one queue attempts transmitting at a given time slot, the packet is considered to be correctly received [3], [6]. In the random access phase, the first queue accesses the channel with probability p_1 whenever it has packets to be sent and the second queue will access the channel with probability p_2 whenever it has packets to send. If at any time slot some queue is empty, it will not attempt any channel access.

In this paper, we will consider the use of feedback information that is leveraged at the queues in the case of collision. In the conventional random multiple access system and in the case of collision, the collided packets stay on the head of the queue and retransmissions are attempted employing the same random multiple access scheme. In this paper, we consider

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[1]The maximum service rate in our model is 1 packet/slot, since the slot duration equals one packet transmission time, then the arrival rates must be less than 1 otherwise the system will be unstable [3].
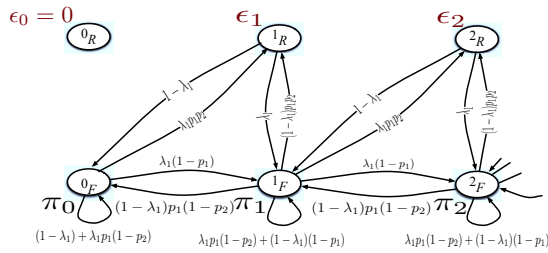
Fig. 2: Queue 1, Q_1, Markov chain model for Dominant System 1.

a system where the feedback information is leveraged at the queues and a priority is set to the first queue; in the next time slot after collision, queue 2 (Q_2) will back off and queue 1 (Q_1) will retransmit its collided packet to allow for collision-free transmission of Q_1; after that the two queues return to the conventional random multiple access scheme. The priority set to queue 1 can be a result of some quality of service (QoS) requirement that is different from the QoS requirement of queue 2 or in a cognitive radio setting where the queue with the higher priority is the primary user queue.

III. THE STABILITY REGION FOR THE FEEDBACK-BASED RANDOM ACCESS PROTOCOL WITH PRIORITIES

Stability can be loosely defined as having a certain quantity of interest kept bounded. In our case, we are interested in the queue size being bounded. For an irreducible and aperiodic Markov chain with countable number of states, the chain is stable if and only if it is positive recurrent, which implies the existence of its stationary distribution. For a rigorous definition of stability under more general scenarios see [6] and [7].

Characterizing the stability region will be a difficult problem due to the interaction of the two queues and due to the fact that the service for one queue will depend on the state of the other queue. We will consider the use of the *Dominant System* concept that was proposed in [6] to characterize the stability region of the conventional RA scheme. We will define two dominant systems tailored to match our feedback-based random access scheme and in each of the two systems we will determine the boundaries of the stability region.

A. Dominant System 1

In any dominant system, we define a system that "stochastically dominates" our system, that is the queues lengths in the dominant system are always larger than the queues lengths in our system if both, the dominant system and our system, start from the same initial state and have the same arrivals and encounter the same packet collisions.

For the first Dominant System, we assume that queue 2 will always have packets to transmit; even if the queue was empty dummy packets will be transmitted from queue 2. Clearly this will set a dominant system to our system since the transmission of dummy packets can only result in more collisions and packet losses. If for a given arrival rate pair (λ_1, λ_2) the first dominant system is stable then clearly our system will be stable. Therefore, the stability region of the first dominant

system will provide an inner bound for our system stability region.

For queue 1, the Markov chain describing the evolution of the queue is shown in Fig. 2. Note that the Markov chain has two classes of states, namely, k_F and k_R and $k = 0, 1, 2, \cdots$. The subscript F denotes first transmission states and the subscript R denotes retransmission states. Note that in the retransmission states, queue 1 packet will always be delivered since there is no collisions in these states (queue 2 is backing off); in these states, either queue 1 length decreases by 1 if no arrival occurs or the queue length will remain the same if an arrival occurs while being in these retransmission states since the packet on the head of the queue is successfully transmitted with probability 1.

The stability condition for queue 1 in Dominant System 1 is given in the following lemma, which is proved in Appendix I.

Lemma 3.1: The arrival rates for queue 1 and queue 2 in Dominant System 1 must satisfy the following two conditions, respectively,

$$\lambda_1 < \frac{p_1}{1 + p_1 p_2}, \quad \lambda_2 < p_2(1 - \lambda_1 - \lambda_1 p_2) \qquad (1)$$

for the system to be stable.

B. Dominant System 2

In the second Dominant System, we assume that queue 1 always has packets to send (dummy packets are sent if the queue decides to transmit while being empty). Again, this will decouple the interaction of the two queues since the service rate of queue 2 will be independent of the state of queue 1.

The Markov chain for the evolution of queue 2 is shown in Fig. 3. Two classes of states are defined in Fig. 3 and denoted by the subscripts ON and OFF. The ON states denote the states where queue 2 can access the channel. The OFF states denote the back off states where queue 1 is retransmitting its collided packets. Note that the transitions from the k_{OFF} state can be either to the k_{ON} state, if no arrival occurs in the slot, or to the $(k+1)_{\text{ON}}$ state, if one arrival occurs in the slot. The OFF states can never make a transition to a state with a lower number of packets since in the OFF states queue 2 is in the back off mode and no access is attempted.

The stability condition for queue 2 in Dominant System 2 is given in the following lemma, proved in Appendix II using the theory of homogeneous quasi birth-and-death (QBD) Markov chains [8].

Lemma 3.2: The arrival rates for queue 1 and queue 2 in Dominant System 2 must satisfy the following two conditions, respectively,

$$\lambda_1 < \frac{p_1(1 - p_1 - \lambda_2 p_1)}{(1 - p_1)}, \quad \lambda_2 < \frac{p_2(1 - p_1)}{1 + p_1 p_2} \qquad (2)$$

for the system to be stable.

C. The Stability Region of the Random Access Protocol with Priorities

In this section, we derive the expression for the stability region of the random access scheme with feedback exploitation
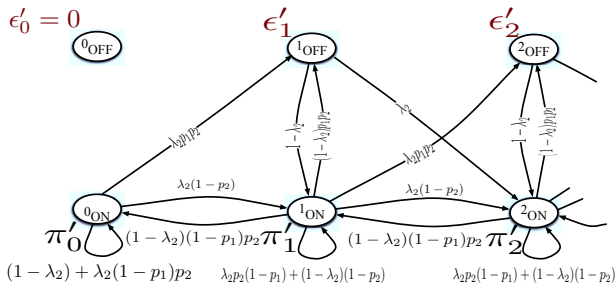
Fig. 3: Queue 2, Q_2, Markov chain model for Dominant System 2.

where a priority is set to one of the two queues. The following Lemma characterizes the stability region for fixed random access probabilities, p_1 and p_2, for queue 1 and queue 2, respectively. The proof of the lemma is omitted due to space limitations (a sketch of the proof is given in [9]).

Lemma 3.3: For a fixed random access probability vector $\mathbf{p} = [p_1\ p_2]^T$, the stability region $\mathcal{R}(\mathbf{p})$ of the random access with priorities is the union of the two regions described by

$$\lambda_2 < p_2(1 - \lambda_1 - \lambda_1 p_2) \text{ when } \lambda_1 < \frac{p_1}{1 + p_1 p_2}$$

and

$$\lambda_1 < \frac{p_1(1 - p_1 - \lambda_2 p_1)}{(1 - p_1)} \text{ when } \lambda_2 < \frac{p_2(1 - p_1)}{1 + p_1 p_2}.$$

The next theorem characterizes the entire stability region for the random access protocol with priorities (proof is omitted due to space limitations and can be found in [9]).

Theorem 3.4: The boundary of the stability region, $\mathcal{R}$, of the random access protocol with priorities, which is defined as the union of the $\mathcal{R}(\mathbf{p})$ regions for the different $\mathbf{p} = [p_1\ p_2]^T$ as

$$\mathcal{R} = \bigcup_{\mathbf{p} \in [0,1]^2} \mathcal{R}(\mathbf{p}) \tag{3}$$

can be characterized as

$$\lambda_2 = \begin{cases} 1 - 2\lambda_1 & \lambda_1 \leq \frac{1}{3} \\ \frac{(1-\lambda_1)^2}{4\lambda_1} & \lambda_1 > \frac{1}{3}. \end{cases} \tag{4}$$

In Fig. 4, we have plotted the derived stability region boundary given in the previous theorem. Fig. 4 also shows the stability region of the random access scheme, whose boundary is given by the following relation [6]:

$$\sqrt{\lambda_1} + \sqrt{\lambda_2} = 1. \tag{5}$$

In Fig. 4, we also show the boundary of the stability region for the time division (TD) based scheme (genie-aided), which serves as the stability region upper bound, given by[2]

$$\lambda_1 + \lambda_2 = 1. \tag{6}$$

It is clear, and straightforward to analytically prove from the closed-form stability region boundary expressions, that the stability region for the RA scheme with priorities encloses the

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[2]Time Division (TD) corresponds to full coordination between the two queues and requires knowledge of the queues arrival rates a priori.
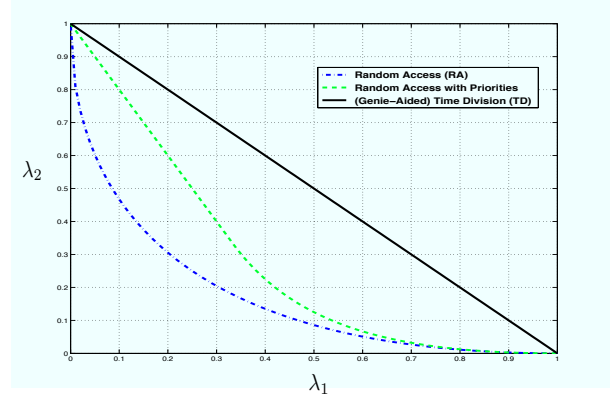

Fig. 4: The stability regions for the Random Access, Random Access with Priorities, and Time Division schemes.

stability region of the RA scheme. This can be explained as follows. For a given arrival rate at the first queue, λ_1, the RA with priority scheme will provide a better service rate to that queue if compared to the RA scheme and this means that queue 1 will be empty with a higher probability and this means that queue 2 will have a higher service rate as well under the RA with priority scheme as compared to the RA scheme.

IV. CONCLUSIONS

In this paper, we consider the problem of deriving the stability region for random access protocol with feedback exploitation. We consider the case of two interacting queue with priority set to one of the two queues. The two queues will access the channel through a conventional random access protocol and in the case of collision the higher priority queue will access the channel in the next slot with probability 1 while the other queue will back off. We derive the stability region for the random access with priorities protocol and show that it contains the stability region for the conventional random access protocol.

APPENDIX I
PROOF OF LEMMA 3.1

The steady state distribution of the Markov chain shown in Fig. 2 can be written as follows (details are omitted due to space limitations and can be found in [9]).

- $\epsilon_0 = 0$.
- $\pi_k = \rho^k \pi_0$, $k \geq 1$ and $\rho = \frac{\lambda_1(1 - p_1 + \lambda_1 p_1 p_2)}{p_1(1-\lambda_1)(1-\lambda_1 p_2)}$.
- $\epsilon_1 = \frac{\lambda_1 p_2}{1 - \lambda_1 p_2}\pi_0$, $\epsilon_k = \rho^{k-1}\epsilon_1$ for $k \geq 2$.

To get the value of the steady state probabilities, we apply the following normalization requirement.

$$\sum_{k=0}^{\infty}(\pi_k + \epsilon_k) = 1 \rightarrow \pi_0\left(1 + \frac{\lambda_1 p_2}{1 - \lambda_1 p_2}\right)\sum_{k=0}^{\infty}\rho^k = 1, \tag{7}$$

where $\rho = \frac{\lambda_1(1 - p_1 + \lambda_1 p_1 p_2)}{p_1(1-\lambda_1)(1-\lambda_1 p_2)}$ as defined above.

Note that for the steady state distribution to exist, i.e. to have π_0 to be non zero, then we must have $\rho < 1$, which is the stability condition for queue 1 in this dominant system. Therefore, the stability condition can be stated as

$$\rho < 1 \rightarrow \lambda_1 < \frac{p_1}{1 + p_1 p_2}. \tag{8}$$

From the normalization condition in (7), we can get the value of π_0 as

$$\pi_0 = \frac{p_1 - \lambda_1(1 + p_1 p_2)}{p_1(1 - \lambda_1)}. \quad (9)$$

In Dominant System 1, queue 2 will be served only in the states denoted by the subscript F in Fig. 2 since in the retransmission states, denoted by the subscript R in Fig. 2, queue 2 will be in the back off mode. Hence, the service rate, μ_2, for queue 2 in Dominant System 1 is given by

$$\mu_2 = p_2(1 - \lambda_1)\pi_0 + p_2(1 - p_1)\lambda_1\pi_0 + \sum_{k=1}^{\infty} p_2(1 - p_1)\pi_k$$
$$= p_2(1 - p_1\lambda_1)\pi_0 + \sum_{k=1}^{\infty} p_2(1 - p_1)\pi_k, \quad (10)$$

where in the 0_F state, and with the arrival at the beginning of the slot assumption, queue 2 is served with a rate of $p_2(1 - \lambda_1)\pi_0$ with no arrival at the beginning of the slot since queue 1 will not attempt any random access since it is empty, and $p_2(1 - p_1)\lambda_1\pi_0$ with arrival at the slot beginning; for the other first transmission states, queue 2 will be served if it decides to access the medium, which occurs with probability p_2, and queue 1 decides not to access the medium, which occurs with probability $(1 - p_1)$. After some manipulation, we can write the expression for μ_2 as

$$\mu_2 = p_2(1 - \lambda_1 - \lambda_1 p_2). \quad (11)$$

For the stability of queue 2, we must have

$$\lambda_2 < \mu_2 = p_2(1 - \lambda_1 - \lambda_1 p_2). \quad (12)$$

APPENDIX II
PROOF OF LEMMA 3.2

We start by calculating the steady state distribution of the Markov chain shown in Fig. 3. The state transition matrix, $\boldsymbol{\Phi}$, of the Markov chain shown in Fig. 3 can be written as

$$\boldsymbol{\Phi} = \begin{pmatrix} \mathbf{B} & \mathbf{A}_0 & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_2 & \mathbf{A}_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (13)$$

where

$$\mathbf{B} = \begin{pmatrix} (1 - \lambda_2) + \lambda_2(1 - p_1)p_2 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\mathbf{A}_0 = \begin{pmatrix} (1 - \lambda_2)(1 - p_1)p_2 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{A}_2 = \begin{pmatrix} \lambda_2(1 - p_2) & \lambda_2 \\ \lambda_2 p_1 p_2 & 0 \end{pmatrix},$$

$$\mathbf{A}_1 = \begin{pmatrix} \lambda_2 p_2(1 - p_1) + (1 - \lambda_2)(1 - p_2) & 1 - \lambda_2 \\ (1 - \lambda_2)p_1 p_2 & 0 \end{pmatrix}.$$

The steady state distribution vector is given by $\mathbf{v} = [\pi'_0\ \epsilon'_0\ \pi'_1\ \epsilon'_1\ \pi'_2\ \epsilon'_2\ \cdots]^T$ and $\mathbf{v} = \boldsymbol{\Phi}\mathbf{v}$.

The state transition matrix $\boldsymbol{\Phi}$ is a block-tridiagonal matrix; therefore the Markov chain shown in Fig. 3 is a homogeneous quasi birth-and-death (QBD) Markov chain [8]. Note that to make the state transition matrix a block-tridiagonal matrix we
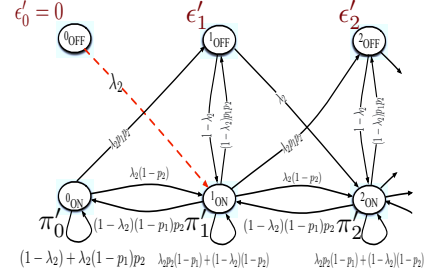

Fig. 5: The queue 2 Markov chain with added transition between 0_{OFF} and 1_{ON}.

have added a transition from the 0_{OFF} state to the 1_{ON} state as shown in Fig. 5 and this will preserve the structure of the state transitions between the different stages in the Markov chain. Note that adding this transition will not affect the stationary state distribution of the Markov chain as well as the balance equations since $\epsilon'_0 = 0$ even with the added transition since the Markov chain will never enter the 0_{OFF} state.

Define the vector $\mathbf{v}'_k = [\pi'_k\ \epsilon'_k]^T$. Note that $\mathbf{v}'_0 = [\pi'_0\ 0]^T$. The steady state distribution of the Markov chain shown in Fig. 3 satisfies the following equation [8]

$$\mathbf{v}'_k = \mathbf{R}^k \mathbf{v}'_0, \quad k \geq 1, \quad (14)$$

where the 2×2 matrix $\mathbf{R}$ is given by the solution to the following equation (which can be readily derived from the states balance equation: $\mathbf{v}'_k = \mathbf{A}_2\mathbf{v}'_{k-1} + \mathbf{A}_1\mathbf{v}'_k + \mathbf{A}_0\mathbf{v}'_{k+1}$, $k \geq 1$).

$$\mathbf{A}_2 + (\mathbf{A}_1 - \mathbf{I}_2)\mathbf{R} + \mathbf{A}_0\mathbf{R}^2 = \mathbf{0}_{2 \times 2}, \quad (15)$$

where $\mathbf{I}_2$ is the 2×2 identity matrix and $\mathbf{0}_{2 \times 2}$ is the all zeros 2×2 matrix.

To get the stationary distribution, we have to find the matrix

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}.$$

Note that for $\mathbf{v}'_1 = \mathbf{R}\mathbf{v}'_0$, where $\mathbf{v}'_0 = [\pi'_0\ 0]^T$ and $\mathbf{v}'_1 = [\pi'_1\ \epsilon'_1]^T$. Therefore, we have

$$r_{11} = \frac{\pi'_1}{\pi'_0} \text{ and } r_{21} = \frac{\epsilon'_1}{\pi'_0}. \quad (16)$$

Writing the balance equation around the 0_{ON} in Fig. 3, we have

$$(\lambda_2 p_1 p_2 + \lambda_2(1 - p_2))\pi'_0 = (1 - \lambda_2)(1 - p_1)p_2\pi'_1$$
$$\rightarrow \pi'_1 = \frac{\lambda_2(1 - p_2 + p_1 p_2)}{(1 - \lambda_2)(1 - p_1)p_2}\pi'_0. \quad (17)$$

Therefore, we have

$$r_{11} = \frac{\lambda_2(1 - p_2 + p_1 p_2)}{(1 - \lambda_2)(1 - p_1)p_2}. \quad (18)$$

Writing the balance equation around 1_{OFF}, we have

$$\epsilon'_1 = \lambda_2 p_1 p_2\pi'_0 + (1 - \lambda_2)p_1 p_2\pi'_1 \rightarrow \epsilon'_1 = \frac{\lambda_2 p_1}{1 - p_1}\pi'_0. \quad (19)$$

Therefore, we have

$$r_{21} = \frac{\lambda_2 p_1}{1 - p_1}. \quad (20)$$

$$\mathbf{sp}(\mathbf{R}) = \frac{\lambda_2 \left(1 - p_2 - \lambda_2 p_1 p_2 + 2p_1 p_2 + \sqrt{1 - 2p_2 + p_2^2 + 4p_1 p_2 - 2\lambda_2 p_1 p_2 - 2\lambda_2 p_1 p_2^2 + \lambda_2^2 p_1^2 p_2^2}\right)}{2p_2 \left(1 - \lambda_2 - p_1 + \lambda_2 p_1\right)}. \tag{27}$$
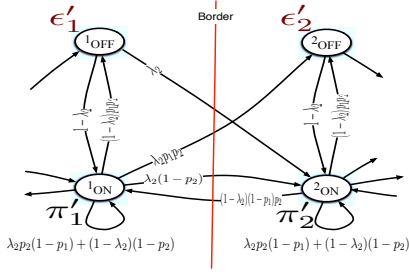

Fig. 6: The segment of the Markov chain used to calculate the values of r_{12} and r_{22}.

To get the values of r_{12} and r_{22}, we consider the transition across the border shown in Fig. 6. For the Markov chain to be positive recurrent then the probability of going across the border in both directions must be the same; hence, we have

$$(1 - \lambda_2)(1 - p_1)p_2 \pi_2' = (\lambda_2 p_1 p_2 + \lambda_2(1 - p_2))\pi_1' + \lambda_2 \epsilon_1'. \tag{21}$$

But we have $\mathbf{v}_2' = \mathbf{R}\mathbf{v}_1'$, from which we have $\pi_2' = r_{11}\pi_1' + r_{12}\epsilon_1'$; using (18) and (21), we can easily show that

$$r_{12} = \frac{\lambda_2}{(1 - \lambda_2)(1 - p_1)p_2}. \tag{22}$$

Finally, the balance equation around 2_{OFF} can be written as

$$\epsilon_2' = \lambda_2 p_1 p_2 \pi_1' + (1 - \lambda_2)p_1 p_2 \pi_2' = r_{21}\pi_1' + r_{22}\epsilon_2'. \tag{23}$$

Substituting for π_2' from (21), we can easily show that

$$r_{22} = \frac{\lambda_2 p_1}{1 - p_1}. \tag{24}$$

Now the matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{pmatrix} \frac{\lambda_2(1 - p_2 + p_1 p_2)}{(1 - \lambda_2)(1 - p_1)p_2} & \frac{\lambda_2}{(1 - \lambda_2)(1 - p_1)p_2} \\ \frac{\lambda_2 p_1}{1 - p_1} & \frac{\lambda_2 p_1}{1 - p_1} \end{pmatrix}, \tag{25}$$

which can be easily checked to satisfy the balance equation given by (15).

To get the stationary distribution of the Markov chain shown in Fig. 3, we apply the following normalization requirement.

$$\sum_{k=0}^{\infty}(\pi_k' + \epsilon_k') = 1 \rightarrow [1\ 1]\left(\sum_{k=0}^{\infty}\mathbf{R}^k\right)\mathbf{v}_0' = 1. \tag{26}$$

For the summation $\left(\sum_{k=0}^{\infty}\mathbf{R}^k\right)$ to converge we must have the spectral radius of the matrix $\mathbf{R}$, $\mathbf{sp}(\mathbf{R})$, to be less than one [8][3]. From (25), we can easily get $\mathbf{sp}(\mathbf{R})$ to be given by equation (27) on the top of this page.

The requirement that $\mathbf{sp}(\mathbf{R}) < 1$ can be used in the last expression to get the stability condition of the second queue arrival rate λ_2 as

$$\lambda_2 < \frac{p_2(1 - p_1)}{1 + p_1 p_2}. \tag{28}$$

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[3]The spectral radius of a matrix is the maximum over the magnitudes of its eigenvalues.

Going back to the normalization requirement in (26), we have

$$[1\ 1]\left(\sum_{k=0}^{\infty}\mathbf{R}^k\right)\mathbf{v}_0' = [1\ 1](\mathbf{I}_2 - \mathbf{R})^{-1}\begin{bmatrix} \pi_0' \\ 0 \end{bmatrix} = 1. \tag{29}$$

From the last expression, we can easily prove that π_0' is given by

$$\pi_0' = \frac{p_2 - \lambda_2 - p_1 p_2 - \lambda_2 p_1 p_2}{(1 - \lambda_2)(1 - p_1)p_2}. \tag{30}$$

Note that the requirement that $\pi_0' > 0$, i.e. a non-zero empty queue probability, is satisfied if $\lambda_2 < \frac{p_2(1 - p_1)}{1 + p_1 p_2}$, which is the queue stability condition.

The service rate, μ_1', for queue 1 in Dominant System 2 can now be expressed as

$$\mu_1' = p_1(1 - \lambda_2)\pi_0' + p_1(1 - p_2)\lambda_2 \pi_0' + p_1(1 - p_2)\sum_{k=2}^{\infty}\pi_k' + \sum_{k=2}^{\infty}\epsilon_k'$$
$$= \frac{p_1(1 - p_1 - \lambda_2 p_1)}{(1 - p_1)}, \tag{31}$$

where in the OFF states, queue 1 is served with probability 1 since queue 2 will be in the back off mode. For the stability of queue 1 in Dominant System 2 we must have

$$\lambda_1 < \mu_1' = \frac{p_1(1 - p_1 - \lambda_2 p_1)}{(1 - p_1)}, \tag{32}$$

which completes the proof.

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