CENSORING FOR IMPROVED PERFORMANCE OF DISTRIBUTED DETECTION IN WIRELESS SENSOR NETWORKS

Karim G. Seddik

Nile University Wireless Intelligent Networks Center (WINC) Smart Village, Egypt

ABSTRACT

In this paper, we consider the problem of binary hypothesis testing for distributed detection in wireless sensor networks in which a transmission censoring scheme is employed. The sensor nodes transmit binary decisions to the fusion center (FC) for final decision making. Sensor nodes with unreliable observation samples censor transmission to FC. By having two thresholds at each sensor node, a sensor node censors transmission if its log-likelihood ratio (LLR) falls between the two thresholds, whereas the more informative sensor nodes transmit their decisions to the FC. In this case of censoring some of the less-informative sensor nodes, and under our system model assumptions, we demonstrate that censoring can lower the probability of error at FC even if there is enough power and rate to support the transmissions of the less-informative sensor nodes.

1. INTRODUCTION

Sensor networks have gained a lot of interest due to their wide range of applications and this has increased the thrill toward their study. The applications of sensor networks include monitoring environmental conditions such as temperature, military applications such as battlefield surveillance, health monitoring, and many other applications. These diverse applications of the sensor networks have raised a lot of challenges due to the fact that the sensor nodes are prone to failures, and have limited power, limited computational capacities, and limited memory [1].

In this paper, we consider the problem of distributed detection for sensor nodes sending binary decisions over wireless fading channels to a fusion center (FC) where the final decision is made. We investigate the possibility of censoring some of the sensor nodes according to the quality of the local sensor observations measured by the local log-likelihood ratios (LLR). The idea of censoring some of the sensor nodes with unreliable observations have been considered before, for example, in [2, 3, 4, 5, 6]. Censoring has been proposed for energy efficiency or information rate reduction assuming that there is a tradeoff between the amount of censoring (used Ahmed K. Sultan and Amr A. El-Sherif

Alexandria University Department of Electrical Engineering Alexandria 21544, Egypt

for energy and/or data rate reduction) and the detection error performance. For instance, in [2] where the idea of sensor censoring has been introduced and where the LLR values are forwarded to FC, the probability of error is minimized under a communication rate constraint. In [4], the detection problem is formulated with constraints on the expected energy cost arising from transmission and measurement.

In this paper, we consider binary decisions sent from the sensor nodes to an FC that does not have knowledge of sensor reliability measured by its instantaneous local LLR. We demonstrate that censoring can indeed enhance the overall system performance in terms of FC error probability even without rate or energy constraints. The idea is that when the sensor nodes send binary decisions to the FC, the quality of the observation is lost and the FC treats all of the sensors decisions equally or perhaps only taking into account the quality of the channels between itself and the sensors. In this case it is better for only the more-informative sensors to send information to FC and for the less-informative sensors to censor transmission. In conventional binary hypothesis testing, there is a single threshold at each sensor node against which the sensor's LLR is compared for local binary decision making. In order to implement the censoring idea as in [2], each sensor node should have two thresholds. If the local sensor's LLR falls between the two thresholds the sensor node censors transmission, otherwise it transmits its binary decision to FC.

The main contribution of the paper is showing that even if sufficient rate and power are available to allow transmissions from the less-informative sensor nodes, it is better for them to refrain from transmitting their local measurements, because censoring can reduce the required rate and power, and simultaneously enhance the global error performance at FC. We provide a theoretical analysis of both the one-threshold and censoring schemes in a large sensor network, and obtain the corresponding error exponents using large deviations theory in order to demonstrate the improved performance of distributed detection when censoring is employed.

The paper is organized as follows. In Section 2, the system and data models are introduced. In Section 3, we present the error exponent analysis for large sensor networks. In Sec-

tion 4, we provide numerical results, whereas Section 5 concludes the paper.

Notations: The symbol \doteq is used to denote equality in the exponential decay rate, that is $f(N) \doteq g(N)$ means that $\lim_{N\to\infty} \frac{1}{N} \log \frac{f(N)}{g(N)} = 0$. The notation $x \sim \mathcal{N}(m, \sigma^2)$ is used to denote that x is a Gaussian random variable with mean m and variance σ^2 . We use $\Re(y)$ to denote the real part of the complex number y. $Q(\cdot)$ denotes the Q-function defined as $Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{\frac{-z^2}{2}} dz$ and $\operatorname{erf}(\cdot)$ denotes the error function defined as $\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-z^2} dz$. $E\{\cdot\}$ denotes the expectation operator.

2. SYSTEM AND DATA MODELS

The sensor network is assumed to have N sensor nodes that are used to monitor a certain phenomenon. The sensor nodes send their local decisions to an FC to make decisions about the state of nature observed by the sensor network. The wireless sensor networks is as depicted in Fig. 1. We assume a binary hypotheses detection problem, i.e., the FC makes decisions between two hypotheses, namely, H_0 and H_1 .

The *i*-th sensor node measurement is x_n , $n = 1, \dots, N$. The x_n 's are assumed to be mutually independent under each hypothesis. We assume that the data model under each hypothesis is given by

$$H_0: x_n \sim \mathcal{N}\left(0, \sigma_0^2\right)$$

$$H_1: x_n \sim \mathcal{N}\left(0, \sigma_1^2\right),$$
(1)

where we assume that $\sigma_1^2 > \sigma_0^2$. The analysis below can be readily extended to other H_0 and H_1 distributions.

Based on its observation, and using an orthogonal transmission scheme, each sensor node transmits one symbol, u_n , $n = 1, \dots, N$, to the FC. Each u_n is either +1 or -1 depending on the sensor local observation. The received signal at the destination due to the *n*th sensor node transmission is given by

$$y_{n,F} = h_{n,F}u_n + v_{n,F},$$
 (2)

where $h_{n,F}$ is the channel gain between the *n*th and the fusion center and is modeled as complex Gaussian random variable with zero-mean and variance 1/2 per dimension and $v_{n,F}$ is the fusion center noise and is modeled as complex Gaussian random variable with variance $N_0/2$ per dimension.

Next, we consider the conventional system, where each sensor compares the observed LLR to a certain threshold to decide whether to transmit +1 or -1. Then we present the two-threshold scheme. If the LLR is larger than the first, and higher threshold, the sensor node transmits $u_n = +1$, and if the LLR is less than the second threshold, the sensor node transmits $u_n = -1$. If the LLR is between the two thresholds, the sensor node censors transmission as the observation at the sensor node is not highly informative to the FC.



Fig. 1. A schematic diagram for the wireless sensor network.

3. LARGE SENSOR NETWORKS

We consider the case of a large sensor network where the number of sensor nodes, N, tends to infinity. We employ the equal gain combining (EGC) scheme presented in [7] for combining the binary decisions sent from the different sensor nodes, which is shown to achieve a close performance to the optimal fusion rule at the FC for a wide range of SNR's¹.

3.1. Conventional (One-Threshold Based) Scheme for Large Sensor Networks

In the conventional scheme, each sensor compares the LLR to a threshold η , and if the LLR is larger than η the sensor sends +1 to FC, otherwise it sends -1. The probability of having $u_n = +1$ given hypothesis H_0 is given by

$$Pr(u_n = +1|H_0)$$

$$= Pr\left(\log\left(\frac{\sigma_0}{\sigma_1}\right) + \frac{x_n^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) > \eta|H_0\right)$$

$$= Pr\left(x_n^2 > \gamma|H_0\right)$$

$$= 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right),$$
(3)

and

$$\gamma = \left(2 / \left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)\right) \left(\eta - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right). \tag{4}$$

¹The optimal fusion rule at the FC requires some *average* local sensor information which we do assume available at the fusion center. What is unavailable at FC is the instantaneous local LLR.

Parameter γ is the detection threshold with which x_n^2 is compared in order to make a decision between the two hypotheses. The probability of having $u_n = -1$ given hypothesis H_0 is given by

$$\Pr(u_n = -1|H_0) = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right).$$
 (5)

Similarly, The probability of having $u_n = +1$ given hypothesis H_1 is given by

$$\Pr(u_n = +1|H_1) = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right),\tag{6}$$

and the probability of having $u_n = -1$ given hypothesis H_1 is given by

$$\Pr(u_n = -1|H_1) = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right).$$
(7)

At the FC, an EGC is applied to the received signals; the output from the EGC is given by

$$y_F = \sum_{\{n:u_n=+1\}} |h_{n,F}| - \sum_{\{n:u_n=-1\}} |h_{n,F}| + \sum_{n=1}^N \Re\{v_{n,F}\},$$
(8)

The FC applies the following decision rule to determine the estimated hypothesis

$$\Re(y_F) \underset{\hat{H}=H_0}{\overset{\hat{H}=H_1}{\gtrless}} 0.$$
(9)

Recall that $\Re(.)$ denotes the real part of a complex number.

Next, we calculate the error exponent under each hypothesis using large deviations theory techniques [8, 9]. Under hypothesis H_0 , the variable $\Re(y_F)$ can be represented as

$$\Re(y_F) = \sum_{n=1}^{N} Z_{n,F},$$
 (10)

where

$$Z_{n,F} = \begin{cases} |h_{n,F}| + \Re(v_{n,F}), & \text{w.p. } P_f = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right); \\ -|h_{n,F}| + \Re(v_{n,F}), & \text{w.p. } 1 - P_f = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right). \end{cases}$$
(11)

and the relation between γ and η is given by (4).

The conditional probability of error, conditioned on H_0 , is given by

$$P_{\mathcal{E}/H_0} = \Pr\left(\Re(y_F) > 0 | H_0\right).$$
 (12)

Using Chernoff's formula, the error probability conditioned on H_0 can be written as

$$P_{\mathcal{E}/H_0} \doteq e^{-N \cdot \max_{\theta} \{-\lambda(\theta)\}},\tag{13}$$

where $\lambda(\theta) = \ln E \{e^{\theta Z_{n,F}}\}\$ is the cumulant generating function (CGF). Averaging over $h_{n,F}$, the error exponent, $\Delta_0(\gamma)$, for the error probability conditioned on H_0 can be derived as

$$\Delta_0(\gamma)$$

$$= -\lim_{N \to \infty} \frac{1}{N} \log P_{\mathcal{E}/H_0}$$

= $-\log\left(\inf_{\theta > 0} \left\{ e^{\frac{\theta^2 N_0}{4}} \left(1 + \frac{\sqrt{\pi\theta}}{2} e^{\frac{\theta^2}{4}} \left(2P_f - 1 + \operatorname{erf}\left(\frac{\theta}{2}\right) \right) \right) \right\} \right)$ (14)

where $P_f = \Pr(u_n = +1|H_0)$.

The conditional probability of error, conditioned on H_1 , is given by

$$P_{\mathcal{E}/H_1} = \Pr\left(-\Re(y_F) > 0|H_1\right).$$
 (15)

Following a similar approach, to the one considered for the case of error conditioned on H_0 , the error exponent for the conditional error probability conditioned on H_1 , $\Delta_1(\gamma)$, can be shown to be given by

$$\begin{aligned} \Delta_1(\gamma) \\ &= -\lim_{N \to \infty} \frac{1}{N} \log P_{\mathcal{E}/H_1} \\ &= -\log\left(\inf_{\theta > 0} \left\{ e^{\frac{\theta^2 N_0}{4}} \left(1 + \frac{\sqrt{\pi \theta}}{2} e^{\frac{\theta^2}{4}} \left(1 - 2P_d + \operatorname{erf}\left(\frac{\theta}{2}\right) \right) \right) \right\} \right) \end{aligned}$$
(16)

where $P_d = \Pr(u_n = +1|H_1)$. The probability of error at FC is

$$P_e \doteq e^{-N \cdot \Delta_0(\gamma)} + e^{-N \cdot \Delta_1(\gamma)}.$$
 (17)

Note the use of the equal sign with a dot. The error exponent is limited by the minimum of $\Delta_0(\gamma)$ and $\Delta_1(\gamma)$. Investigating the expressions given in (14) and (16) we can see that $\Delta_0(\gamma)$ is monotonically increasing in γ and $\Delta_1(\gamma)$ is monotonically decreasing in γ (which is intuitively clear since as γ is increased, the FC is more directed towards selecting hypothesis H_0). Therefore, the optimal value for γ , γ^* , that maximizes the error exponent must satisfy $\Delta_0(\gamma^*) = \Delta_1(\gamma^*)$. It can be proved (sketch of the proof is in the Appendix) that γ^* is the solution of

$$1 - 4Q\left(\frac{\sqrt{\gamma^*}}{\sigma_0}\right) = 4Q\left(\frac{\sqrt{\gamma^*}}{\sigma_1}\right) - 1.$$

3.2. Two-Threshold-Based Scheme for Large Sensor Networks

For the two-threshold based scheme we define two thresholds, η_1 and η_2 with $\eta_1 \ge \eta_2$, at any sensor node. If the sensor observation LLR is larger than the higher threshold, then the sensor node sends $u_n = +1$ to the FC. If the LLR is less than the lower threshold then the sensor sends $u_n = -1$ to the FC. If the LLR falls between the two thresholds, the sensor censors transmission to the FC as the observation is not highly informative. We note that the conventional scheme presented in the previous section can be considered as a special case of the two-threshold based scheme with $\eta_1 = \eta_2 = \eta$. In the two-threshold-based scheme each sensor node has a state in the following set $\{u_n = +1, \text{ censor}, u_n = -1\}$.

At the FC, EGC is applied to the received signals, where we assume that we combine the signals from the active sensors only. The output from EGC is given by

$$y_F = \sum_{\{n:u_n=\pm1\}} |h_{n,F}| - \sum_{\{n:u_n=\pm1\}} |h_{n,F}| + \sum_{\{n:u_n=\pm1\}} \Re\{v_{n,F}\}$$
(18)

Under hypothesis H_0 , the variable $\Re(y_F)$ can be represented as

$$\Re(y_F) = \sum_{n=1}^{N} Z_{n,F},$$
 (19)

where

$$Z_{n,F} = \begin{cases} |h_{n,F}| + \Re(v_{n,F}), & \text{w.p. } 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right); \\ -|h_{n,F}| + \Re(v_{n,F}), & \text{w.p. } 1 - 2Q\left(\frac{\sqrt{\gamma_2}}{\sigma_0}\right); \\ 0, & \text{w.p. } 2Q\left(\frac{\sqrt{\gamma_2}}{\sigma_0}\right) - 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right), \end{cases}$$
(20)

where

$$\gamma_1 = \left(2 / \left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)\right) \left(\eta_1 - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right)$$

and

$$\gamma_2 = \left(2/\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)\right) \left(\eta_2 - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right).$$

Following a similar approach to the one considered for the one-threshold case, the error exponent under H_0 for the twothreshold based scheme, $\delta_0(\gamma_1, \gamma_2)$, can be proved to be given by (21), whereas the error exponent under H_1 , $\delta_1(\gamma_1, \gamma_2)$, is given by (22). For any selection of γ_1 and γ_2 , the error exponent at the FC is equal to the minimum of $\delta_0(\gamma_1, \gamma_2)$ and $\delta_1(\gamma_1, \gamma_2)$. We get the error exponents as well as the optimal thresholds via numerical search.

4. SIMULATION RESULTS

In this section, we present some simulation results for largesensor networks. Fig. 2 shows the error exponent as a function of the average SNR of the reporting channels between the local sensors and FC. We can see that the scheme with censoring achieves better performance in terms of a higher error exponent relative to the conventional scheme. A higher error exponent means a faster decay of the average probability of error as the number of sensors of the network increases. Fig. 3 shows the optimal thresholds versus the average SNR of sensor-FC channels for the conventional and twothreshold-based schemes for a large sensor network with N going to infinity and $\sigma_0^2 = 0.25$ and $\sigma_1^2 = 1$. If one sensor's local LLR falls between the two thresholds of the censoring scheme, it is better for this sensor not to transmit hoping for the other sensors to have more reliable observations. If it sends its binary decision, it would be treated by the FC as a reliable observation, thereby causing a performance degradation in the conventional one-threshold scheme.



Fig. 2. Error exponents for large sensor network with $\sigma_0^2 = 0.25$ and $\sigma_1^2 = 1$. The horizontal axis is the average SNR of the channels from the sensors to FC.



Fig. 3. Thresholds' values for large sensor network with $\sigma_0^2 = 0.25$ and $\sigma_1^2 = 1$.

5. CONCLUSIONS

In this paper, we have considered the problem of binary hypothesis distributed detection over wireless sensor networks. The main result of this work is that if the sensor nodes send binary decisions to the FC, it is better to censor the sensor nodes that are less-informative to the FC even if we have enough rate and/or energy to allow binary transmissions from those

$$\delta_{0}(\gamma_{1},\gamma_{2}) = -\log\left(\inf_{\theta>0}\left\{e^{\frac{\theta^{2}N_{0}}{4}}\left(1+2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{0}}\right)-2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{0}}\right)+\frac{\sqrt{\pi\theta}}{2}e^{\frac{\theta^{2}}{4}}\left(2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{0}}\right)+2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{0}}\right)-1\right)\right)\right)\right)\right)$$

$$+\left(1+2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{0}}\right)-2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{0}}\right)\right)\operatorname{erf}\left(\frac{\theta}{2}\right)\right)+2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{0}}\right)-2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{0}}\right)\right)\right).$$

$$\delta_{1}(\gamma_{1},\gamma_{2}) = -\log\left(\inf_{\theta>0}\left\{e^{\frac{\theta^{2}N_{0}}{4}}\left(1+2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{1}}\right)-2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{1}}\right)+\frac{\sqrt{\pi\theta}}{2}e^{\frac{\theta^{2}}{4}}\left(1-2Q\left(\frac{\sqrt{\gamma_{1}}}{\sigma_{1}}\right)-2Q\left(\frac{\sqrt{\gamma_{2}}}{\sigma_{1}}\right)\right)\right)\right).$$

$$(21)$$

 $+\left(1+2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_1}\right)-2Q\left(\frac{\sqrt{\gamma_2}}{\sigma_1}\right)\right)\operatorname{erf}\left(\frac{\theta}{2}\right)\right)+2Q\left(\frac{\sqrt{\gamma_2}}{\sigma_1}\right)-2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_1}\right)\right\}\right).$

sensor nodes. Allowing all of the sensor nodes to transmit binary decisions can lead to performance degradation in terms of probability of error since the quality of observations is lost after a sensor node quantizes its LLR to one bit. Unreliable sensor nodes' decisions can highly degrade the system performance when the FC considers only the channels between itself and the sensors, but not the *instantaneous* reliability of local sensor decisions.

6. APPENDIX

We start from the expression for the conditional error exponents $\Delta_0(\gamma)$ and $\Delta_1(\gamma)$ given in (14) and (16), respectively. It can be easily seen that $\Delta_0(\gamma)$ is a monotonic increasing function in γ , and $\Delta_1(\gamma)$ is a monotonic decreasing function in γ , which are intuitively clear.

To prove by contradiction, let's assume that for the optimal threshold, γ^* , we have $(2P_f - 1) > (1 - 2P_d)$, then for any θ we will have

$$e^{\frac{\theta^2 N_0}{4}} \left(1 + \frac{\sqrt{\pi\theta}}{2} e^{\frac{\theta^2}{4}} \left(2P_f - 1 + \operatorname{erf}\left(\frac{\theta}{2}\right) \right) \right)$$

> $e^{\frac{\theta^2 N_0}{4}} \left(1 + \frac{\sqrt{\pi\theta}}{2} e^{\frac{\theta^2}{4}} \left(1 - 2P_d + \operatorname{erf}\left(\frac{\theta}{2}\right) \right) \right).$ (23)

From (23) then we can easily see that $\Delta_0(\gamma^*) > \Delta_1(\gamma^*)$. Then, the error exponent will be dominated by $\Delta_1(\gamma^*)$. However, we can select another threshold $\gamma < \gamma^*$, which achieves a better exponent (since $\Delta_0(\gamma)$ is a monotonic increasing function in γ and $\Delta_1(\gamma)$ is a monotonic decreasing function in γ) as we have a room between $\Delta_0(\gamma^*)$ and $\Delta_1(\gamma^*)$; this contradicts the assumption that γ^* is the optimal threshold.

The other case of having an optimal threshold γ^* with $(2P_f - 1) < (1 - 2P_d)$ can be proved to be unattainable following a similar approach to the one considered in the previous case. Therefore, at the optimal threshold we must have $(2P_f - 1) = (1 - 2P_d)$ which leads to the condition given by

$$1 - 4Q\left(\frac{\sqrt{\gamma^*}}{\sigma_0}\right) = 4Q\left(\frac{\sqrt{\gamma^*}}{\sigma_1}\right) - 1$$

which results in a unique solution for γ^* .

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