Collaborative Compressive Spectrum Sensing Using Kronecker Sparsifying Basis

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Abstract—Spectrum sensing in wideband cognitive radio networks is challenged by several factors such as hidden primary users (PUs), overhead on network resources, and the requirement of high sampling rate. Compressive sensing has been proven effective to elevate some of these problems through efficient sampling and exploiting the underlying sparse structure of the measured frequency spectrum. In this paper, we propose an approach for collaborative compressive spectrum sensing. The proposed approach achieves improved sensing performance through utilizing Kronecker sparsifying bases to exploit the two dimensional sparse structure in the measured spectrum at different, spatially separated cognitive radios. Experimental analysis through simulation shows that the proposed scheme can substantially reduce the mean square error (MSE) of the recovered power spectrum density over conventional schemes while maintaining the use of a low-rate ADC. We also show that we can achieve dramatically lower MSE under low compression ratios using a dense measurement matrix but using Nyquist rate ADC.

Index Terms—Spectrum Sensing, Cognitive Radios, Kronecker Compressive Sensing

I. INTRODUCTION

The problem of spectrum sensing has gained a new dimension with the arrival of cognitive radios and the concept of opportunistic spectrum access [1]. In such systems, a cognitive radio (CR) is expected to obtain awareness about the spectrum usage and the existence of primary users in a geographical area, and then utilize this awareness to restrict their transmission to frequency bands that are not occupied by primary users. Spectrum sensing task is considered one of the most challenging tasks in cognitive radio systems, specially with the wideband that these systems are expected to scan and utilize.

There are many challenging issues that face wideband spectrum sensing for CR applications. One of theses issues is the hidden primary user problem [2], where the cognitive radio device fails to detect the primary transmitter's signal, causing unwanted interference at the primary user receiver. This problem occurs due to many factors, such as the relative locations of devices, severe multipath fading and shadowing. Cooperative sensing is proposed as a solution for handling the hidden primary user problem [3], where exploiting spatial diversity among several collaborating CRs was shown to be an effective method to improve the detection performance, but at the expense of cooperation overhead on the network resources [4].

Another major challenge of wideband spectrum sensing is the need of high sampling rate, and high resolution analog to digital converters (ADCs) with large dynamic range to deal with wideband spectrum sensing [1]. Compressive sampling has been used as one of the preferred solutions to overcome ADCs constraints in cognitive radio networks, as it enables the sampling of the wideband signals at sub-Nyquist rate as long as the spectrum is underutilized, which is a valid assumption in cognitive radio networks [5].

One of the first attempts to use compressive spectrum sensing was [6], where the authors proposed a scheme which samples the spectrum at Nyquist rate and then performs compressive sampling on the autocorrelation function of the sampled signal. A coarse spectrum is recovered and used to estimate the location of spectrum edges, and then smoothed over the recovered edges to find accurate spectrum. The main underlying assumption is that the spectrum is piecewise constant, so there are small number of edges in the frequency domain. A following approach was proposed in [7], where the wideband analog signal is directly captured by an analog to information converter, solving the bottleneck in the sampling rate presented in the previous scheme.

Several papers tried to elicit solutions for both problems simultaneously, such as the work in [8], which uses distributed compressive spectrum sensing recovery algorithm and shows a performance gain over the single CR scheme presented in [7]. Instead of assuming a piecewise constant spectrum, it assumes that spectrum has a slotted frequency segmentation structure like OFDM. They also develop distributed fusion techniques for multi-hop large networks operating in frequency selective fading channels .

All of the previous approaches have only considered the signal structure at each CR, utilizing different sparse structure assumptions to reduce the number of measurements needed. In this paper, we propose a novel approach to the problem by making use of the correlation between the measurements of different cognitive radios in addition to the special structure of the spectrum at each CR. This is achieved by using a Kronecker product matrix as a sparsifying basis, which enables us to jointly model the different types of structures present in the signal and to exploit the two dimensional correlation

nature of the collaborative observations. We build a new recovery approach that utilizes Kronecker spectrum sensing to recover the spectrum with fewer number of samples and with improved noise behavior, which alleviates the drawbacks of the cooperative spectrum sensing represented by the overhead data mentioned earlier.

We experiment our approach under a modified signal ensemble model that addresses the problem of hidden primary user. We propose several different measurements matrices, and show through simulation results that the proposed approach can substantially reduce the mean square error of the recovered power spectrum density by about 50% over conventional schemes at low compression ratio (around 0.2) while maintaining the use of low-rate ADC. We also show that by using a dense measurement matrix we can achieve a dramatically low MSE under low compression ratios. However, such matrix can only be used under Nyquest rate ADC.

The rest of the paper is organized as follows. In section II we explain the signal model and the spectrum sensing problem of interest. An overview of different trends in Compressive Sensing (CS) is presented in section III. Section IV develops the Kronecker approach for compressive collaborative spectrum sensing. Simulation results are presented in Section V to show the effectiveness of the proposed compressed sensing and distributed fusion techniques. We finish with concluding remarks in Section VI.
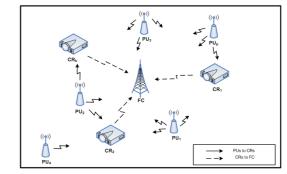
II. SIGNAL MODEL AND PROBLEM STATEMENT

We consider a network of K cognitive radio terminals, distributed randomly in a certain geographic area and performing collaborative spectrum sensing through a centralized fusion center (FC), as shown in Fig. 1a. Each CR locally monitors P non-overlapped channels, where each channel is either occupied by a primary user or unoccupied. We assume a low ratio of occupied channels U/P, where U is the number of active PUs and P is the total number of PUs; this low ratio of occupied channels is due to the low percentage of spectrum occupancy by active radios, which is normally in the range of 15% to 85% [9].

Each CR captures a wideband analog N dimensional signal $\boldsymbol{x_k} \in \mathbb{R}^N$, where the subscript k denotes a specific CR terminal. The signal $\boldsymbol{x_k}$ is captured using the pseudo random demodulation scheme for compressive sampling presented in [10]. The autocorrelations of the measurements are sent synchronously to the fusion center over Additive White Gaussian Noise (AWGN) wireless channel as shown in Fig. 1b.

The recovered signals at FC can be modeled as an $N \times K$ matrix, which can be written as:

$$\boldsymbol{X} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_K}] = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_K^1 \\ x_1^2 & x_2^2 & \cdots & x_K^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^N & x_2^N & \cdots & x_K^N \end{bmatrix}, \quad (1)$$

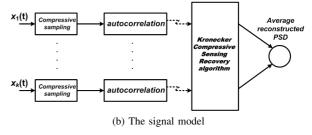(a) An overview of the considered system model

(b) The signal model

Fig. 1. Compressive sensing acquisition and recovery scheme

where x_i^j represents the j-th measurment of the i-th CR. This way, the columns of the matrix represent the individual signals of each CR corresponding to different snapshots, while the rows of the matrix represent the same snapshot of the signal at different cognitive radio terminals.

We use a signals ensemble that is a modified version of the joint sparsity model (JSM2) presented in [8], [11]. The original (JSM2) model assumes only common sparse support, neglecting the possible effect of hidden users, while the (JSM1) model assumes common signal amplitude which is valid only if CRs are distributed in very small area. The proposed modified model (JSM2M) relaxes these assumptions and generates signals which have a common sparse support in frequency domain with different amplitudes plus innovations due to hidden primary user problem.

III. APPLYING CS: A MATHEMATICAL OVERVIEW

In this section, we briefly overview the mathematical tools that are used in the compressive acquisition of signals and their recovery algorithms. We start by describing Compressive Sensing (CS), then we explain in details how to use distributed compressive sensing. Finally, we discuss how to obtain the transformation bases required by CS through Kronecker product bases.

A. Compressive sensing

Compressive sensing is a data acquisition technique that allows for the recovery of a signal from a small number of non-adaptive linear measurements, under the condition that the signal is sparse in some domain [12]–[14]. Assume a signal $\boldsymbol{x_1} \in \mathbb{R}^N$ that has a sparse representation in some domain $\boldsymbol{\Psi}$ such as Fourier transform, Wavelet, or Discrete Cosine Transform (DCT), such that

$$\boldsymbol{x_1} = \boldsymbol{\Psi s_1}, \quad (2)$$

where the N dimensional vector s_1 is a sparse vector. It is said that s_1 is K sparse if it has at most K non-zero entries and $K << N$. The signal x_1 can be acquired through M linear measurements of x_1 where, $M < N$, according to the following equation:

$$y_1 = \Phi x_1 = \Phi\Psi s_1 = \Theta s_1, \qquad (3)$$

where Φ is $M \times N$ measurement matrix.

The CS theory proves that under certain conditions on the measurement matrix Φ, the sparse vector s_1 can be recovered by solving the following ℓ_1 minimization problem,

$$\hat{s}_1 = \arg\min \|s_1\|_1 \text{ such that } \Theta s_1 = y_1. \qquad (4)$$

Which can be solved efficiently using linear programming approaches such as Basis Pursuit [12], [14].

B. Distributed compressive sensing

Distributed compressive sensing (DCS) is a new distributed coding algorithm for simultaneously acquiring and recovering multiple signals which share the same sparsity order and locations of non zero components [15], [11]. The DCS theory relies on a new notion that exploits the joint sparsity of a signal ensemble.

$$Y_2 = \Phi X_2 = \Phi\Psi S_2, \qquad (5)$$

where Y_2 is an $M \times J$ matrix, J is the number of signals in the ensemble, and S_2 is $N \times J$ sparse matrix with a common sparse support and different coefficient values, which might not be a valid assumption in all cases. We can recover the signals by using Simultaneous Orthogonal Matching Pursuit (SOMP) algorithm presented in [15]. DCS-SOMP is an efficient greedy algorithm for joint signal recovery based on the SOMP algorithm for simultaneous sparse approximation with only K measurements not M like the conventional ℓ_1 algorithm.

C. Kronecker compressive sensing

Kronecker Compressive Sensing (KCS) is one of the recently developed compressive sensing techniques. In contrast to DCS, which exploits joint sparsity in one dimension, KCS exploits the structure of a multidimensional signal in every dimension [16], [17]. Kronecker product bases are well suited for CS applications concerning multidimensional signals, i.e., signals that capture information from an event that spans multiple dimensions, such as space, time, frequency, etc. These bases can be used both to obtain sparse or compressible representations of many real signals. KCS depends on the concept that every dimension has its own sparsifying basis, so we can jointly apply these sparsifying bases by getting the Kronecker product of all sparsifying matrices.

We assume that a three dimensional signal is represented by a 3-D matrix X_3, where $X_3 \in \mathbb{R}^{N1 \times N2 \times N3}$. This signal can be captured using Kronecker product measurement matrix $\Phi_k = \Phi_{k1} \otimes \Phi_{k2} \otimes \Phi_{k3}$, where Φ_{k1}, Φ_{k2}, and Φ_{k3} are the measurement matrices that operate individually on portions of the multidimensional signal and $\otimes$ denotes the Kronecker product. The measurement vector y_3 can be written as

$$y_3 = \Phi_k \overline{x}_3 = \Phi_k \Psi_k \overline{s}_3 = \Theta \overline{s}_3. \qquad (6)$$

The sparsifying basic $\Psi_k = \Psi_{k1} \otimes \Psi_{k2} \otimes \Psi_{k3}$ is the Kronecker product of all individual bases, where Ψ_{k1}, Ψ_{k2}, and Ψ_{k3} are the sparsifying bases for the first, second, and third dimensions, respectively, and $\overline{x}_3$ and $\overline{s}_3$ are the column vector-reshaped representation of the matrix X_3 and the sparse coefficient matrix S_3, respectively.

We may then recover $\overline{s}_3$ by solving ℓ_1 minimization program using Basis Pursuit.

IV. KRONECKER COMPRESSIVE COLLABORATIVE SPECTRUM SENSING

In our proposed approach, we utilize the Kronecker CS technique to exploit the different structures embedded in the signal, which allows us to recover the spectrum measurements with a better accuracy using fewer number of samples. We use a sparsifying basis which makes use of the piecewise constant structure of the power spectrum density across different frequency channels, and another sparsifying basis which exploits the correlation structure of the measurements from different CR terminals. Finally, we propose a combined structure which exploits both dimensions for signal recovery.

A. Sparsifying basis along columns (time) of the matrix X

The signal is captured at each CR terminal using the method stated in [7], the signal x_k is sampled at sub-Nyquist rate by analog to information converter according to

$$y_k = \Phi_A x_k, \qquad (7)$$

where y_k is the measurement vector and Φ_A is the $M \times N$ random measurement matrix with i.i.d. Gaussian entries. Denote the autocorrelation at lag j by $r_x(j) = E(x_n x_{j-n}^*)$. Denote the $2N \times 1$ autocorrelation vector of x_k by $r_{kx} = [0, r_x(-N+1), \ldots, r_x(N-1)]^T$. The Nyquist rate autocorrelation vector and the compressed autocorrelation vector are related by the following equation

$$r_{ky} = \Phi_{II} r_{kx}, \qquad (8)$$

where r_{kx} and r_{ky} denote the autocorrelation vectors for x_k and y_k, respectively. The matrix Φ_{II} is formed as follows:

$$\Phi_{II} = \begin{bmatrix} \overline{\Phi}_A \Phi_{11} & \overline{\Phi}_A \Phi_{22} \\ \overline{\Phi}_A \Phi_{33} & \Phi_A \Phi_{44} \end{bmatrix}. \qquad (9)$$

Let $\phi_{i,j}^*$ denote the (i,j)-th element of Φ_A. The $M \times N$ matrix $\overline{\Phi}_A$ has its (i,j)-th elements given by

$$[\overline{\Phi}_A]_{i,i} = \begin{cases} 0 & i = 1, j = 1, \cdots, N, \\ \phi_{M+2-i} & i \neq 1, j = 1, \cdots, N, \end{cases}$$

and the $N \times N$ matrices $\Phi_{11}, \Phi_{22}, \Phi_{33}$, and Φ_{44} are given, respectively, by

$$\Phi_{11} = hankel([0_{N \times 1}], [0 \ \dot{\phi}_{1,1} \cdots \phi_{1,N}^*]),$$

$$\Phi_{22} = hankel([\phi_{1,1}^* \cdots \phi_{1,N}^*], [\phi_{1,N}^* \ 0_{1 \times (N-1)}]),$$

$$\mathbf{\Phi_{33}} = teoplitz([0_{N \times 1}], [0 \; \phi_{1,N} \cdots \phi_{1,2}]),$$

$$\mathbf{\Phi_{44}} = teoplitz([\phi_{1,1} \cdots \phi_{1,N}], [\phi_{1,1} \; 0_{1 \times (N-1)}]),$$

where $hankel(c, r)$ is a hankel matrix (i.e., symmetric and constant across the anti-diagonals) whose first column is c and last row is r, $toeplitz(c, r)$ is a toeplitz matrix (i.e., symmetric and constant across the diagonals) whose first column is c and first row is r, $0_{N \times 1}$ is a column vector of N zeros, and $0_{1 \times (N-1)}$ is a row vector of $N - 1$ zeros.

The power spectrum density (PSD) is the Fourier transform of the autocorrelation function as

$$\mathbf{S_k}(f) = \mathbf{F} r_{kx}, \tag{10}$$

where $\mathbf{F}$ denotes $2N \times 2N$ discrete Fourier transform matrix. The edges of the spectrum are sparse in frequency domain and can be approximated using the differentiation of PSD as follows

$$\mathbf{z_k} = \mathbf{\Gamma} \mathbf{S_k}(f) = \mathbf{\Gamma} \mathbf{F} r_{kx}, \tag{11}$$

where $\mathbf{\Gamma}$ is the first order difference matrix given by

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

The structure of the signals $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K$ which is observable on each column of the matrix could be sparsified by the sparsifying matrix $\mathbf{\Psi_1}$ as

$$r_{kx} = (\mathbf{\Gamma} \mathbf{F})^{-1} \mathbf{z_k} = \mathbf{\Psi_1} \mathbf{z_k}. \tag{12}$$

B. Sparsifying basis along rows (space) of the matrix $\mathbf{X}$

We assume that the values of the measurements which span all the nearby cognitive radio stations $\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^N$ are expected to be highly correlated [19]. Therefore, it is quite reasonable to assume that $\mathbf{x}^j$ is compressible, where the piecewise smooth signals tend to be compressible in wavelet basis as [20]

$$\mathbf{x}^j = \mathbf{\Psi_2} \mathbf{s}^j, \tag{13}$$

where $\mathbf{x}^j$ is a $K \times 1$ column vector, $\mathbf{\Psi_2}$ is a $K \times K$ wavelet sparsifying basis matrix and $\mathbf{s}^j$ is a $K \times 1$ column vector representing the sparse coefficient of vector $\mathbf{x}^j$ in the basis $\mathbf{\Psi_2}$.

C. Combined structure using the Kronecker product

The Kronecker product is used not only for the generation of sparsifying basis that combines both structures presented in the signal, but also for the formation of the measurement matrix used in compressive sensing. Assuming that we use the same Analog to Information Converter with the same measurement matrix at each CR terminal $\mathbf{\Phi_A}$, then from equation (8) we can find the compressed autocorrelation matrix for all CRs through the following equation

$$\mathbf{R_y} = \mathbf{\Phi_{II}} \mathbf{R_x}, \tag{14}$$

where $\mathbf{R_x}$ and $\mathbf{R_y}$ matrices are given by

$$\mathbf{R_x} = [r_{1x}, r_{2x}, \cdots, r_{Kx}]_{2N \times K},$$

$$\mathbf{R_y} = [r_{1y}, r_{2y}, \cdots, r_{Ky}]_{2M \times K}.$$

The joint $2MK \times 2NK$ measurement matrix $\mathbf{\Phi_1}$ can be given as follows:

$$\mathbf{\Phi_1} = \mathbf{I_K} \otimes \mathbf{\Phi_{II}}, \tag{15}$$

$$\mathbf{\Phi_1} = \begin{bmatrix} \mathbf{\Phi_{II}} & 0_{2M \times 2N} & \cdots & 0_{2M \times 2N} \\ 0_{2M \times 2N} & \mathbf{\Phi_{II}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0_{2M \times 2N} & \cdots & \cdots & \mathbf{\Phi_{II}} \end{bmatrix}, \tag{16}$$

where $\mathbf{I_K}$ denotes the $K \times K$ identity matrix.

From Eq. (14) we can express the reshaped vector of $\mathbf{R_y}$ in the following form

$$\overline{r}_y = \mathbf{\Phi_1} \overline{r}_x, \tag{17}$$

where $\overline{r}_x$ and $\overline{r}_y$ denote the reshaped vectors of $\mathbf{R_x}$ and $\mathbf{R_x}$, respectively, given as follows.

$$\overline{r}_y = [\mathbf{R}_y^T(:,1), \mathbf{R}_y^T(:,2), \cdots, \mathbf{R}_y^T(:,K)]^T.$$

$$\overline{r}_x = [\mathbf{R}_x^T(:,1), \mathbf{R}_x^T(:,2), \cdots, \mathbf{R}_x^T(:,K)]^T.$$

We can now generate the sparsifying matrix $\mathbf{\Psi}$, which combines the structures of both the rows and columns of the matrix $\mathbf{X}$. The global sparsifying basis is the Kronecker product of both sparsifying bases presented in equations (12) and (13) and is given by

$$\mathbf{\Psi} = \mathbf{\Psi_1} \otimes \mathbf{\Psi_2}. \tag{18}$$

The reshaped vector of autocorellation matrix $\mathbf{R_x}$ can be viewed in $\mathbf{\Psi}$ domain as follows

$$\overline{r}_x = \mathbf{\Psi} \overline{e}, \tag{19}$$

where $\overline{e}$ is the sparse coefficient vector in the basis $\mathbf{\Psi}$, also the reshaped vector of autocorellation matrix $\mathbf{R_y}$ can be viewed in $\mathbf{\Psi}$ domain from (17) as

$$\overline{r}_y = \mathbf{\Phi_1} \mathbf{\Psi} \overline{e}. \tag{20}$$

The edges of the spectrum $\overline{e}$ can be recovered by the Basis Pursuit algorithm using the following ℓ_1 minimization:

$$\hat{\overline{e}} = \arg \min \|\overline{e}\|_1 \text{ subject to } \mathbf{\Phi_1} \mathbf{\Psi} \overline{e} = \overline{r}_y. \tag{21}$$

The reshaped recovered vector can be estimated by

$$\hat{\overline{r}}_x = \mathbf{\Psi} \hat{\overline{e}}. \tag{22}$$

The recovered power spectrum densities seen by the K cognitive radio terminals at FC $\hat{\mathbf{S}}_{N \times K}$ can be estimated by

$$\hat{\mathbf{S}} = \mathbf{F} \hat{\mathbf{R}}_x, \tag{23}$$

where $\hat{\mathbf{R}}_x$ is the reshaped matrix from the vector $\hat{\overline{r}}_x$.

TABLE I
THE SPARSITY ORDER OF DIFFERENT SIGNAL MODELS

Signal Model	$JSM2$	$JSM2M$
Kronecker basis $\mathbf{\Psi}$	7	17
traditional basis $\mathbf{\Psi_1}$	47	47

V. SIMULATION RESULTS

In this section, we perform numerical simulations to illustrate the performance of the proposed Kronecker approach. In all of our experiments, we consider a wideband spectrum of interest with a bandwidth in the range $[50, 150]$ MHz centred around a carrier frequency f_c. The PSD is smooth within each subband, but exhibits a discontinuous change between adjacent subbands similar to [6], with number of samples $N = 128$. The network consists of $K = 8$ CRs. The total number of PU $P = 6$, and the number of active PU $U = 3$. The Daubechies wavelet "db4" is used in forming the sparsifying basis $\mathbf{\Psi}_2$.

The received signal is corrupted by additive white Gaussian noise and the signal to noise ratio is considered as the inverse of the noise variance and is set to $SNR = 3$ dB.

In order to compare the performance of our proposed approach with current approaches like [8], we use the normalized mean square error between the average of the recovered spectrum and the average of the noise free spectrum versus the Compression Ratio (M/N) as our performance criterion.

In the first experiment, we examine the effect of ensemble model selection and recovery algorithm on the sparsity order K (the number of non zero elements). The sparsity orders of the different ensembles models under both Kronecker and traditional sparsifying bases are shown in Table I. It is obvious that the signal has the sparsest representation (lowest sparsity order) under the proposed Kronecker basis $\mathbf{\Psi}$. This proves that the proposed approach can exploit more underlying sparsity than the 1-D based approach since it exploits different structure presented in the ensemble (like the strong correlation between the measurements at different CRs), as compared to the traditional sparsifying basis $\mathbf{\Psi_1}$ which only works on a single dimension of the received signals (intra-signal).

In the second experiment, we compare the performance of different recovery algorithms using the same Kronecker measurement matrix presented in (16). Fig. 2 elucidates a noticeable performance improvement while using a very low rate compressive sampler, as we can achieve a lower MSE using the Kronecker approach specially under low compression ratio (about 50% reduction at $M/N = 0.2$) as compared to both the Simultaneous Orthogonal Matching Pursuit (SOMP) presented in [8] with $\mathbf{\Psi_1}$ as sparsifying basis, and the Independent Recovery using Basic Pursuit(BP) algorithm [12].

In the third experiment, we evaluate the performance of collaborative compressive sensing under different measurement matrices such as:

1) Kronecker measurement matrix in the proposed approach $\mathbf{\Phi_1} = \mathbf{I_K} \otimes \mathbf{\Phi_{II}}$ where $\mathbf{\Phi_{II}}$ is the matrix given in (8)
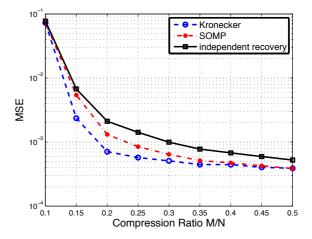2) Kronecker measurement matrix $\mathbf{\Phi_2} = \mathbf{I_K} \otimes \mathbf{\Phi_{BB}}$ where

Fig. 2. MSE performance versus Compression Ratio under different recovery algorithms

$\mathbf{\Phi_{BB}}$ is a matrix formed in the same way as $\mathbf{\Phi_{II}}$ but by substituting for $\mathbf{\Phi_A}$ by a Bernoulli measurement matrix rather than a Gaussian matrix as in (8).

3) Global random dense measurement $2KM \times 2KN$ matrix $\mathbf{\Phi_3}$ with i.i.d. Gaussian entries.

The performance results for these different measurement matrices are shown in Fig. 3. It is evident from the figure that the dense matrix $\mathbf{\Phi_3}$ achieves a significant improvement in the MSE performance as compared to the Kronecker measurement matrices, especially at very low compression ratios (less than 0.2). However, we cannot theoretically find a dense matrix that relates Nyquist autocorrelation vector and the compressed autocorrelation vectors in the way shown in Eq. (14). This results in a computational bottleneck, as we have to sample the signals at Nyquist rate, find the autocorrelation function, compress it using a compressive sampler, and then send it compressed to the FC. While this may deprive our system from the advantage of using a low-rate ADC, it can significantly reduce the cooperation overload on some of the radio resources needed to achieve communication between the CRs and the FC like the number of multiple access channels. A similar observation can be made for Bernoulli measurement matrix $\mathbf{\Phi_2}$, which results in the worst MSE performance. However, it has the advantage of putting some CRs at sleep mode which saves both power and bandwidth.

In the fourth experiment, we evaluate the performance of collaborative compressive sensing using the dense measurement matrix $\mathbf{\Phi_3}$ under different sparsifying bases. The four different sparsifying bases used in our simulation are as follows. 1) SOMP algorithm with sparsifying basis $\mathbf{\Psi_1}$ [8] 2) Kronecker CS with $\mathbf{\Psi} = \mathbf{\Psi_1} \otimes \mathbf{\Psi_2}$. 3) $\mathbf{\Psi_a} = \mathbf{\Psi_1} \otimes \mathbf{IDCT}$. 4) $\mathbf{\Psi_b} = \mathbf{\Psi_1} \otimes \mathbf{I_K}$.

Fig. 4 depicts the effect of our choice of the sparsifying basis in exploring the underlying structure of the measurements. The Kronecker basis $\mathbf{\Psi}$ in Section IV performs significantly better than all other bases. The parsifying bases $\mathbf{\Psi_1}$ and $\mathbf{\Psi_b}$ have the largest MSE since these bases exploit the sparsity in only
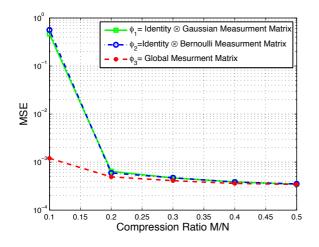
Fig. 3. MSE versus Compression Ratio using different measurements matrices

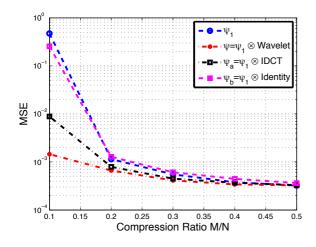
Fig. 4. MSE versus Compression Ratio under different sparsifying bases

one dimension. On the other hand, the sparsifying basis Ψ_a exploits sparsity of 2-D signals, where the first dimension is the ordinary sparsifying matrix Ψ_1 of the individual signals and the second is the DCT sparsifying basis. The signal in the DCT domain is sparse only under the assumption that the measurements have strong correlation without sudden peaks. However, this is not the case when there is a hidden primary user, since there will be sudden peaks which would destroy the sparsity. Therefore, wavelet basis is more suitable for that case and evinced the lowest MSE at very low compression ratio.

VI. CONCLUSION

In this paper, we have developed an approach for collaborative spectrum sensing for cognitive radio networks. The proposed approach exploits the underlying multi-dimensional sparse structure in the measured spectrum observations at different cognitive radios. This is achieved by formulating the problem as a Kronecker compressive sensing recovery problem, and carefully designing suitable measurement and sparsi-

fying bases. We propose a modified signals ensemble model, that accounts for the scenario of hidden primary user problem. The performance using MSE under different sparsifying bases and measurement matrices evinced a significant improvement, as the Kronecker sparsifying basis exploits different structures presented in the signal, which allows for significant reduction in sampling rate, relaxes constraints put on the ADCs, and finally reduces the amount of radio resources needed for the communication between the CRs and the FC.

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