Power Optimization for Layered Transmission Over Decode-and-Forward Relay Channels

Mohamed Adel Attia [1], Mohammad Shaqfeh [2], Karim Seddik [1] and Hussein Alnuweiri [2]

[1] Electronics Engineering Department, American University in Cairo, New Cairo 11835, Egypt,
E-mail: {madel, kseddik}@aucegypt.edu

[2] Department of Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar.
E-mail: {Mohammad.Shaqfeh, Hussein.Alnuweiri}@qatar.tamu.edu

***Abstract*—In this paper, we consider a fading relay channel where the source uses two layers source coding with successive refinement. The two source layers are transmitted using superposition coding at the source and relay with optimal power allocation, and successive interference cancellation at the receivers (i.e. relay and destination). The power allocation for the two layers at the source and relay is subject to optimization in order to maximize the expected user satisfaction that is defined by a utility function of the total decoded rates at the destination. We assume that only the channel statistics are known. The relay is half-duplex and applies decode and forward. We characterize the expected utility function in terms of the channel statistics of the fading channels, and we solve the optimization problem using the numerical random search method. We provide many numerical examples to show the prospected gains of using the relay on the expected utility for different channel conditions. Furthermore, we obtain that for some conditions, it is optimal to send only one layer.**

***Keywords*—Broadcast approach, multilayer transmission, relay channel, selection relaying decode-and-forward, utility maximization**

I. Introduction

The topic of this paper is on the application of "multilayer transmission" using the broadcast approach on a "relay" channel. Unlike single layer transmission, where all transmitted information bits have the same protection level by the channel coding scheme, multilayer transmission schemes combine successive refinement layered source coding [1] with ordered protection levels of the source layers. Therefore, the "base" source layer is given higher priority and protected more than the "enhancement" source layers. Consequently, the receiver will be able to decode "some" information when the channel is faded and "all" information when the channel is good.

In particular, we are interested in the broadcast approach in which the source layers are protected using different channel codewords and transmitted jointly using superposition coding at the physical layer [2], [3]. The receiver decodes the layers in order, up to the supported layer by its channel condition, using successive interference cancellation (SIC). These schemes have been studied in the literature from different perspectives. We give few examples here[1]. In [4], the broadcast approach was applied and the layers power and rate allocation were optimized under the objective of minimizing the expected distortion of a Gaussian source from an information-theoretic perspective. Utility maximization for layered transmission with known rates of the layers was considered in [5], which was originally presented in [6]. Furthermore, utility maximizing for layered transmission with joint power and rate allocation for any finite number of layers over Rayleigh fading channels was solved recently in [7]–[9].

Our contribution of this paper is on the investigation of multilayer transmission on a relay channel [10], [11]. The topic of relaying/cooperative communication have recently become an active research area due to the potential deployment of relay nodes in fourth generation wireless systems. Few examples of the papers on relaying strategies among many others include [12]–[16].

In this paper, we consider a two-layer transmission scheme with optimal power allocation at the source and the relay. We assume that the relay applies "selection relaying" decode-and-forward (SDF) [12]. We formulate the optimization problem as utility maximizing with known layers rates similar to the problem formulation in [5]. The expected utility function in our problem is a function of the channel statistics of the three links in the channel (i.e. source-destination, source-relay and relay-destination). We characterize the expected utility function and use it in the power optimization problem. We apply the random search method [17, Chapter 14] to solve this optimization problem and we provide several numerical examples to show the gains in the maximum expected utility when relaying is applied.

The outline of the paper is as follows; we describe the system model and the used transmission scheme and we formulate the optimization problem in Section II. Then, we characterize the expected utility in terms of the channels' statistics in Section III. After that, we show the numerical results in Section IV. Finally, we summarize the main conclusions in Section V.

This publication was made possible by NPRP grant # 05-401-2-161 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

[1] The list of related references presented in this paper is not exhaustive due to the scope of this paper.

II. System Model and Problem Formulation

A. System Model and Transmission Scheme

We consider a system that consists of three nodes; source, destination and relay. We assume that the source is Gaussian and it is encoded into two layers, L_1 and L_2, with rates R_1 and R_2 respectively. L_1 is the base layer, and L_2 is the enhancement layer that refines the description in L_1. The relay is half-duplex and applies selection relaying decode-and-forward (SDF) [12]. Therefore, the transmission is carried over two consecutive time slots of equal duration and bandwidth. In the first time slot, the source broadcasts L_1 and L_2 to the relay and the destination using superposition coding, where L_1 should be decoded first then L_2 using successive interference cancellation (SIC). If the relay is able to decode one or both layers, it forwards the decoded layer(s) to the destination using new complementary[2] codewords in the second time slot. Otherwise, the source re-transmits L_1 and L_2 in the second time slot using new complementary codewords. In both cases, the destination tries to decode L_1 first and then L_2 based on the received codewords in the two time slots of the transmission.

We assume that the three nodes are equipped with a single antenna. We denote the Signal-to-Noise-Ratio (SNR) over the three links of the relay channel using γ_{sr}, γ_{sd} and γ_{rd} for the source-relay, source-destination, and relay-destination links, respectively. We assume that the source and the relay transmit using constant power. Furthermore, we assume that the channel gain, and consequently the SNR, stay constant for the duration of one transmission block, which consists of two consecutive time slots. However, γ_{sr}, γ_{sd} and γ_{rd} varies from one channel block to another randomly. Furthermore, we assume that the source and the relay do not know the instantaneous values of the SNRs.

In the numerical results in Section IV, we assume that the variation (i.e. fading) of the channels' gain is Rayleigh distributed[3]. Hence, the probability density function (PDF) of the channels follow an exponential distribution, and they are given as

$$f_{\mathsf{sd}}\left(\gamma_{\mathsf{sd}}\right) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{sd}}}{\bar{\gamma}}\right), \tag{1a}$$

$$f_{\mathsf{sr}}\left(\gamma_{\mathsf{sr}}\right) = \frac{1}{m\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{sr}}}{m\bar{\gamma}}\right), \quad f_{\mathsf{rd}}\left(\gamma_{\mathsf{rd}}\right) = \frac{1}{m\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{rd}}}{m\bar{\gamma}}\right), \tag{1b}$$

for the source-destination, source-relay and relay-destination links, respectively. In (1), $\bar{\gamma}$ is the average SNR for the direct source-destination link and m is ratio between the average SNR of the source-relay and the relay-destination links to the source-destination link. We assume that $\bar{\gamma}$ and m are known at the source and the relay and they are used in the optimization of the power allocation over the two layers L_1 and L_2 at these two nodes to maximize the expected utility function, denoted U, of the total decoded rate, denoted $\bar{R}$, at the destination.

The optimization variables are denoted α_1 and α_2 for the ratios of the total power at the source that are allocated to L_1 and L_2, respectively. Additionally, we have β_1 and β_2 for the ratios of the total power at the relay that are allocated to L_1 and L_2, respectively, when the relay transmits both layers. However, when the relay can decode only L_1, it forwards only this layer, and hence it allocates all of its power to it.

B. Mathematical Notation and Problem Formulation

First, we define the following functions because we will use them frequently in the sequel.

$$\mathcal{C}_1(\gamma, \epsilon) = \log\left(1 + \frac{(1-\epsilon)\gamma}{1+\epsilon\gamma}\right), \tag{2a}$$

$$\mathcal{C}_2(\gamma, \epsilon) = \log\left(1 + \epsilon\gamma\right). \tag{2b}$$

The functions in (2) define the information-theoretic maximum achievable rates for the transmission of two layers over a Gaussian channel with SNR γ, and power ratio of the enhancement layer equals ϵ.

$$\Gamma_1(x, \epsilon) = \frac{x-1}{1-\epsilon x}, \tag{3a}$$

$$\Gamma_2(x, \epsilon) = \frac{x-1}{\epsilon}. \tag{3b}$$

Also, we use the functions in (3) to denote the information-theoretic minimum SNR threshold that is required in order to be able to decode two layers over a Gaussian channel with x a function of the layer rate as will be explained in the sequel, and ϵ is the power ratio allocated to the enhancement layer.

In this paper, our objective is to maximize the expected user satisfaction determined by the utility function $U(\bar{R})$. The utility function $U(\bar{R})$ can be flexibly defined to employ many special cases such as minimizing the expected distortion of a Gaussian source or maximizing the expected rate. The optimization problem is to optimally allocate the power among the two layers such that the expectation of the utility function $E[U(\bar{R})]$ is maximized. The expectation of the utility function can be described as

$$E\left[U(\bar{R})\right] = U(R_1).P_{\mathsf{d1}} + U(R_1+R_2).P_{\mathsf{d2}}, \tag{4}$$

where P_{d1} and P_{d2} are defined as

$$P_{\mathsf{d1}} \equiv \Pr\left(\text{Destination can decode } \mathrm{L}_1 \text{ only}\right), \tag{5a}$$

$$P_{\mathsf{d2}} \equiv \Pr\left(\text{Destination can decode both } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right). \tag{5b}$$

These probabilities can be characterized as

$$P_{\mathsf{d1}} = P_{\mathsf{d1|r0}}.P_{\mathsf{r0}} + P_{\mathsf{d1|r1}}.P_{\mathsf{r1}} + P_{\mathsf{d1|r2}}.P_{\mathsf{r2}}. \tag{6}$$

Similarly, we have

$$P_{\mathsf{d2}} = P_{\mathsf{d2|r0}}.P_{\mathsf{r0}} + P_{\mathsf{d2|r1}}.P_{\mathsf{r1}} + P_{\mathsf{d2|r2}}.P_{\mathsf{r2}}, \tag{7}$$

where the following notations are used

$$P_{\mathsf{r0}} \equiv \Pr\left(\text{Relay cannot decode } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right), \tag{8a}$$

$$P_{\mathsf{r1}} \equiv \Pr\left(\text{Relay can decode } \mathrm{L}_1 \text{ only}\right), \tag{8b}$$

$$P_{\mathsf{r2}} \equiv \Pr\left(\text{Relay can decode both } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right) \tag{8c}$$

[2]We assume that the codewords achieves the information-theoretic maximum achievable rates of decode-and-forward over the relay channel.

[3]The extension of the results of this paper into other channel fading models is straightforward.

$$P_{\mathsf{d1|r0}} \equiv \Pr\left(\text{Destination can decode } \mathrm{L}_1 \text{ only}|\text{Relay} \right.$$
$$\left. \text{cannot decode } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right), \quad (9a)$$
$$P_{\mathsf{d1|r1}} \equiv \Pr\left(\text{Destination can decode } \mathrm{L}_1 \text{ only}|\text{Relay} \right.$$
$$\left. \text{can decode } \mathrm{L}_1 \text{ only}\right), \quad (9b)$$
$$P_{\mathsf{d1|r2}} \equiv \Pr\left(\text{Destination can decode } \mathrm{L}_1 \text{ only}|\text{Relay} \right.$$
$$\left. \text{can decode both } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right) \quad (9c)$$

$$P_{\mathsf{d2|r0}} \equiv \Pr\left(\text{Destination can decode both } \mathrm{L}_1 \right.$$
$$\left. \text{and } \mathrm{L}_2|\text{Relay cannot decode } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right), \quad (10a)$$
$$P_{\mathsf{d2|r1}} \equiv \Pr\left(\text{Destination can decode both } \mathrm{L}_1 \right.$$
$$\left. \text{and } \mathrm{L}_2|\text{Relay can decode } \mathrm{L}_1 \text{ only}\right), \quad (10b)$$
$$P_{\mathsf{d2|r2}} \equiv \Pr\left(\text{Destination can decode both } \mathrm{L}_1 \right.$$
$$\left. \text{and } \mathrm{L}_2|\text{Relay can decode both } \mathrm{L}_1 \text{ and } \mathrm{L}_2\right) \quad (10c)$$

Notice that $P_{\mathsf{r}i}$ depends on γ_{sr} while $P_{\mathsf{d}i|\mathsf{r}j}$ depends on γ_{sd} and γ_{rd}. We assume that the channels are fading independently.

The main optimization problem is

$$\max_{\alpha_2,\beta_2} E[U(\bar{R})], \qquad \text{subject to} \quad (11a)$$
$$0 \le \alpha_2 \le 1, \qquad 0 \le \beta_2 \le 1, \quad (11b)$$

where α_1 and β_1 are equal to

$$\alpha_1 = 1 - \alpha_2, \qquad \beta_1 = 1 - \beta_2 \quad (12)$$

The first step to solve (11) is to find the probability that the destination is able to decode only layer L_1 correctly, then find the probability that the destination is able to decode both layers L_1 and L_2 correctly. These two probabilities will be substituted in (4), and then it is required to find optimal power ratios α_2 and β_2 to solve (11) using "random search" numerical method.

III. Characterizing the Successful Decoding Probabilities

A. Successful Decoding Probabilities at the Relay

$$P_{\mathsf{r0}} = \Pr\left(R_1 > \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sr}}, \alpha_2)\right) \quad (13a)$$
$$= \Pr\left(\gamma_{\mathsf{sr}} < \Gamma_1(2^{2R_1}, \alpha_2)\right) \quad (13b)$$
$$= F_{\mathsf{sr}}\left(\Gamma_1(2^{2R_1}, \alpha_2)\right) \quad (13c)$$

where F_{sr} denoted the cumulative distribution function (CDF) of γ_{sr}.

$$P_{\mathsf{r1}} = \Pr\left(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sr}}, \alpha_2) \text{ and } R_2 > \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sr}}, \alpha_2)\right) \quad (14a)$$
$$= \Pr\left(\Gamma_1(2^{2R_1}, \alpha_2) \le \gamma_{\mathsf{sr}} < \Gamma_2(2^{2R_2}, \alpha_2)\right) \quad (14b)$$
$$= F_{\mathsf{sr}}\left(\max\left(\Gamma_1(2^{2R_1}, \alpha_2), \Gamma_2(2^{2R_2}, \alpha_2)\right)\right) - F_{\mathsf{sr}}\left(\Gamma_1(2^{2R_1}, \alpha_2)\right) \quad (14c)$$

The maximum of $\Gamma_1(2^{2R_1}, \alpha_2)$ and $\Gamma_2(2^{2R_2}, \alpha_2)$ is taken in order to take into consideration the cases when $\Gamma_2(2^{2R_2}, \alpha_2) < \Gamma_1(2^{2R_1}, \alpha_2)$. In this case, the probability of decoding L_1 only is zero.

$$P_{\mathsf{r2}} = \Pr\left(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sr}}, \alpha_2) \text{ and } R_2 \le \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sr}}, \alpha_2)\right) \quad (15a)$$
$$= \Pr\left(\gamma_{\mathsf{sr}} \ge \max\left(\Gamma_1(2^{2R_1}, \alpha_2), \Gamma_2(2^{2R_2}, \alpha_2)\right)\right) \quad (15b)$$
$$= 1 - F_{\mathsf{sr}}\left(\max\left(\Gamma_1(2^{2R_1}, \alpha_2), \Gamma_2(2^{2R_2}, \alpha_2)\right)\right) \quad (15c)$$

B. Case: The Relay cannot decode any Layer

Next, we characterize the conditional probabilities starting with the case when the relay is not able to decode any layer. In this case, the source re-transmits L_1 and L_2 using new codewords and with the same power ratios α_1 and α_2.

$$P_{\mathsf{d1|r0}} = \Pr\left(R_1 \le \mathcal{C}_1(\gamma_{\mathsf{sd}}, \alpha_2) \text{ and } R_2 > \mathcal{C}_2(\gamma_{\mathsf{sd}}, \alpha_2)\right) \quad (16a)$$
$$= \Pr\left(\Gamma_1(2^{R_1}, \alpha_2) \le \gamma_{\mathsf{sd}} < \Gamma_2(2^{R_2}, \alpha_2)\right) \quad (16b)$$
$$= F_{\mathsf{sd}}\left(\max\left(\Gamma_1(2^{R_1}, \alpha_2), \Gamma_2(2^{R_2}, \alpha_2)\right)\right) - F_{\mathsf{sd}}\left(\Gamma_1(2^{R_1}, \alpha_2)\right) \quad (16c)$$

$$P_{\mathsf{d2|r0}} = \Pr\left(R_1 \le \mathcal{C}_1(\gamma_{\mathsf{sd}}, \alpha_2) \text{ and } R_2 \le \mathcal{C}_2(\gamma_{\mathsf{sd}}, \alpha_2)\right) \quad (17a)$$
$$= \Pr\left(\gamma_{\mathsf{sd}} \ge \max\left(\Gamma_1(2^{R_1}, \alpha_2), \Gamma_2(2^{R_2}, \alpha_2)\right)\right) \quad (17b)$$
$$= 1 - F_{\mathsf{sd}}\left(\max\left(\Gamma_1(2^{R_1}, \alpha_2), \Gamma_2(2^{R_2}, \alpha_2)\right)\right) \quad (17c)$$

C. Case: The Relay can decode Only One Layer

In this case when the relay can decode only L_1, it transmits only this layer using a new codeword in the second time slot. Therefore, L_1 is allocated the full power of the relay in this case.

$$P_{\mathsf{d1|r1}} = \Pr\left(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}}, \alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}}, 1) \text{ and } R_2 > \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}}, \alpha_2)\right) \quad (18)$$

The region defined by the condition $R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}}, \alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}}, 1)$ can be characterized in terms of γ_{sd} and γ_{rd} as

$$\gamma_{\mathsf{rd}} \ge \max\left(0, \Gamma_2\left(\frac{2^{2R_1}(1+\alpha_2\gamma_{\mathsf{sd}})}{1+\gamma_{\mathsf{sd}}}, 1\right)\right) \quad (19)$$

Equivalently, this region can be characterized as

$$\gamma_{\mathsf{sd}} \ge \max\left(0, \Gamma_1\left(\frac{2^{2R_1}}{1+\gamma_{\mathsf{rd}}}, \alpha_2\right)\right) \quad (20)$$

Furthermore, the condition $R_2 > \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}}, \alpha_2)$ is equivalent to

$$\gamma_{\mathsf{sd}} < \Gamma_2\left(2^{2R_2}, \alpha_2\right) \quad (21)$$

Therefore, based on (19) and (21), we can write (18) as

$$P_{\mathsf{d1|r1}} = \int_0^{\Gamma_2\left(2^{2R_2},\alpha_2\right)} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) \int_{\omega(\gamma_{\mathsf{sd}})}^{\infty} f_{\mathsf{rd}}(\gamma_{\mathsf{rd}}) d\gamma_{\mathsf{rd}} d\gamma_{\mathsf{sd}} \tag{22a}$$

$$= \int_0^{\Gamma_2\left(2^{2R_2},\alpha_2\right)} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) \left(1 - F_{\mathsf{rd}}\left(\omega(\gamma_{\mathsf{sd}})\right)\right) d\gamma_{\mathsf{sd}}, \tag{22b}$$

where $\omega(\gamma_{\mathsf{sd}})$ is defined as

$$\omega(\gamma_{\mathsf{sd}}) = \max\left(0, \Gamma_2\left(\frac{2^{2R_1}(1+\alpha_2\gamma_{\mathsf{sd}})}{1+\gamma_{\mathsf{sd}}}, 1\right)\right) \tag{23}$$

Equivalently, based on (20) and (21), we can write (18) as

$$P_{\mathsf{d1|r1}} = \int_0^{\infty} f_{\mathsf{rd}}(\gamma_{\mathsf{rd}}) \int_{\zeta(\gamma_{\mathsf{rd}})}^{\Gamma_2\left(2^{2R_2},\alpha_2\right)} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) d\gamma_{\mathsf{sd}} d\gamma_{\mathsf{rd}}, \tag{24}$$

where $\zeta(\gamma_{\mathsf{rd}})$ is defined as

$$\zeta(\gamma_{\mathsf{rd}}) = \min\left(\Gamma_2\left(2^{2R_2},\alpha_2\right), \max\left(0, \Gamma_1\left(\frac{2^{2R_1}}{1+\gamma_{\mathsf{rd}}}, \alpha_2\right)\right)\right) \tag{25}$$

In a similar way, we can obtain $P_{\mathsf{d2|r1}}$

$$P_{\mathsf{d2|r1}} = \Pr\Big(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}},1) \text{ and } R_2 \le \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}},\alpha_2)\Big) \tag{26a}$$

$$= \int_{\Gamma_2\left(2^{2R_2},\alpha_2\right)}^{\infty} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) \left(1 - F_{\mathsf{rd}}\left(\omega(\gamma_{\mathsf{sd}})\right)\right) d\gamma_{\mathsf{sd}}, \tag{26b}$$

where $\omega(\gamma_{\mathsf{sd}})$ is defined in (23). For the sake of brevity, we do not show the equivalent expressions to characterize $P_{\mathsf{d2|r1}}$.

D. Case: The Relay can decode Both Layers

When the relay can decode both layers, it forwards both layers using power ratios β_1 and β_2, respectively. The conditional probabilities in this case are characterized as follows.

$$P_{\mathsf{d1|r2}} = \Pr\Big(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{rd}},\beta_2) \text{ and } R_2 > \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}},\beta_2)\Big) \tag{27}$$

The region defined by the condition $R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{rd}},\beta_2)$ can be characterized in terms of γ_{sd} and γ_{rd} as

$$\gamma_{\mathsf{rd}} \ge \max\left(0, \Gamma_1\left(\frac{2^{2R_1}(1+\alpha_2\gamma_{\mathsf{sd}})}{1+\gamma_{\mathsf{sd}}}, \beta_2\right)\right) \tag{28}$$

Equivalently, this region can be characterized as

$$\gamma_{\mathsf{sd}} \ge \max\left(0, \Gamma_1\left(\frac{2^{2R_1}(1+\beta_2\gamma_{\mathsf{rd}})}{1+\gamma_{\mathsf{rd}}}, \alpha_2\right)\right) \tag{29}$$

Similarly, the region defined by the condition $R_2 > \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}},\beta_2)$ can be characterized in terms of γ_{sd} and γ_{rd} as

$$\gamma_{\mathsf{rd}} < \max\left(0, \Gamma_2\left(\frac{2^{2R_2}}{1+\alpha_2\gamma_{\mathsf{sd}}}, \beta_2\right)\right) \tag{30}$$

Equivalently, this region can be characterized as

$$\gamma_{\mathsf{sd}} < \max\left(0, \Gamma_2\left(\frac{2^{2R_2}}{1+\beta_2\gamma_{\mathsf{rd}}}, \alpha_2\right)\right) \tag{31}$$

Therefore, based on (28) and (30), we can write (18) as

$$P_{\mathsf{d1|r2}} = \int_0^{\Gamma_2\left(2^{2R_2},\alpha_2\right)} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) \Big(F_{\mathsf{rd}}\left(\max\left(\sigma(\gamma_{\mathsf{sd}}), \xi(\gamma_{\mathsf{sd}})\right)\right) - F_{\mathsf{rd}}\left(\sigma(\gamma_{\mathsf{sd}})\right)\Big) d\gamma_{\mathsf{sd}}, \tag{32a}$$

where $\sigma(\gamma_{\mathsf{sd}})$ is defined as

$$\sigma(\gamma_{\mathsf{sd}}) = \max\left(0, \Gamma_1\left(\frac{2^{2R_1}(1+\alpha_2\gamma_{\mathsf{sd}})}{1+\gamma_{\mathsf{sd}}}, \beta_2\right)\right), \tag{33}$$

and $\xi(\gamma_{\mathsf{sd}})$ is defined as

$$\xi(\gamma_{\mathsf{sd}}) = \max\left(0, \Gamma_2\left(\frac{2^{2R_2}}{1+\alpha_2\gamma_{\mathsf{sd}}}, \beta_2\right)\right) \tag{34}$$

Equivalently, based on (29) and (31), we can write (27) as

$$P_{\mathsf{d1|r2}} = \int_0^{\Gamma_2\left(2^{2R_2},\beta_2\right)} f_{\mathsf{rd}}(\gamma_{\mathsf{rd}}) \Big(F_{\mathsf{sd}}\left(\max\left(\theta(\gamma_{\mathsf{rd}}), \phi(\gamma_{\mathsf{rd}})\right)\right) - F_{\mathsf{sd}}\left(\theta(\gamma_{\mathsf{rd}})\right)\Big) d\gamma_{\mathsf{rd}}, \tag{35a}$$

where $\theta(\gamma_{\mathsf{rd}})$ is defined as

$$\theta(\gamma_{\mathsf{rd}}) = \max\left(0, \Gamma_1\left(\frac{2^{2R_1}(1+\beta_2\gamma_{\mathsf{rd}})}{1+\gamma_{\mathsf{rd}}}, \alpha_2\right)\right), \tag{36}$$

and $\phi(\gamma_{\mathsf{rd}})$ is defined as

$$\phi(\gamma_{\mathsf{rd}}) = \max\left(0, \Gamma_2\left(\frac{2^{2R_2}}{1+\beta_2\gamma_{\mathsf{rd}}}, \alpha_2\right)\right) \tag{37}$$

In a similar way, we can obtain $P_{\mathsf{d2|r2}}$

$$P_{\mathsf{d2|r2}} = \Pr\Big(R_1 \le \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_1(\gamma_{\mathsf{rd}},\beta_2) \text{ and } R_2 \le \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{sd}},\alpha_2) + \frac{1}{2}\mathcal{C}_2(\gamma_{\mathsf{rd}},\beta_2)\Big) \tag{38a}$$

$$= \int_0^{\infty} f_{\mathsf{sd}}(\gamma_{\mathsf{sd}}) \Big(1 - F_{\mathsf{rd}}\Big(\max\left(\sigma(\gamma_{\mathsf{sd}}), \xi(\gamma_{\mathsf{sd}})\right)\Big)\Big) d\gamma_{\mathsf{sd}} \tag{38b}$$

where $\sigma(\gamma_{\mathsf{sd}})$ and $\xi(\gamma_{\mathsf{sd}})$ are defined in (33) and (34). For the sake of brevity, we do not show the equivalent expressions to characterize $P_{\mathsf{d2|r2}}$.

IV. Numerical Results

We present several numerical results in this section with the assumption that the fading distribution of the channels follows (1), where the average SNR for the source-relay and the relay-destination links are m times the average SNR for the source-destination link. The rates of the two source layers are respectively 1 and 2 bps/Hz. We consider two different utility functions; namely, $U(\bar{R}) = 1 - 2^{-2\bar{R}}$, which corresponds to minimizing the expected distortion of a Gaussian source and $U(\bar{R}) = \bar{R}$, which corresponds to maximizing the expected total rate at the destination [5].

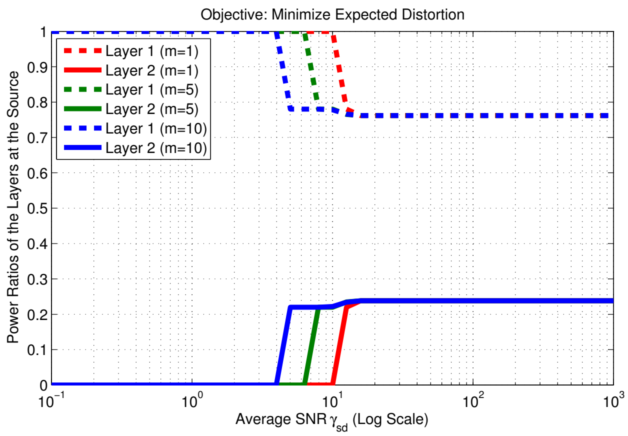

Fig. 1. The relative power ratios of the layers at the source versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.

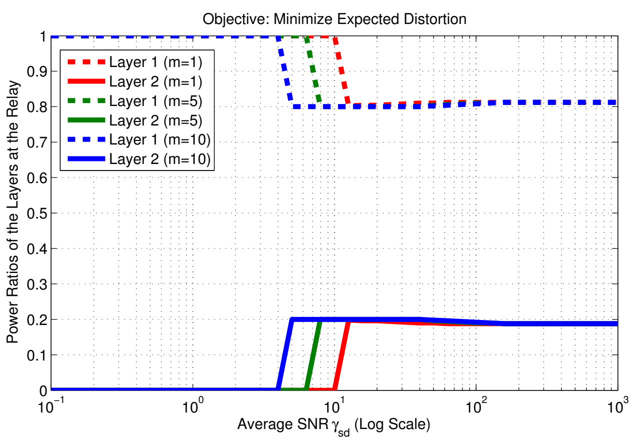

Fig. 2. The relative power ratios of the layers at the relay versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.

Figs. 1 and 2 show the relative power ratios of the layers at the source and the relay respectively, for different values of m, with the target of minimizing the expected distortion of a Gaussian source over Rayleigh fading channels. The optimal power ratios are plotted against the average SNR of the source-destination channel. It can be seen that for low average SNR values it is optimal to send only one layer. This is because the enhancement layer cannot be decoded reliably in this case. Therefore, it is better to discard it in order to get rid of its interference on the base layer, which enables the reception of the base layer at lower SNR values. On the other hand, when the average SNR is above a certain value, it becomes optimal to send the two layers. This value for the average SNR is the same for the source and the relay. It is obvious that as the ratio m increases, the curves are shifted to the left, which means that for higher values of m it is optimal to send the two layers for lower values of the average SNR. This is intuitive because as m increases, the relay becomes more capable of enhancing the end-to-end performance, and hence the destination becomes more capable of decoding the enhancement layer even when its direct channel with the source has low SNR.

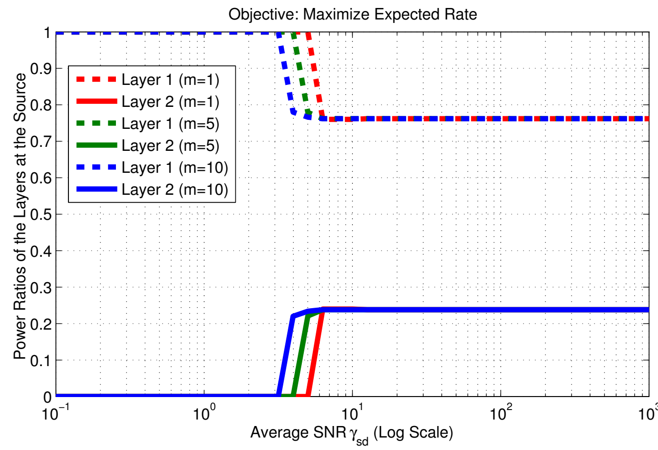

Fig. 3. The relative power ratios of the layers at the source versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = \bar{R}$.

Figs. 3 and 4 show the relative power ratios of the layers at the source and the relay respectively, for different values of m, with the target of maximizing the expected rate over Rayleigh fading channels. Since the utility function for maximizing the rate is linear, the enhancement layer has more importance than in the case of distortion minimization. Consequently, a higher ratio of the power is allocated to the enhancement layer in this case. Furthermore, it becomes optimal to send both layers for lower values of the average SNR. In comparison, the solution for minimizing the average distortion gives more importance to the base layer, and hence it becomes optimal to send both layers for higher values of the average SNR. That is why the curves are shifted to the left in Figs. 3 and 4 compared to Figs. 1 and 2. Moreover, it is obvious that as the the ratio m increases, the curves are shifted to the left similar to the distortion minimization case.

In a comparison between relay-assisted transmission and direct channel with no relay assistance, over Rayleigh fading channels with different values of m, we can see from Figs. 5 and 6 that there is an obvious gain in the maximum expected utility when the relay is involved. This is valid for both cases $U(\bar{R}) = 1 - 2^{-2\bar{R}}$ and $U(\bar{R}) = \bar{R}$. Furthermore, as m increases, the gain with respect to the no-relay case increases as well, as expected. For the case when $m = 1$, it can be seen that the maximum expected utility is close (and maybe less than) the no-relay case. This is because the channel gains of the relay channel are not high in this case. Therefore, the prospected gain due to channel diversity of the relay channel will be opposed by the multiplexing loss due to the transmission over two time slots.

V. Conclusion

In this paper, we have considered layered source coding with two layers transmitted using superposition coding at the transmitter with successive interference cancellation at the receiver. A relay has been considered to assist the channel using selection relaying decode and forward strategy. The random search method has been applied to find the optimal power allocation at the source and at the relay in order to maximize the expected user satisfaction that is defined by a

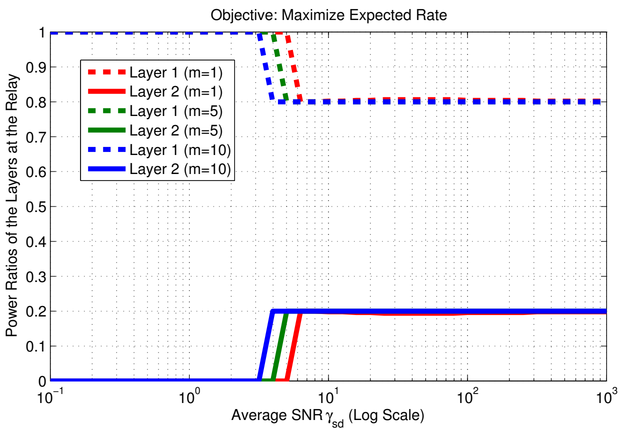

Fig. 4. The relative power ratios of the layers at the relay versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = \bar{R}$.

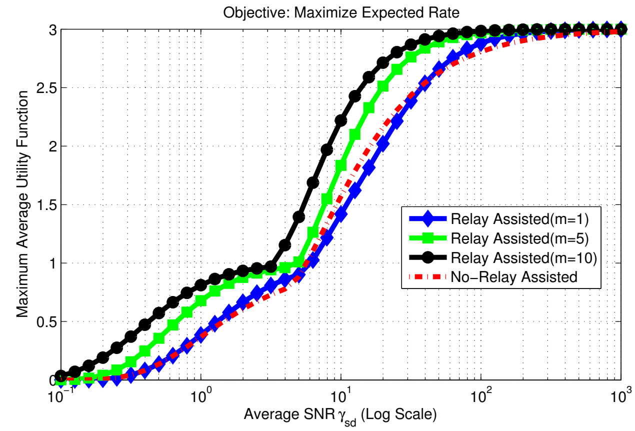

Fig. 6. The maximized average utility function versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = \bar{R}$.

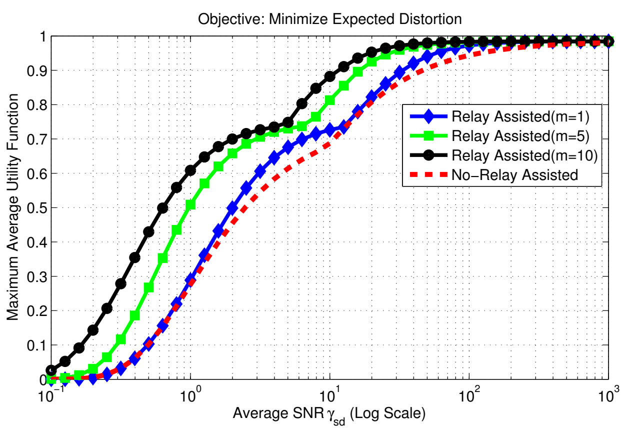

Fig. 5. The maximized average utility function versus the average SNR value of a Rayleigh fading channel with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.

utility function of the total decoded rate at the destination. Several numerical examples were obtained for two different utility functions, which were maximizing the expected rate and minimizing the expected distortion of a Gaussian source. It has been shown that it may be optimal not to transmit both layers for low average SNR values of the channels. In this case, all the power is allocated to the base layer and the enhancement layer is discarded. An obvious gain was observed for the relay channel in comparison with the direct transmission case with no relay assistance. These gain increases as the ratio between the average SNR of the source-relay and relay-destination links to the average SNR of the source-destination link increases.

References

[1] W. Equitz and T. Cover, "Successive refinement of information," *IEEE Transactions on Information Theory*, vol. 37, no. 3, pp. 269–275, Mar. 1991.

[2] P. Bergmans, "Random coding theorem for broadcast channels with degraded components," *IEEE Transactions on Information Theory*, vol. 19, no. 2, pp. 197–207, Mar. 1973.

[3] T. Cover and J. Thomas, *Elements of Information Theory*, 2nd ed. Wiley-Interscience, July 2006.

[4] C. Ng, D. Gunduz, A. Goldsmith, and E. Erkip, "Distortion minimization in Gaussian layered broadcast coding with successive refinement," *IEEE Transactions on Information Theory*, vol. 55, no. 11, pp. 5074–5086, Nov. 2009.

[5] M. Shaqfeh, W. Mesbah, and H. Alnuweiri, "Utility maximization for layered transmission using the broadcast approach," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1228–1238, Mar. 2012.

[6] ——, "Utility maximization for layered broadcast over Rayleigh fading channels," in *Proceedings IEEE International Conference on Communications (ICC)*, Cape Town, South Africa, May 2010, pp. 1–6.

[7] W. Mesbah, M. Shaqfeh, and H. Alnuweiri, "Jointly optimal rate and power allocation for multilayer transmission," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 834–845, Feb. 2014.

[8] ——, "Rate maximization of multilayer transmission over Rayleigh fading channels," in *Proceedings IEEE International Symposium on Information Theory (ISIT)*, Istanbul, Turkey, Jul. 2013, pp. 2074–2078.

[9] ——, "Distortion minimization in layered broadcast transmission of a Gaussian source over Rayleigh channels," in *Proceedings IEEE Information Theory Workshop (ITW)*, Seville, Spain, Sep. 2013.

[10] E. C. V. D. Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 120–154, 1971.

[11] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[12] J. N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[13] L. Lai, K. Liu, and H. E. Gamal, "The three node wireless network: Achievable rates and cooperation strategies," *IEEE Transactions on Information Theory*, vol. 52, no. 3, pp. 805–828, Mar. 2006.

[14] M. Shaqfeh and H. Alnuweiri, "Joint power and resource allocation for block-fading relay-assisted broadcast channels," *IEEE Transactions on Wireless Communications*, vol. 10, no. 6, pp. 1904–1913, Jun. 2011.

[15] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[16] A. Zafar, M. Shaqfeh, M.-S. Alouini, and H. Alnuweiri, "Exploiting multi-user diversity and multi-hop diversity in dual-hop broadcast channels," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3314–3325, Jul. 2013.

[17] S. Chapra and R. Canale, *Numerical Methods for Engineers*, 6th ed. McGraw-Hill Science/Engineering/Math, 2009.