1. (20) A child twirls a small 50-g ball attached to the end of a 1-m rope so that the ball traces a circle in a vertical plane as shown. What is the minimum speed \( v \) which the ball must have when in position 1? If this speed is maintained throughout the circle, determine the tension \( T \) in the rope when the ball is in position 2.

2. (20) A package having a mass of 2 kg is delivered from a conveyor to a smooth circular ramp with a velocity \( v_0 = 1 \) m/s as shown. If the radius of the ramp is 0.5 m, determine the angle \( \theta \) at which the package begins to leave the surface of the ramp.

3. (20) Show that the equation of the path of a projectile fired from the origin with an initial velocity \( v_0 \) at an angle \( \theta_0 \) for a freely moving projectile is given by:

\[
y = x \tan \theta_0 - \frac{1}{2} \frac{g}{v_0^2 \cos^2 \theta_0} x^2
\]

4. (20) A plane travels at a constant speed \( v \), and at a constant altitude. The radar tracks the plane and computes the distance \( D \), the angle \( \theta \) and the rate of change of \( \theta \) at all times. Find \( v \), the speed of the plane, in terms of \( \theta \), \( d\theta/dt \) and \( D \).

5. (20) A thin disc of mass 5 kg and a radius of 0.2 m is released from rest on an inclined surface. The disc is allowed to roll without slipping down a distance of 5 m as shown. Determine the time required for the disc to reach that distance.
Equation Sheet

\[ \frac{dx}{dt} = v \]

\[ \frac{dv}{dt} = a \]

\[ \frac{d^2x}{dt^2} = \frac{dv}{dx} \]

Uniform rectilinear motion

\[ x = x_0 + vt \]

Uniformly accelerated rectilinear motion

\[ v = v_0 + at \]

\[ x = x_0 + v_0t + \frac{1}{2}at^2 \]

\[ v^2 = v_0^2 + 2a(x-x_0) \]

Acceleration components

**Tangential & normal:**

\[ a_t = \frac{dv}{dt} \]

\[ a_n = \frac{v^2}{\rho} \]

**Radial and transverse:**

\[ a_r = \ddot{r} - r\ddot{\theta} \]

\[ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \]

Work and energy:

\[ T_1 + U_{1\rightarrow2} = T_2 \]

Conservation of energy:

\[ T_1 + V_1 = T_2 + V_2 \]

Impulse and momentum:

\[ m\ddot{v}_1 + \int_{t_1}^{t_2} \vec{F} \, dt = m\ddot{v}_2 \]

Coefficient of restitution:

\[ e = \frac{v'_B - v'_A}{v_A - v_B} \]

\[ \ddot{v}_B = \ddot{v}_A + \ddot{\vec{r}}_{B/A} \]

\[ \ddot{a}_B = \ddot{a}_A + \ddot{\vec{r}}_{B/A} + (\ddot{a}_{B/A})_n + (\ddot{a}_{B/A})_t \]

\[ (\ddot{a})_t = r\ddot{\alpha} \]

\[ (\ddot{a})_n = r\omega^2 \]

\[ \sum \vec{F} = m\ddot{\vec{a}}_G \]

\[ \sum \vec{M}_G = I_G\ddot{\vec{\omega}}_G \]

Impulse and momentum for rigid bodies:

\[ m\ddot{v}_{G1} + \int_{t_1}^{t_2} \vec{F} \, dt = m\ddot{v}_{G2} \]

\[ I_G\ddot{\vec{\omega}}_G + \int_{t_1}^{t_2} \vec{M}_G \, dt = I_G\ddot{\vec{\omega}}_G \]