1. (30) A wheel of radius $r$ rolls without slipping and has an angular velocity $\omega$. Write an expression for the velocity of point $A$ in terms of $\omega$, $r$, and $\theta$. Show that the velocities of $A$ and $B$ are perpendicular to each other.

Ans. $v_A = 2\omega r \sin \frac{\theta}{2}$

2. (30) Determine the angular acceleration and the forces on the bearing at $O$ for (a) the hoop of mass $m$ and (b) the circular disc of mass $m$ immediately after each is released from rest in the vertical plane with $OG$ horizontal.

Ans. (a) $\alpha = \frac{g}{2r}$, $F_y = \frac{mg}{2}$

(b) $\alpha = \frac{2g}{3r}$, $F_y = \frac{mg}{3}$

3. (40) A drum $A$ is given a constant angular acceleration $\alpha_0 = 3$ rad/s$^2$ and causes the 70-kg wheel $B$ to move on the horizontal surface by means of the cable which is wrapped around the inner hub of the wheel. The moment of inertia of the wheel is 4.375 kgm$^2$ and the coefficient of friction between the wheel and the horizontal surface is 0.25. Determine the tension $T$ in the cable and the friction force $F$ exerted by the horizontal surface on the wheel.

Ans. $T = 154.58$ N, $F = 75.83$ N

Useful equations

$\ddot{v}_B = \ddot{v}_A + \ddot{v}_{B/A} = \ddot{v}_A + \dot{\omega} \times \dot{r}_{B/A}$

$\ddot{a}_B = \ddot{a}_A + \ddot{a}_{B/A} = \ddot{a}_A + (\ddot{a}_{B/A})_n + (\ddot{a}_{B/A})_t$

$(a)_t = r\alpha \quad (a)_n = r\omega^2$

$\sum F = m\ddot{a}_G \quad \sum M_G = I_G\ddot{\alpha}$
For a hoop, $I_G = mr^2$

For a disc, $I_G = \frac{1}{2}mr^2$