Expectations of Random Variables

Variables (X, Y, … etc) denote quantities that change. Variables may be discrete (counts) or continuous (measures). A random variable is a variable whose values cannot be predicted with accuracy. For example:
X: The number of students who will show up for class out of 50 who are registered for the course (discrete).
Y: The temperature today (continuous).

Since a random variable cannot be predicted with accuracy, some measures are used to characterize its behavior. These measures include the mean (μ) and the variance (σ^2). The mean is also known as the expected value and is denoted by E[]. In other words, E[X] = μ_x. The following expectation rules are important to our discussion of forecasting techniques. In these rules, we assume that *c* is a constant, and that *X* and *Y* are random variables.

1. E[*c*] = *c*. In other words, the average of a constant value is that constant value.
2. 2. E[*cX*] = *c*E[*X*]. In other words, multiplying every value of a random variable by a constant c and taking the average of the results is that same as multiplying that constant *c* by the average of the random variable.
3. E[*X* ± *Y*] = E[*X*] ± E[*Y*]. The average of a total or a difference is the same as the total or difference of averages.

As an example assume that the bookstore has 20 copies of the textbook for MENG 445 and 10 copies of the textbook for MENG 447. The probability of selling a copy of each book is 0.9 and 0.8 at the cost of LE 280 and 240, respectively. The number of textbooks sold for each course is a random variable. Assume that X is the number of 445 textbooks sold and Y is the number of MENG 447 textbooks sold. It is easy to show that E[X] = 18, E[Y] = 8. The income is also a random variable given by 280X + 240Y. The average income is 280E[X] + 240E[Y] = LE6,960.

The variance is defined as the average square distance from the mean. Using our discussion above, we could define the variance as: $V[X] = \sigma_x^2 = E[(X - \mu_x)^2] = E[(X - E[X])^2]$

Again, assuming that *c* is a constant, and that *X* and *Y* are independent random variables, the following rules apply to the variance:

1. V[c] = 0. The variance of a constant is zero. It is a constant, so it doesn't vary.
2. V[cX] = c^2 V[X]
3. V[*X* ± *Y*] = V[*X*] + V[*Y*], assuming the X and Y are independent.