MENG 445
Forecasting- Linear Model

$$x_t = a + bt + \varepsilon_t,\ \varepsilon_t \approx NID(0, \sigma^2)$$

a. Least Square

$$\hat{b} = \frac{N\sum_{t=T-N+1}^{T} t\,x_x - \sum_{t=T-N+1}^{T} x_x \sum_{t=T-N+1}^{T} t}{N\sum_{t=T-N+1}^{T} t^2 - (\sum_{t=T-N+1}^{T} t)^2}, \quad \hat{a} = \bar{x} - \hat{b}\bar{t}$$

$$\hat{x}_{T+\tau}(T) = \hat{a} + \hat{b}(T+\tau) \pm 2\sigma\sqrt{1 + \frac{1}{N} + \frac{(T+\tau-\bar{t})^2}{S_{tt}}}$$

b. Double Moving Average

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} x_t, \quad E[M_T] = E[x_T] - \frac{N-1}{2}b$$

$$M_T^{[2]} = \frac{1}{N}\sum_{t=T-N+1}^{T} M_t, \quad E[M_T^{[2]}] = E[x_T] - (N-1)b = E[M_T] - \frac{(N-1)}{2}b$$

$$\hat{x}_T(T) = 2M_T - M_T^{[2]}, \quad \hat{x}_{T+\tau}(T) = (2 + \frac{2\tau}{N-1})M_T - (1 + \frac{2\tau}{N-1})M_T^{[2]}, \quad \text{(point estimate)}$$

Estimating the variance of the DMA forecasting error is not simple. Therefore, we'll use the variance of the LS as an approximation. Accordingly, the PI may be defined as:

$$\hat{x}_{T+\tau}(T) = (2 + \frac{2\tau}{N-1})M_T - (1 + \frac{2\tau}{N-1})M_T^{[2]} \pm 2\sigma\sqrt{1 + \frac{1}{N} + \frac{(T+\tau-\bar{t})^2}{S_{tt}}}$$

c. Double Exponential Smoothing

$$S_T = \alpha x_T + (1-\alpha)S_{T-1}, \quad E[S_T] = E[x_T] - \frac{\beta}{\alpha}b$$

$$S_T^{[2]} = \alpha S_T + (1-\alpha)S_{T-1}^{[2]}, \quad E[S_T^{[2]}] = E[x_T] - \frac{2\beta}{\alpha}b = E[S_T] - \frac{\beta}{\alpha}b$$

$$\hat{x}_T(T) = 2S_T - S_T^{[2]}, \quad \hat{x}_{T+\tau}(T) = (2 + \frac{\alpha\tau}{\beta})S_T - (1 + \frac{\alpha\tau}{\beta})S_T^{[2]}, \quad \text{(point estimate)}$$

For initial values, fit the line $\hat{a}(0) + \hat{b}(0)t$ to the first N observations,

$$S_0 = \hat{a}(0) - \frac{\beta}{\alpha}\hat{b}(0),\ S_0^{[2]} = \hat{a}(0) - 2\frac{\beta}{\alpha}\hat{b}(0)$$

Estimating the variance of the DES forecasting error is not simple. Therefore, we'll use the variance of the LS as an approximation. Accordingly, the PI may be defined as:

$$\hat{x}_{T+\tau}(T) = (2 + \frac{\alpha\tau}{\beta}) S_T - (1 + \frac{\alpha\tau}{\beta}) S_T^{[2]} \pm 2\sigma\sqrt{1 + \frac{1}{N} + \frac{(T + \tau - \bar{t})^2}{S_{tt}}}$$