MENG 445 Production and Inventory Control

Forecasting

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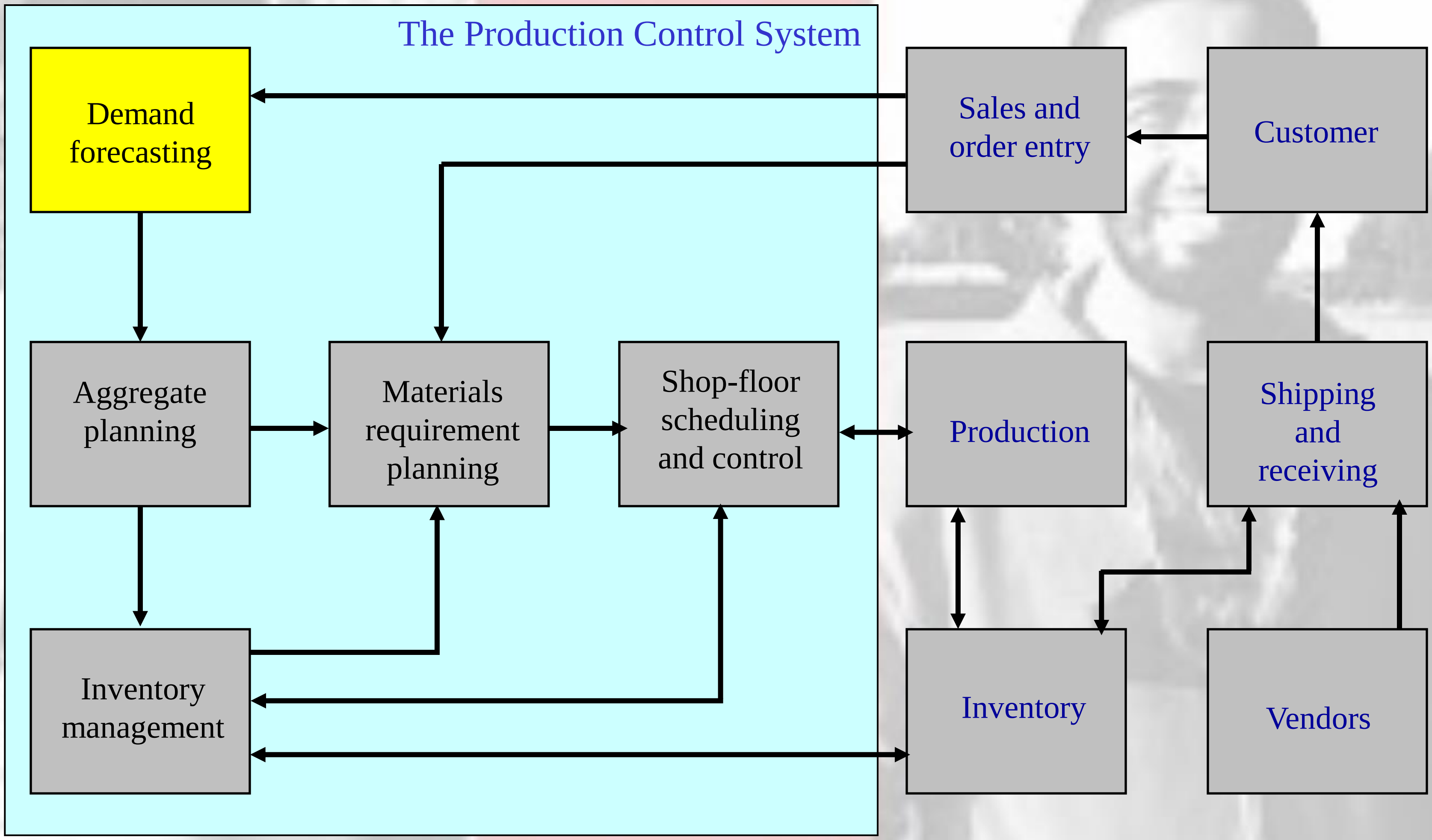
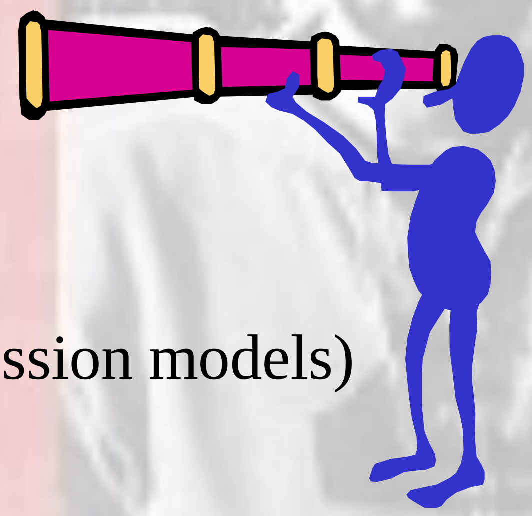
Production Planning and Control



Forecasting

- Basic Problem: **predict demand for planning purposes.**
- Forecasting Tools:
  - ***Qualitative*:**
    - Delphi
    - Analogies
  - ***Quantitative*:**
    - Causal models (e.g., regression models)
    - Time series models

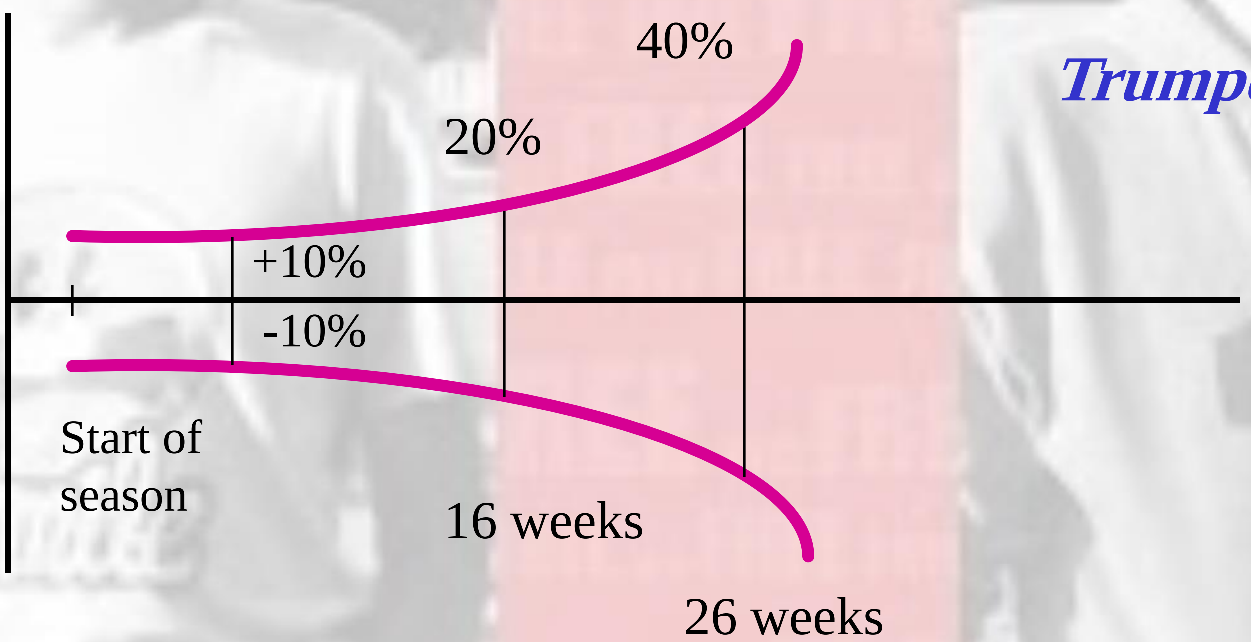


Forecasting "Laws"

1) Forecasts are always wrong!

2) Forecasts always change!

3) The further into the future, the less reliable the forecast!



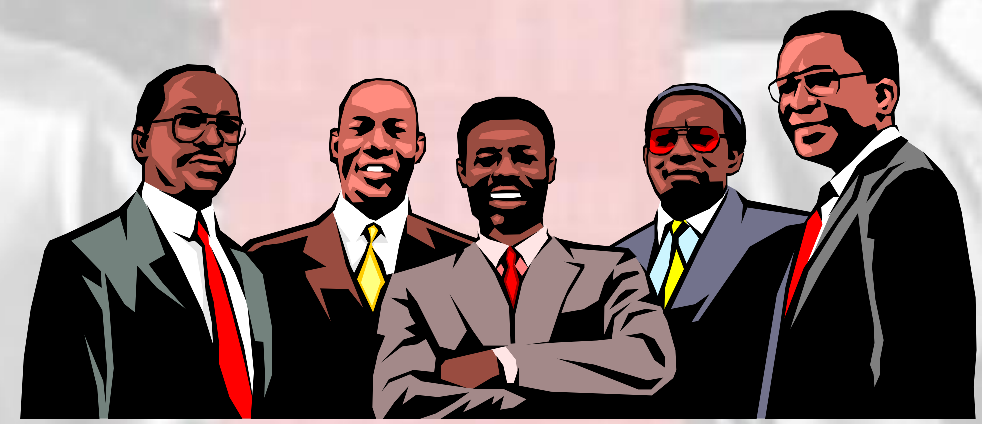
Trumpet of Doom

Why Forecasting?

- Most decisions made in PP&C are "plans for future"
  - How much of a certain product should I make next week?
  - How much inventory should I keep in the May 2001?
- If you forecast the future (more accurately), you will (more likely) have better plans.
- Many decisions (plans) in PPC depend on the forecasts of demand.
- Forecasting demand versus forecasting sales.

Qualitative Forecasting

Delphi Technique: Iteratively tapping multiple experts opinion with continuous comparisons and revisions.

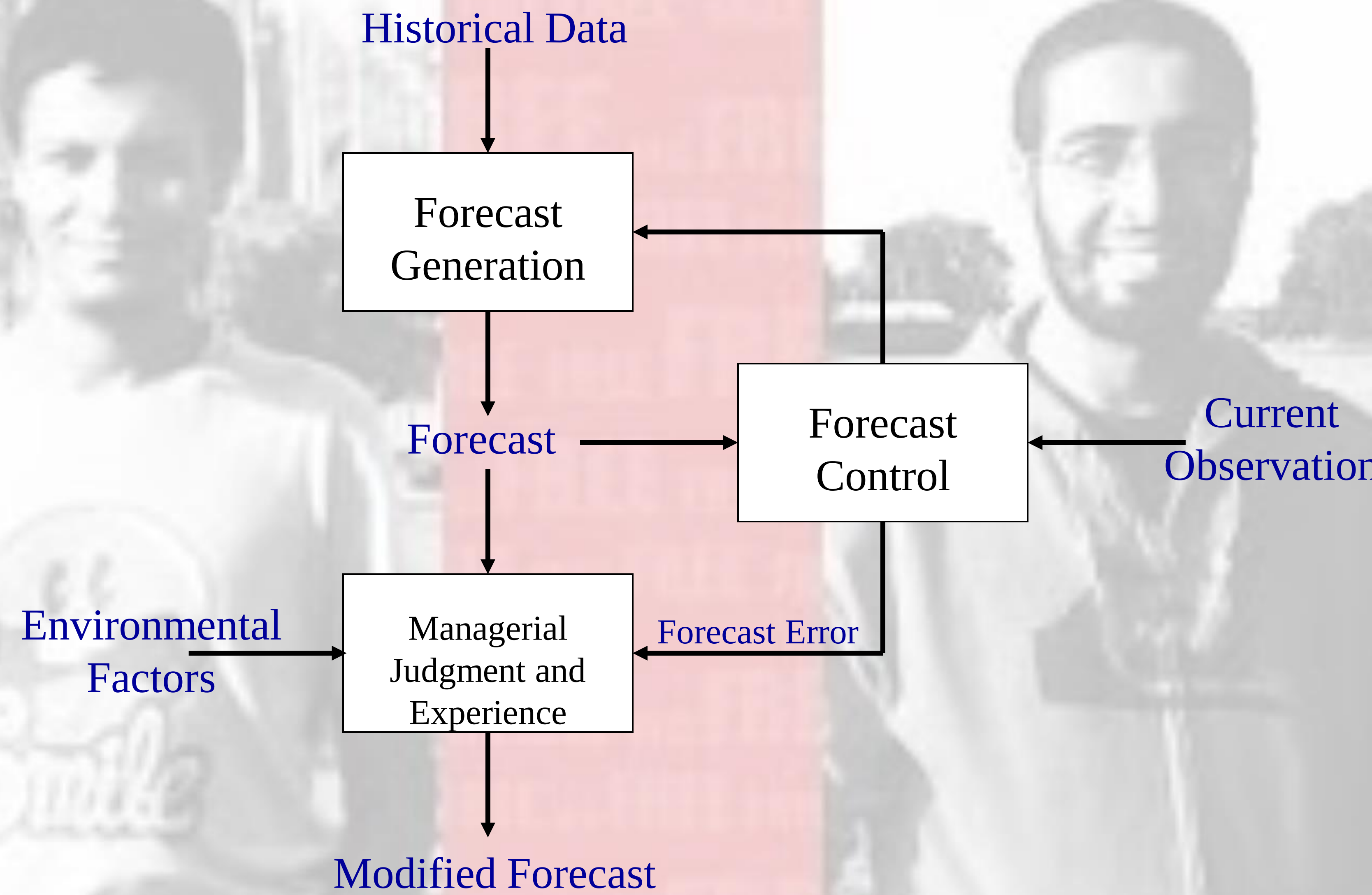


Quantitative Forecasting

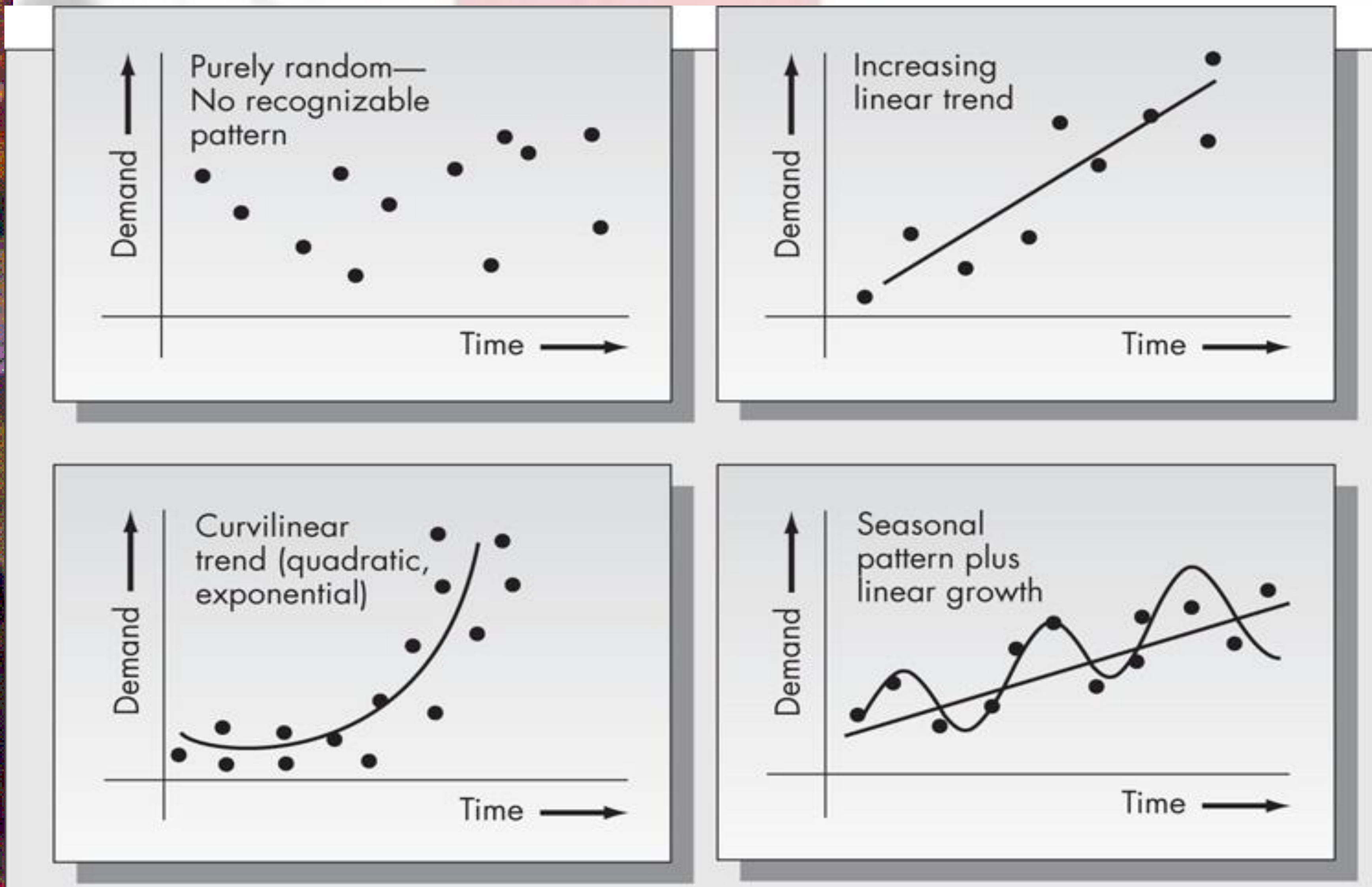
- Goals:
  - –Predict future from past
  - –Smooth out “noise”
  - –Standardize forecasting procedure

- Methodologies:
  - –***Causal Forecasting***:
    - regression analysis
  - –***Time Series Forecasting***:
    - moving average
    - exponential smoothing
    - regression analysis
    - seasonal models

The Forecasting Model



Possible Patterns



Source: Stevenson, Stevenson, Nahmias, J. Production and Operations Analysis, 2009.

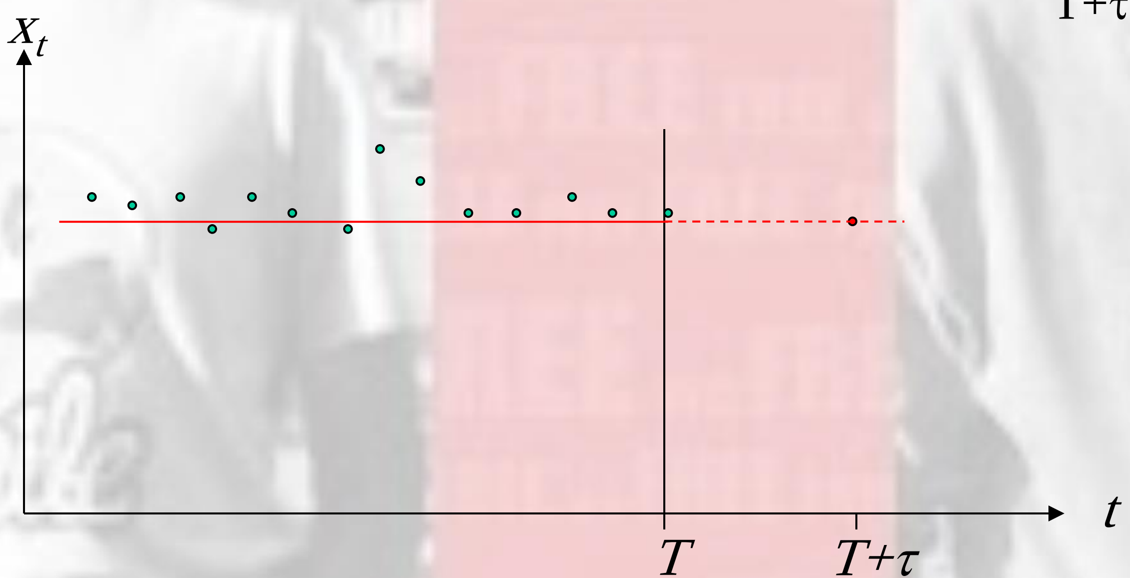
Time Series Forecasting

Historical Data ***Forecast***

$x_t,\ t = 1, \ldots, T$ → **Time series model** → $\hat{x}_{T+\tau}(T)$

Observed value at time *t*

Predicted value for time T+τ made at time T.



The Constant Model

- Model:

$$x_t = a + \varepsilon_t, \ \varepsilon_t \ NID(0, \sigma^2)$$

- Assumptions:

No trend, only random noise

Noise is normally distributed

Noise is independent

Noise has a constant variance (doesn’t change with time)

Noise averages out to zero

The Constant Model

Moving Average (same as least squares)

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} x_t, \quad V(M_T) = \frac{\sigma^2}{N}, \quad \hat{x}_{T+\tau}(T) = M_T \pm 2\sqrt{1+\frac{1}{N}}\,\sigma$$

- Averages the latest N observations.
- N ranges from 2 to 12.
- Larger N provides stability, but less responsive.

➢ Assigns equal weights to the latest N observations

➢ Does not adapt to error

The Constant Model

Moving Average- Example

t	x_t	M_t	$\hat{x}_t(t-1)$	e_t	$\lvert e_t \rvert$
1	20				
2	22				
3	23	21.67			
4	20	21.67	21.67	-1.67	1.67
5	18	20.33	21.67	-3.67	3.67
6	22	20.00	20.33	1.67	1.67
7	22	20.67	20.00	2.00	2.00
8	23	22.33	20.67	2.33	2.33
9	20	21.67	22.33	-2.33	2.33
10	24	22.33	21.67	2.33	2.33
					2.29

The Constant Model

Exponential Smoothing

$$S_T = \alpha\, x_T + (1-\alpha)\, S_{T-1}, \quad V(S_T) = \frac{\alpha}{2-\alpha}\sigma^2, \quad \hat{x}_{T+\tau}(T) = S_T \pm 2\sigma\sqrt{1+\frac{\alpha}{2-\alpha}}$$

- Combines current observation and previous forecasts for future predictions.
- α ranges from 0.1 to 0.5.
- Smaller α provides stability, but less responsive.

➢ Assigns decaying weights to all observations

➢ Adapts to error

The Constant Model

Exponential Smoothing- Example

t	x_t	S_t	$\hat{x}_t(t-1)$	e_t	$\|e_t\|$
1	20	21.67			
2	22	21.83	21.67		
3	23	22.42	21.83	1.17	1.17
4	20	21.21	22.42	-2.42	2.42
5	18	19.60	21.21	-3.21	3.21
6	22	20.80	19.60	2.40	2.40
7	22	21.40	20.80	1.20	1.20
8	23	22.20	21.40	1.60	1.60
9	20	21.10	22.20	-2.20	2.20
10	24	22.55	21.10	2.90	2.90
					2.14

The Linear Model

- Model:

$$x_t = a + bt + \varepsilon_t,\ \varepsilon_t\ NID(0, \sigma^2)$$

- Assumptions:

  Linear trend

  Noise is normally distributed

  Noise is independent

  Noise has a constant variance (doesn’t change with time)

  Noise averages out to zero

The Linear Model

Least Squares

$$\hat{b} = \frac{N \sum_{t=T-N+1}^{T} t\, x_x - \sum_{t=T-N+1}^{T} x_x \sum_{t=T-N+1}^{T} t}{N \sum_{t=T-N+1}^{T} t^2 - \left(\sum_{t=T-N+1}^{T} t \right)^2}, \qquad \hat{a} = \bar{x} - \hat{b}\bar{t}$$

$$\hat{x}_{T+\tau}(T) = \hat{a} + \hat{b}(T+\tau) \pm 2\sigma \sqrt{1 + \frac{1}{N} + \frac{(T+\tau-\bar{t})^2}{S_{tt}}}$$

The Linear Model

Double Moving Average

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} x_t, \quad E[M_T] = E[x_T] - \frac{N-1}{2}b$$

$$M_T^{[2]} = \frac{1}{N}\sum_{t=T-N+1}^{T} M_t, \quad E[M_T^{[2]}] = E[x_T] - (N-1)b = E[M_T] - \frac{(N-1)}{2}b$$

$$\hat{x}_T(T) = 2M_T - M_T^{[2]}, \quad \hat{x}_{T+\tau}(T) = (2 + \frac{2\tau}{N-1})M_T - (1 + \frac{2\tau}{N-1})M_T^{[2]}, \quad \text{(point estimate)}$$

$$\hat{x}_{T+\tau}(T) = (2 + \frac{2\tau}{N-1})M_T - (1 + \frac{2\tau}{N-1})M_T^{[2]} \pm 2\sqrt{1 + \frac{1}{N} + \frac{(T+\tau-\bar{t})^2}{S_{tt}}}\,\sigma$$

The Linear Model

Double Exponential Smoothing

$$S_T^{[2]} = \alpha S_T + (1-\alpha) S_{T-1}^{[2]}, \quad E[S_T^{[2]}] = E[\chi_T] - \frac{2\beta}{\alpha} b = E[S_T] - \frac{\beta}{\alpha} b$$

$$S_T^{[2]} = \alpha S_T + (1-\alpha) S_{T-1}^{[2]}, \quad E[S_T^{[2]}] = E[\chi_T] - \frac{2\beta}{\alpha} b = E[S_T] - \frac{\beta}{\alpha} b$$

$$\hat{\chi}_T(T) = 2 S_T - S_T^{[2]}, \quad \hat{\chi}_{T+\tau}(T) = (2 + \frac{\alpha\tau}{\beta}) S_T - (1 + \frac{\alpha\tau}{\beta}) S_T^{[2]}, \quad \text{(point estimate)}$$

$$S_0 = \hat{a}(0) - \frac{\beta}{\alpha}\hat{b}(0), S_0^{[2]} = \hat{a}(0) - 2\frac{\beta}{\alpha}\hat{b}(0)$$

$$\hat{\chi}_{T+\tau}(T) = (2 + \frac{\alpha\tau}{\beta}) S_T - (1 + \frac{\alpha\tau}{\beta}) S_T^{[2]} \pm 2\sqrt{1 + \frac{1}{N} + \frac{(T + \tau - \bar{t})^2}{S_{tt}}}\,\sigma$$

The Seasonal Model

•Model:

$$x_t = (a + bt)c_L + \varepsilon_t, \varepsilon_t\ NID(0, \sigma^2)$$

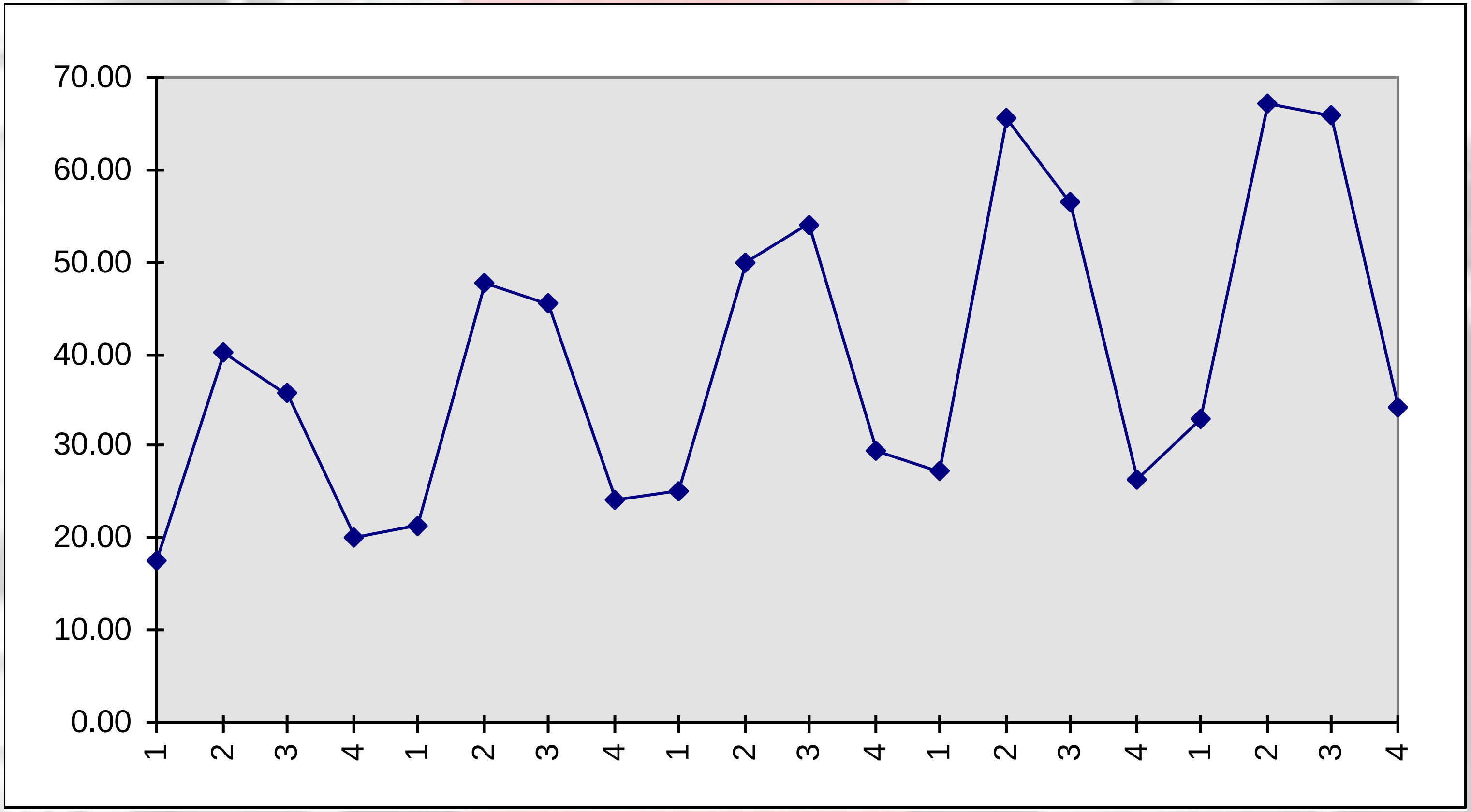
Example

The quarterly sales of company XYZ over the past five years are given below. Use this data to forecast the sales in every quarter of years 6 and 7. Use the same data to forecast the total sales of years 6 and 7.

	Quarter				
Year	I	II	III	IV	
1	17.51	40.24	35.76	20.08	
2	21.33	47.81	45.45	24.11	
3	25.26	50.04	53.92	29.65	
4	27.38	65.50	56.58	26.51	
5	32.99	67.27	65.94	34.34	

The Seasonal Model

Example



The Seasonal Model

Example

Year	I	II	III	IV	
1	17.51	40.24	35.76	20.08	28.40
2	21.33	47.81	45.45	24.11	34.67
3	25.26	50.04	53.92	29.65	39.72
4	27.38	65.50	56.58	26.51	43.99
5	32.99	67.27	65.94	34.34	50.14

a = 23.55
b = 5.28.

Seasonal Indices

Year	I	II	III	IV
1	0.62	1.42	1.26	0.71
2	0.62	1.38	1.31	0.70
3	0.64	1.26	1.36	0.75
4	0.62	1.49	1.29	0.60
5	0.66	1.34	1.32	0.68
Average	0.63	1.38	1.31	0.69

The Seasonal Model

Forecasted Values

Year 6 average = 23.55 + 6 * 5.28 = 55.23

Year 7 average = 23.55 + 7 * 5.28 = 60.51

Seasonal Indices

Quarter	I	II	III	IV
Index	0.63	1.38	1.31	0.69

Forecasted Values

Year	I	II	III	IV
6	34.77	76.05	72.11	37.95
7	38.09	83.33	79.00	41.58