## Signal amplification in time-modulated RF components with infinite superluminality

Mohamed F. Hagag<sup>(1)</sup>, Thomas R. Jones<sup>(2)</sup>, Karim Seddik<sup>(1)</sup>, and Dimitrios Peroulis<sup>(2)</sup>

<sup>(1)</sup> Electronics and Communications Engineering Department, American University in Cairo, Cairo 11835, Egypt

 $^{(2)}$  Elmore Family School of Electrical and Computer Engineering, Purdue University, West

Lafayette, Indiana 47907, USA

Wave propagation in time-varying media is a hot topic at the moment due to the exotic properties the wave experiences, such as magnetless non-reciprocity, parity-time symmetry, and topological aspects. When the speed of modulation exceeds the speed of light in a media (superluminal regime), momentum gaps appear in the real frequency dispersion diagram. Only complex frequencies exist within these gaps with positive values (causing wave growth) and negative values (causing wave decay) imaginary parts. If the media is pure-time modulated (infinite superluminality), signal amplification happens at 0.5 wave/modulation frequency ratio  $(F_s/F_m)$ . Here, we present signal amplification modeling of time-modulation RF components, transmission line (TL) and cavity resonators. The transmission matrix method is utilized to get the dispersion relations which confirmed by circuit modeling transient simulations.

The unit cell shown in Fig. 1(a) is designed to have a Bloch impedance of 50  $\Omega$  at signal frequency 0.5 GHz. The loaded capacitance is modulated following  $C(\omega, t) = C_o(\omega) (1 + M_D \cos(\omega_m t))$  with 1 GHz modulation frequency and  $M_D = 0.7$ . Fig. 1(a) shows the real and complex dispersion diagrams. The momentum band gap is obvious, and positive/negative imaginary part values at 0.5  $F_s/F_m$ . Signal amplification is confirmed by circuit modeling in Fig. 1(c). Compared to TLs, resonators can produce a much stronger momentum kick. As a result, higher levels of amplifications are expected when a resonator property is time-modulated. The cavity resonator shown in Fig. 1(a) is designed to have two overlapped modes at one of the ports with opposite polarities. A time-modulated capacitance is placed to control one of the modes' resonant frequency. Consequently, mode cancellation and the external coupling coefficient are time-modulated. With a sufficient modulation speed, at 0.5  $F_s/F_m$ , signal amplification is achieved, as shown in Fig. 1(e). As expected, considering the same value of  $M_D$ , a higher amplification level is achieved with lower unit cell numbers than time-modulated TLs.



Figure 1: (a) A unit cell of a TL with a length of  $1.5\lambda_m$  and 83  $\Omega$  characteristic impedance loaded with time-modulated capacitor ( $C_o=4\,$  pF), (b) Real and complex frequency dispersion diagram with,  $M_D = 0.7$  and  $F_m = 1$  GHz, (c) Transient simulated output power of nine unit cells with 1 W input power loaded with a 50  $\Omega$  load impedance. (d) A cavity resonator with time modulated external coupling coefficient. (e) Amplified output power of three successive cavities considering 1 W input power and  $M_D = 0.7$ .