PMUs Placement with Max-Flow Min-Cut Communication Constraint in Smart Grids

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***Abstract*—Synchronized phasor measurement units (PMUs) play an important role in the process of monitoring, controlling, and protection of today's smart grid networks. Therefore, a strategic placement of these PMUs is essential to perform these tasks. Previous studies in this area mostly concentrate on minimizing the total number of PMUs while maintaining the system fully observable in various contingency situations. However, most of them neglect the communication constraints among PMUs, e.g., the finite communication capacity[1] available when using power line communications (PLC) among PMUs. In this paper, we introduce two different formulations to solve the problem of optimal PMUs placement (OPP), while taking into consideration a communication constraint for the case where the communication between the PMUs and the control center is established via wired power lines. Therefore, there will be a constraint on the max-flow min-cut in the wired smart grid model. In the first formulation, we find the best location or bus in the grid for the controller to be located in order to support the communication max-flow min-cut constraint. In the second formulation, we fix the controller at a certain bus and find the optimal solution that maintains the networks full observability and also satisfies the communication max-flow min-cut constraint. We also apply our formulations to different IEEE standard bus systems, and our results reveal that there are some buses at which we should avoid placing the controller, and the solution of the conventional OPP problem may not support the max-flow min-cut constraint.**

I. Introduction

Synchronized phasor measurement units (PMUs) play a key role in monitoring and controlling the performance of the smart grid systems. It can be defined as a device which is used to synchronize the ac current and voltage measurements of the smart grid system with a common time reference; in general, this time reference is a GPS signal with accuracy less than 1 μs [1]. The main function of the PMU is to convert the ac current and voltage signals measured at the different buses of the smart grid to complex numbers representing the magnitude and the phase angle of these measured signals or "phasor", and time stamp them [2]. Therefore, these phasors help to improve the performance of the smart grid control and monitoring systems in various fields such as state estimation, cyber attacks and bad data detection [3].

The optimal PMU placement problem has been originally introduced in [4] and [5] as a very important stage in solving the problem of system state estimation, because in order to estimate the system state sufficient data should be collected to make the system fully observable, but it is not economical to fix a PMU device at each bus to achieve the systems full observability. Therefore, the objective of the conventional OPP problem is to find the minimum number of the PMUs to achieve the network full observability.

Actually, the OPP problem can be considered as a combinatorial optimization problem with a solution space that consists of 2^n possible combinations for an n-bus smart grid system. In earlier studies, simulated annealing and graph theory have been used to formulate and solve the conventional OPP problem in [4] and [6], while in [7] and [8] integer programming was used to introduce an alternative approach to find the minimum number of PMUs to make the system fully observable. The approaches that have been introduced in this area can be classified into two classes: the meta-heuristic optimization methods and the conventional deterministic techniques, both of which have been discussed in [9].

A common trend in most of the previous studies is that they have focused on introducing new algorithms and enhancing the performance of the existing approaches towards the OPP problem considering factors such as minimizing the time needed to find the global optimum, and the ability of these algorithms to deal with power systems with large search space. They have, however, neglected addressing communication restrictions on the data transfer in the power grid systems. In [10] and [11], we can find two different approaches that consider certain restrictions on the number of "measurement channels" available for each PMU in the smart grid system; but, to the best of the authors knowledge, there has been no work on PMUs placement with formal restriction on the communication between the nodes in the smart grid network.

In this paper, we introduce two different strategies to solve practical variations on the OPP problem. We add a communication constraint to the conventional OPP problem addressed in the previous studies and obtain a solution. This constraint is formulated based on the max-flow min-cut theorem [12], as we consider that the communication between the different nodes in the smart grid network is established via the connecting power lines. In the first strategy, we seek the best locations at which we can fix the controller unit to allow the communication from the PMUs to the controller under full

[1]Throughout this paper, the word "capacity" refers to the communication channel capacity not the power flow capacity of the smart grid.

observability and communication constraints. In the second strategy, we assume that the location of the controller unit is fixed and resolve the problem to find the places at which we can fix the PMUs to allow the communication for this bus.

The rest of this paper is organized as follows. In Section II, some background on the max-flow min-cut problem is presented. The system model is presented in Section III. In Section IV, the problems formulation is presented. The analysis and the simulation results for the proposed formulations are presented in Sections V and VI, respectively. Conclusions are drawn in Section VII.

II. Max-Flow Min-Cut Background

In this section, before introducing our formulations, we give a brief explanation of the max-flow min-cut theorem which will be used in our analysis. As mentioned before we consider that the communication between the controller and the system buses in our model is established via the power lines (Power Line Communication) that connect the system buses to each other and the controller.

The max-flow min-cut theorem states that the maximum value of the flow that passes from the source node to the sink node in a flow network, such as our smart grid system, is equal to the minimum capacity that, when removed in a specific way from the network, causes the situation that no flow can pass from the source to the sink. In our model, we can consider that the source node s is the node at which a PMU is installed, and the sink node t is the bus at which the controller unit is installed. Therefore, our problem turns to be a multi-source single-sink max-flow min-cut problem [12].

The capacity of the power line that connects node u to node v is denoted by $c(u, v)$, and it also represents the maximum flow that can pass through this edge e, where $e \in E$ and E is the set of edges, while $f(u, v)$ is used to denote the flow passing through this edge. The flow that passes through the edge should be lower than or equal to the maximum capacity of the edge, i.e., $f(u, v) \leq c(u, v)$. Another concept that should be elaborated on is the cut, which can be defined as a partitioning process of the network nodes into two different sets, the source set S and the sink set T, where the cut set $C(S, T)$ of the cut is given by the set of edges $\{(u, v) \in E : u \in S, v \in T\}$, and the capacity of an $S - T$ cut is defined as follows:

$$c(S, T) = \sum_{(u,v)\in S\times T} c(u, v). \tag{1}$$

Equ. (1) states that the capacity of the cut is equal to the summation of the capacities of all the edges or lines across the cut. The max-flow min-cut between two nodes s and t is defined as the minimum flow over all the cuts S-T such that $s \in S$ and $t \in T$.

Therefore, in the max-flow min-cut problem the main objective is to route as much flow as possible from the source to the sink, and at the same time try to determine S and T such

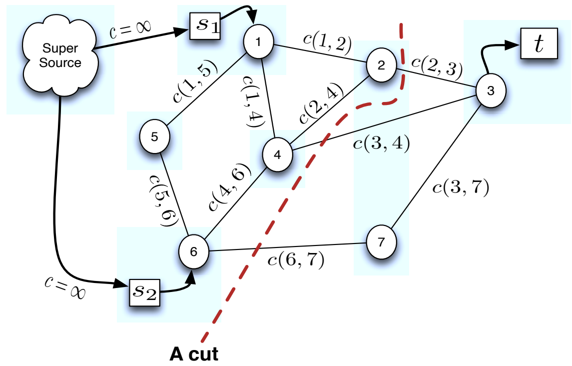

Fig. 1: Multi-source single-sink max-flow min-cut problem: two source nodes (s_1 and s_2) and one sink node, t. A super source is connected to the two sources with infinite capacity edges to have a single-source single-sink problem.

that the capacity of the $S-T$ cut is minimal[2]. It is found that the maximum value of the $s-t$ flow is equal to the minimum capacity over all $S - T$ cuts.

As mentioned before, our problem is a multi-source single-sink max-flow min-cut problem. Therefore, to simplify the problem and convert it to a single-source single-sink problem, a consolidated super source is added and connected to each source with infinite capacity on each edge as shown in Fig. 1.

III. System Model

When a PMU is installed at a certain bus it is able to measure and determine both the voltage phasor of this bus and the current phasors of some or all the buses connected to this bus. Therefore, The objective of the conventional problem of Phasor Measurement Units (PMU) placement is to find the minimum number of PMUs that can be installed on the buses of the smart grid to make the system fully observable.

Then, the optimal PMU placement problem (OPP) can be formulated as follows:

$$\begin{aligned} &\min \sum_{i}^{n} w_i x_i \\ &\text{s.t. } f(\mathbf{x}) = \mathbf{A}\mathbf{x} \geq 1 \end{aligned} \tag{2}$$

where w_i is the cost of installing a PMU at bus i, n indicates the total number of buses in the smart grid system, x_i is a binary decision variable which represents the solution of the problem and can be defined as follows:

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{Otherwise,} \end{cases} \tag{3}$$

$\mathbf{x}$ is an $n \times 1$ vector that represents the solution of the problem, $f(\mathbf{x})$ is the observability function, and is a vector function with that equals zero if the voltage of corresponding bus cannot be solvable by the given measurement set and nonzero otherwise.

[2]The minimum cut will be the limiting cut for the flow from the source to the sink.

This means that when the value of the observability function is greater than or equal to 1 the system will be fully observable and each bus is observable by one or more than one PMU. Finally, $\mathbf{A}$ is the binary connectivity matrix of the smart grid system, it indicates the connectivity between any two buses u and v in the system and can be defined as follows:

$$\mathbf{A}(u,v) = \begin{cases} 1 & \text{if } u = v \\ 1 & \text{if bus } u \text{ and bus } v \text{ are connected} \\ 0 & \text{Otherwise.} \end{cases} \quad (4)$$

IV. Problem Formulation

Our target in this section is to develop new PMUs placement formulations. In the first formulation, we aim at searching for the best location bus to place the controller, or the sink node, at in the smart grid system. This location should support and allow the data flow from the sources to the sink and also make the system fully observable.

We also introduce another formulation in which the controller location is fixed to a certain bus. We minimize the total number of PMUs that can be used to make the smart grid network fully observable, but at the same time we take into consideration the max-flow min-cut (communication) constraint, so that we can guarantee that the solution of our problem will support the data flow that comes out from the sources, which are the PMUs in our case, to the sink or the controller node.

It is known that the power lines that connect the system buses to each other and the sink node can support data rates in the range of hundreds of Kbits/sec [13], while the data rates that are required by the PMU devices are in the range of tens of Kbits/sec. Our communication constraint or the max-flow min-cut constraint can be formulated as follows:

$$N\alpha \leq c(S,T) \quad (5)$$

where N is the total number of the sources or PMUs, α is the data rate that comes out from each PMU, and $c(S,T)$ is the minimum cut which is equivalent to the max flow that can be drawn from the sources to the controller. It also represents the maximum capacity of the network as defined in (1). Our constraint states that the total amount of flow that comes out from the super source should be lower than or equal to the maximum capacity of the network in order to allow the communication and transfer all the data from the PMUs to the controller unit.

Next, we consider two variations of the above problem. In the first one, we try to find the best bus to place the controller at to make sure that the PMUs can communicate their measurements to the controller unit. In the second problem, we assume that the controller location is fixed and the PMU placement problem is formulated with the extra data communication constraint.

A. Controller Placement Formulation

In this subsection, our objective is to find the best location for the controller unit, so that we can satisfy the full observability constraint from (2) and the communication constraint of the max-flow min-cut theorem from (5).

Therefore, our problem can be formulated as a binary integer programming optimization problem as follows:

$$\begin{aligned} &\min \sum_{i}^{n} w_i x_i \\ &\text{s.t. } f(\mathbf{x}) = \mathbf{A}\mathbf{x} \geq 1, \\ &N\alpha \leq c(S,T). \end{aligned} \quad (6)$$

Solving the OPP with the extra max-flow min-cut constraint is very complex[3]. Therefore, we solve the problem in two steps. First, we find the optimal solution of the conventional optimal PMUs placement problem (OPP) from (2) with no communication constraint. This provides us with the locations of our sources. We then place the sink at all the different buses and at each bus we obtain the maximum capacity of the system $c(S,T)$ and compare it to the max flow that comes out from the super source. We use the second constraint from (6) to determine if this bus or location is acceptable for the controller unit. Finally, we can determine the best locations in our IEEE bus system that satisfy the system full observability and the max-flow min-cut communication constraints.

B. Fixed Controller Formulation

In this subsection, we consider the case where the controller location is fixed to a certain bus. We solve the OPP problem but with the extra constraint of making sure that communication can take place between the PMUs and the fixed controller. Here, the optimal solution means finding the best locations at which we can fix the sources in order to allow data flow from the PMUs to the controller unit with full system observability.

Again, due to the complexity of the max-flow min-cut communication constraint, we start our problem by solving the conventional OPP problem from (2) to find the first optimal solution; then, we check our communication constraint as mentioned in (5) to check if this solution satisfies the max-flow min-cut restriction. If this solution does not support the constraint, then we reject it and start to search for another optimal solution in the feasible solution set of the conventional OPP problem excluding the previous solutions that do not support the communication constraint.

Therefore, our problem can be formulated as a binary integer programming optimization problem as follows:

$$\begin{aligned} &\min \sum_{i}^{n} w_i x_i \\ &\text{s.t. } f(\mathbf{x}) = \mathbf{A}\mathbf{x} \geq 1, \\ &N\alpha \leq c(S,T), \\ &\sum_{i \in I_j} x_i + \sum_{i \in n \setminus I_j} (1 - x_i) \geq 1. \end{aligned} \quad (7)$$

where I_j is a set that represents a previous unacceptable solution. I_j is the set of indices, i's, for which $x_i = 0$ from

[3] Solving the max-flow min-cut problem with fixed source(s) and destination locations can be formulated as a linear program. In our formulation, we try to find the optimal locations for the source nodes and this adds more complexity to the problem.

a previous unacceptable solution. Note that every previous unacceptable solution will require one constraint of the form $\sum_{i \in I_j} x_i + \sum_{i \in n \setminus I_j} (1 - x_i) \geq 1$ to make sure that it is excluded from the feasible set in the next iteration of solving the OPP problem.

V. Formulation Analysis

In this section, we apply our two formulations on two different standard IEEE bus systems, namely, the IEEE 9-bus system and the IEEE 14-bus system by considering the worst case in which no PMU is installed at zero-injection buses, so that the number of PMUs which is needed to make the system fully observable is larger than the case where PMUs installed at zero-injection buses. This means that the flow that comes out from the super source is large and this represents the worst case we deal with to allow the communication from the PMUs to the controller unit. We show that in some cases the solutions obtained from the conventional OPP problem cannot support the max-flow min-cut restriction, and we also show that not all the buses are suitable locations to place the controller at in order to achieve this communication constraint. We denote the capacity of the power line that connects two system buses to each other by β.

A. Controller Placement and Fixed Controller Formulations for the IEEE 9-Bus System

In this subsection, we are interested in solving the two problems addressed in (6) and (7) for the case of the IEEE 9-bus system shown in Fig. 2. For the controller or sink placement formulation, we start by solving the OPP problem and finding the optimal solution that achieves the system full observability, which is found to be (4, 6, 8); therefore, our sources, or PMUs, will be placed at buses number 4, 6, and 8.

Now, we move to the second step in this approach in which we seek the best locations for our controller unit to be placed at. This step can be established by solving the max-flow min-cut problem for the 9 different buses of the smart grid system and finding the locations that will support our communication constraint.

It is found that the value of max capacity $c(S, T)$ that the network can handle is 2β if we place the sink at buses 4, 5, 6, 7, 8, or 9 and β if the sink is placed at buses 1, 2, and 3. While, the flow that comes out from the super source has a value that depends on the number of sources or PMUs that makes the system fully observable. In our case, the optimal number of sources is 3 PMUs. Finally, we apply the constraint from (5) to decide whether a certain bus is a suitable location for our controller or not. We have found that the total number of sources N and the value of α control our decision. If α = β this means that the only way to allow the communication is to fix the controller at one of the sources, but if α is in the range of 0.5β or 0.75β (which is the practical case) the results will be discussed in the next section.

For the fixed controller formulation, we solve the problem of finding the PMUs locations in order to support the full observability and communication constraints together. It is found that some of the optimal solutions from the OPP problem feasible solution set support our communication constraint, but this is based on a certain limit on the value of α, which will be discussed in the following section.

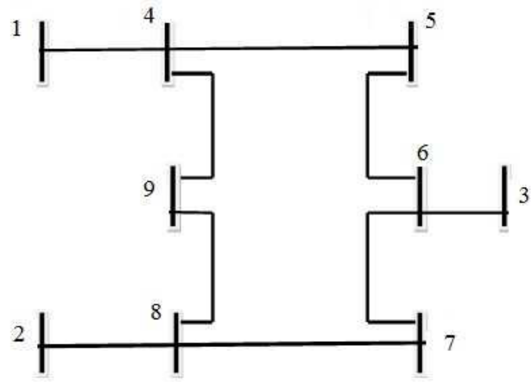

Fig. 2: The IEEE 9-Bus System.

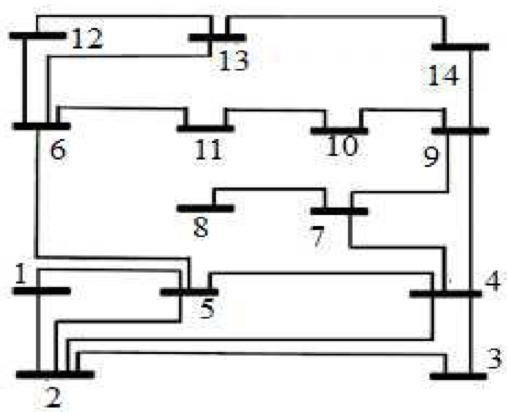

Fig. 3: The IEEE 14-Bus System.

B. Controller Placement Formulation for the IEEE 14-Bus System

For the IEEE 14-bus system shown in Fig. 3, and after solving the conventional OPP problem, it is found that the optimal solution that makes the system fully observable is to place the PMUs at buses 2,6,7, and 9. This means that the sources will be placed at buses number 2, 6 ,7, and 9. After that, the max-flow min-cut theorem is applied at all the 14 buses of the system to find the maximum capacity that the system can handle at each bus; the maximum flow at each bus is compared to the value of the flow that comes out from the PMUs to determine the best locations for the controller unit. Again, the value of α as compared to β plays an important role in determining the best locations for the controller as will be discussed in the next section.

C. Fixed Controller Formulation for the IEEE 14-Bus System

In this formulation, we fix the controller at one of the buses, and solve the conventional OPP problem to find the first optimal solution. Then, we place the PMUs in the network according to this optimal solution, and find the max-flow min-cut solution for this placement and compare it with the flow that comes out from the super source; then, this solution is checked to whether it supports the communication requirement or not. If not, then we start to find the second optimal solution to relocate the PMUs again and solve the problem again till finding the solution that achieves our communication constraint. This can be done by solving the problem addressed in (7). We resolve the problem by adding the following constraint which is mentioned before in (7)

$$\sum_{i \in I_j} x_i + \sum_{i \in n \setminus I_j} (1 - x_i) \geq 1.$$

What we actually do here is that we obtain the feasible solution set for the conventional OPP problem for the standard IEEE 14-bus system, and arrange them in an ascending order according to the number of ones in each solution, which is the number of PMUs, to make the IEEE 14-bus system fully observable. Then, the previous constraint is applied to reject the solutions that do not satisfy our communication constraint. The first summation $\sum_{i \in I_j} x_i$ is over the ones elements in some previous optimal solution, which does not support the data flow, and I_j is the set of indices of elements that equal 0, while the second summation $\sum_{i \in n \setminus I_j} (1 - x_i)$ is over the elements whose values are 1 in the same previous optimal solution. We need one more constraint for every previously rejected solution. We repeat this process till reaching a solution that satisfies the full observability and communication constraints.

VI. Simulation Results

In this section, we introduce the simulation results of our proposed strategies. These simulations are performed on the IEEE 9-, and 14-bus systems using MATLAB binary integer programming function, because as we mentioned before, our problem is a binary integer programming problem.

First, we solved the conventional OPP problem for the IEEE 9-, and 14-bus systems. We found that the feasible solution set consists of 186, and 6168 solutions for the 9-, and 14-bus systems, respectively. These solutions have been organized in an ascending order from the best to the worst solution, and it is found that there is no unique optimal solution for this problem. For the 9-bus system we have 3 optimal solution (4, 6, 8), (2, 4, 6), and (1, 6, 8), while the 14-bus system has 5 optimal solutions (2, 6, 7, 9), (2, 6, 8, 9), (2, 7, 10, 13), (2, 8, 10, 13), and (2, 7, 11, 13). This means that there is more than one solution that achieve the system full observability. Then, we applied the controller or sink placement formulation for the 9-, and 14-bus systems.

For the IEEE 9-bus system, we have chosen one of the three optimal solutions, (4, 6, 8), and fixed the sources at buses number 4, 6, and 8. Then, the max-flow min-cut theorem has been applied at each bus from the system buses and the results are given in Table. I. Note that we assume that all connections (edges) in the network have the same capacity β; therefore, the maximum flow in any cut equals the number of edges across the cut multiplied by β as given in Table I.

TABLE I: Controller placement formulation simulation results for the the IEEE 9-, and 14-bus systems

IEEE System	Controller at bus No.	Total flow out of source	No. of edges across the min cut	Network capacity
9-Bus	1	3α	1	β
	2	3α	1	β
	3	3α	1	β
	4	2α	2	2β
	5	3α	2	2β
	6	2α	2	2β
	7	3α	2	2β
	8	2α	2	2β
	9	3α	2	2β
14-Bus	1	4α	2	2β
	2	3α	3	3β
	3	4α	2	2β
	4	4α	5	5β
	5	4α	4	4β
	6	3α	3	3β
	7	3α	2	2β
	8	4α	1	β
	9	3α	4	4β
	10	4α	2	2β
	11	4α	2	2β
	12	4α	2	2β
	13	4α	3	3β
	14	4α	2	2β

From Table. I, it can be seen that the value of the total flow that comes out from the super source alternates between 2α and 3α, and this depends on whether the sink is placed at one of buses connected to a PMU or not; if the controller is placed at a bus connected to a PMU then this PMU can directly communicate to the controller and the total flow that needs to be transferred to the controller node will be 2α. If we fix the controller at one of the following buses: 5, 7, or 9, we should make sure that the value of α is less than or equal to $\frac{2}{3}\beta$ to satisfy the communication constraint; this depends on the type of the PMUs and the power lines used in the smart grid system. For controller placed at buses 1, 2, or 3, the value of α should be less than or equal $\frac{1}{3}\beta$ to allow communication from the PMUs to the controller.

Therefore, we can conclude that the best locations in the IEEE 9-bus system at which we can place the controller are the buses at which the PMUs are placed, namely, buses number 4, 6, and 8; this will minimize the total flow that comes out from the PMUs, because the value of total flow will be 2α and the network maximum capacity will be 2β at any of these buses (as given in Table I); therefore, in the worst conditions, if the power lines used in system have small capacity which approximately equals α we will still guarantee that the measurements will flow from the PMUs to the controller.

After that, we have applied the fixed controller formulation to the same IEEE 9-bus system by fixing the controller unit at one bus. We have found that it is impossible to allow the data flow if we fix the controller at buses number 1, 2, and 3, unless we have the condition $\alpha \leq \frac{1}{3}\beta$. The same case can be concluded for the buses 5, 7, and 9, but here we should have $\alpha \leq \frac{2}{3}\beta$ to allow the data flow. Therefore, α is the only parameter that controls the decision of determining which solution is the best solution for the problem, and this depends on the types of the PMUs and the power lines used in the smart grid system. If we try one of the non optimal solutions for the IEEE 9-bus system, in which the number of PMUs that makes the system fully observable is greater than 3, then the total flow that comes out from the PMUs will increase, and obviously this might not allow the data flow from the PMUs to the controller node.

For the IEEE 14-bus system, we have considered the following optimal solution: 2, 6, 7, and 9 and fixed the PMUs at these buses; then, we have considered the controller placement formulation. From Table. I, it can be shown that bus No. 4 is the best location at which we can fix the controller, as the network capacity will reach its maximum value at this bus. Buses No. 2, 5, 6, and 9 will also support the data flow, but with lower capacity than bus No. 4. The rest of the buses will not support the data flow unless we make sure that α will satisfy (5).

After that, we have considered the fixed controller formulation on the IEEE 14-bus system by fixing the controller at one of the buses that does not allow the data flow from the PMUs to the sink given the previous solution (2, 6, 7, 9), namely, buses 1, 3, 7, 8, 10, 11, 12, 13, and 14, and tried to find another solution from the feasible set in order to allow the communication for this bus. Our results show that for the buses No. 1, 3, 7, 8, 10, 11, 12, and 14 no solution in the feasible set will allow the communication; therefore, we should avoid fixing the controller unit at these buses. However, for bus No. 13 we have found that the following solution (2, 7, 10, 13), will allow the communication. Therefore, if it is desired to construct a smart grid network based on the IEEE standard 14-bus system, the controller may be fixed at one of the following buses: 2, 5, 6, 9, and 13, but the places of the PMUs will vary depending on the location of the controller. Also, the number of PMUs will vary depending on the location of the controller because of the communication constraint.

VII. Conclusion

In this paper, two new formulations for addressing the problem of optimal PMUs placement (OPP) have been introduced. A communication constraint, based on the max-flow min-cut theorem, has been included in the conventional (OPP) problem; this constraint has been formulated to guarantee that the communication between the PMUs and the controller unit can be established via the power lines. The first formulation aims at finding the best locations in the smart grid system at which we can place the controller to allow data (measurements) transfer from the PMUs to the controller unit. While, in the second formulation, we assume that the controller is placed at a fixed bus and the OPP is formulated with the extra communication constraint. We have applied our formulations to different IEEE standard bus systems, and we have concluded that not all the buses in the smart grid system are good places to locate the controller at. Our new formulations allowed the placement of the controller at locations where measurements flow to the controller is guaranteed. Also, for the fixed controller OPP problem, our formulation allowed for optimal placement of the PMUs to ensure the measurements flow.

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