

On the Role of Finite Queues in Cooperative Cognitive Radio Networks with Energy Harvesting

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Abstract—This paper studies the problem of cooperative communications in cognitive radio networks where the secondary user is equipped with finite length relaying queue as well as finite length battery queue. The major hurdle towards fully characterizing the stable throughput region stems from the sheer complexity associated with solving the two-dimensional Markov Chain (MC) model for both finite queues. Motivated by this, we relax the problem and focus on two energy constrained systems, namely, finite battery queue with infinite relay queue and finite relay queue with infinite battery queue. We characterize the stable throughput regions for the two proposed simpler systems. For each proposed system, we investigate the maximum service rate of the cognitive node subject to stability conditions. Despite the complexity of the formulated optimization problems attributed to their non-convexity, we exploit the problems' structure to transform them into linear programs. Thus, we manage to solve them efficiently using standard known linear programming solvers. Our numerical results reveal interesting insights about the role of finite data queues as well as energy limitations on the network performance, compared to baselines with unlimited energy sources and infinite data queues.

I. INTRODUCTION

One of the prominent challenges in wireless communication networks is to efficiently utilize the spectrum. The cognitive radio technology is one approach to tackle the hurdle of spectrum scarcity. In cognitive radio networks, the unlicensed users (secondary users (SUs)) are allowed to exploit the unused spectrum by the licensed users (the primary users (PUs)) to improve the utilization of the spectrum [1], [2]. Nevertheless, the spectrum occupation by the SUs is tied with a minimum quality of service guaranteed for the PUs.

Cooperative cognitive radio networks has recently attracted considerable attention [3]–[5]. [3] introduced a full cooperation protocol in a wireless multiple-access system for a system composed of N users in which each user is a source and at the same time a potential relay. [4] proposed a cooperative strategy with probabilistic relaying. In this strategy, the SU is equipped with two infinite length queues; one is for storing its own packets and the other is for relaying the PU packets. If the PU's packet is not successfully decoded by the destination, whereas it is successfully decoded by the SU, the SU admits the PU's packet with probability a . On the other hand, when the PU is sensed idle, the SU serves its own data queue with probability

b or the relaying queue with probability $1-b$. Furthermore, [5] characterized the throughput region when the relaying buffer at the secondary user has finite length. In [4], [5], it was implicitly assumed that the SU is equipped with unlimited energy supply, i.e., the SU can access the channel whenever the PU is inactive without any energy limitations. Differently from [4], [5], in this work, we study the scenario when the SU is equipped with limited energy source. Furthermore, we investigate the effects caused by the finiteness of queue lengths for both the relaying queue as well as the battery queue. It can be contemplated that the proposed system model constitutes an important step towards real systems.

Our main contribution in this paper is three-fold. First, we show the challenges of fully characterizing the stable throughput region when having finite relaying and battery queues. Second, we characterize the stable throughput region for two energy constrained systems, namely, finite battery queue with infinite relay queue and finite relay queue with infinite battery queue. Third, we formulate two optimization problems to investigate the maximum achievable throughput of the SU, subject to queue stability conditions, for the two simpler systems. Despite the complexity of the formulated optimization problems caused by their non-convexity, we exploit the problems' structure to cast them as linear programs. This, in turn, leads to efficiently solve the formulated optimization problems using standard optimization tools. Our numerical results reveal interesting insights about the effects of finite relay and energy queues as well as the energy limitations on the achievable stable throughput region.

II. SYSTEM MODEL

In this paper, we study cooperative cognitive radio network as shown in Fig. 1. The network consists of a PU and a SU transmitting their packets to a common destination d . The PU is equipped with an infinite queue (Q_p) for storing its data packets. On the other hand, the SU is equipped with an infinite queue (Q_s) for storing its data packets and a finite queue (Q_{sp}) of length $L_{sp} = N$ for storing packets overheard from the PU. The arrival processes at the data queues, Q_p and Q_s , are modeled as Bernoulli processes with means λ_p and λ_s [6], respectively, where $0 \leq \lambda_p, \lambda_s \leq 1$. The arrival processes at both users are assumed to be independent of each other, and are independent and identically distributed across

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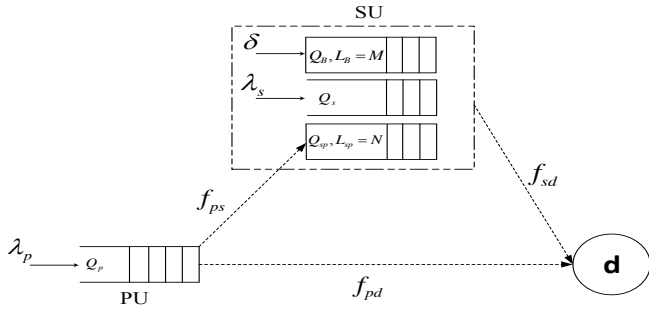


Fig. 1. System model.

time slots. It is assumed that the SU is equipped with radio frequency energy harvesting circuitry. The harvested energy from the surrounding environment is stored in a finite battery queue (Q_B) of length $L_B = M$. In addition, the harvested energy is assumed to be harvested in a certain unit and one energy unit is consumed for one transmission attempt. The energy harvesting process at the SU is modeled as a Bernoulli process with mean δ , where $0 \leq \delta \leq 1$.

Time is slotted and one slot duration is equal to one packet transmission time. It is assumed that the SU has perfect sensing. Therefore, the system is collision-free since at most one user transmits one packet each time slot. For a successful transmission, the entire transmitted packet must be received at the destination without error. In addition, the channel must not be in outage, i.e., the received signal-to-noise ratio (SNR) at the destination must not be less than a pre-specified threshold required to successfully decode the received packet. Let f_{pd} , f_{sd} and f_{ps} denote the probability of successful transmission between the PU and destination, the SU and destination, and the PU and SU, respectively. We assume that $f_{pd} < f_{sd}$ throughout the paper. This assumption characterizes the effective relaying role of the SU for the PU overheard packets in cooperative cognitive radio networks. Moreover, it is assumed that acknowledgement packets (ACKs) are sent either by the destination for successfully-decoded packets from the PU or SU, or by the SU for successfully-decoded overheard packets from the PU. These ACKs are assumed to be instantaneous, error-free and can be heard by all the nodes in the network.

The proposed channel access policy is as follows. It is assumed that the PU has the priority to transmit a packet whenever Q_p is non empty. If the packet is successfully decoded by the destination, the destination sends back an ACK heard by both users (PU and SU). Therefore, the packet is dropped from Q_p and exits the system. If the packet is not successfully decoded by the destination but successfully decoded by the SU, Q_{sp} either admits the packet with probability $a_{i,j}$ or discards it with probability $(1 - a_{i,j})$, $i = 0, \dots, M$ and $j = 0, \dots, N$. The packet admission probabilities depend on the number of packets in Q_B and Q_{sp} , i.e., $a_{i,j}$ is the admission probability when Q_B has i

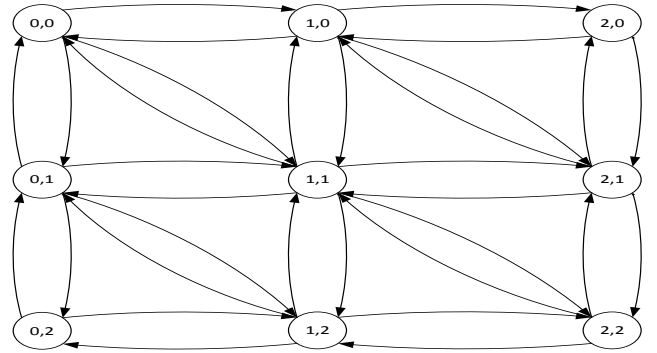


Fig. 2. Discrete time two-dimensional MC model for Q_{sp} and Q_B , where $M = N = 2$.

packets and Q_{sp} has j packets. This admission strategy, in turn, constitutes the probabilistic admission relaying policy. If the packet is buffered in Q_{sp} , the SU sends back an ACK to announce successful reception of PU's packet. Thus, the packet is dropped from Q_p and the SU becomes responsible of delivering the PU's packet to the destination. Finally, if the packet is neither successfully decoded by the destination nor decoded by the SU or decoded but not admitted to Q_{sp} , then the packet is kept at Q_p for retransmission in the next time slot.

When the PU is idle, the SU's packet transmission depends on the battery and data queues status. If the battery queue is empty, then the SU is unable to transmit a packet. On the other hand, if the battery queue is not empty, the SU either transmits a packet from Q_s with probability $b_{i,j}$ or from Q_{sp} with probability $(1 - b_{i,j})$, $i = 0, \dots, M$ and $j = 0, \dots, N$. Also, we notice that the queue selection probability depends on the number of packets in Q_B and Q_{sp} . If the destination successfully decodes the packet, it sends back an ACK heard by the SU. Therefore, the packet is dropped from either Q_s or Q_{sp} and exits the system. Otherwise, the packet is kept at its queue for later retransmission. In the next section, we characterize the stability conditions of all infinite queues in the network.

III. STABLE THROUGHPUT REGION

In this section, we characterize the stable throughput region of the proposed system model. The system is stable if all of its queues are stable. Loyens' theorem [7] provides the stability condition for an infinite size queue. The theorem states that if the queue arrival and service processes are stationary, the queue is stable if and only if the packet arrival rate is strictly less than the packet service rate. Note that Q_B and Q_{sp} are finite queues; therefore, the number of packets in each of them will never grow to infinity since it is upper bounded by M and N , respectively.

A packet leaves Q_p if it is either successfully decoded by the destination or successfully decoded by the SU and admitted to the relaying buffer (Q_{sp}). Therefore, the service rate of Q_p is given by

$$\mu_p = f_{pd} + (1 - f_{pd}) f_{ps} \sum_{i=0}^M \sum_{j=0}^N a_{i,j} \pi_{i,j}, \quad (1)$$

where $\pi_{i,j}$ denotes the steady state probability that Q_B has i packets and Q_{sp} has j packets at a given time slot, $i = 0, 1, \dots, M$ and $j = 0, 1, \dots, N$. Therefore, the stability condition for Q_p is given by

$$\lambda_p < f_{pd} + (1 - f_{pd}) f_{ps} \sum_{i=0}^M \sum_{j=0}^N a_{i,j} \pi_{i,j}. \quad (2)$$

Similarly, a packet leaves Q_s if Q_p is empty which has a probability of $1 - \frac{\lambda_p}{\mu_p}$, Q_B is not empty or empty but there is an energy packet arrival, Q_s is selected for transmission which happens with probability $b_{i,j}$ and the destination successfully decodes the packet which has a probability of f_{sd} . Thus, the stability condition for Q_s can be expressed as

$$\lambda_s < \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \left(\sum_{i=1}^M \sum_{j=0}^N b_{i,j} \pi_{i,j} + \delta \sum_{j=0}^N b_{0,j} \pi_{0,j} \right). \quad (3)$$

It is worth nothing that the service rate of packets in both queues Q_s and Q_{sp} depends on the state of the battery queue (Q_B) at the secondary user and vice versa. This dependency, in turn, leads to an interacting system of queues and complicates the characterization of the stable throughput region. In sequel, we use the Dominant System concept [8] in order to tackle this problem such that we assume that Q_s and Q_{sp} have dummy packets to transmit when they are empty and, hence, the service rate of Q_B becomes only dependent on the PU's status. This system stochastically dominates our system since the SU queues' lengths in the dominant system are always larger than that of our system if both systems start from the same initial state, have the same arrivals and encounter the same packet losses.

Now, our objective is to calculate the steady state distribution of Q_B and Q_{sp} ($\pi_{i,j}$) in order to fully characterize the stable throughput region. And, hence, be ready to investigate the optimal admission and selection probabilities ($a_{i,j}$ and $b_{i,j}$) in order to achieve the maximum SU's service rate subject to queue stability conditions. We start by showing the complexity of fully characterizing the steady state distribution, which arises from the nature of discrete time multidimensional MC with diagonal transitions. Afterwards, we relax the assumption of having both Q_B and Q_{sp} with finite lengths, and study two different settings, namely, finite battery queue with infinite relay queue and finite relay queue with infinite battery queue, with completely characterizing of their stable throughput regions.

Q_B and Q_{sp} can be modeled as a discrete time two-dimensional MC. The MC is shown in Fig. 2 where state i, j denotes that the number of packets in Q_B and Q_{sp} are i and j ,

respectively. The probability of moving from state i, j to state $i+1, j+1$ is the probability that Q_p is non empty, an energy packet arrives at Q_B , the PU's packet is not successfully decoded at the destination, the SU successfully decodes the packet and Q_{sp} admits the packet. Hence, $P_{i,j \rightarrow i+1, j+1}$ can be expressed as

$$P_{i,j \rightarrow i+1, j+1} = \frac{\lambda_p}{\mu_p} \delta (1 - f_{pd}) f_{ps} a_{i,j}. \quad (4)$$

Similarly, the probability of moving from state i, j to state $i-1, j-1$ is the probability that Q_p is empty, there is no energy packet arrival at Q_B , Q_{sp} is selected for transmission and the packet is successfully decoded at the destination. Therefore, $P_{i,j \rightarrow i-1, j-1}$ is given by

$$P_{i,j \rightarrow i-1, j-1} = \left(1 - \frac{\lambda_p}{\mu_p}\right) (1 - \delta) f_{sd} (1 - b_{i,j}). \quad (5)$$

Using the same rationale, the remaining transition probabilities can be expressed as follows

$$P_{i,j \rightarrow i+1, j} = \frac{\lambda_p}{\mu_p} \delta (f_{pd} + (1 - f_{pd}) (1 - f_{ps} a_{i,j})), \quad (6)$$

$$P_{i,j \rightarrow i, j+1} = \frac{\lambda_p}{\mu_p} (1 - \delta) (1 - f_{pd}) f_{ps} a_{i,j}, \quad (7)$$

$$P_{i,j \rightarrow i, j-1} = \left(1 - \frac{\lambda_p}{\mu_p}\right) \delta f_{sd} (1 - b_{i,j}), \quad (8)$$

$$P_{i,j \rightarrow i-1, j} = \left(1 - \frac{\lambda_p}{\mu_p}\right) (1 - \delta) (b_{i,j} + (1 - b_{i,j}) (1 - f_{sd})). \quad (9)$$

Indeed, the existence of diagonal transitions, i.e., $P_{i,j \rightarrow i-1, j-1}$ and $P_{i,j \rightarrow i+1, j+1}$, complicates the solution of the balance equations along with the normalization equation $\sum_{i=0}^M \sum_{j=0}^N \pi_{i,j} = 1$. Consequently, the product-form solution of the MC is inapplicable and there is no closed form expressions for the steady state probabilities. It is worth mentioning that various approximation techniques for solution of multidimensional MCs have been extensively used in the literature [9], e.g., equilibrium point analysis (EPA). However, these approaches depend on approximating the stationary probability distribution of the MC by a unit impulse located at a point in the state space where the system is in equilibrium. Thus, they do not fit with our objective of having closed form expressions for the steady state probabilities to be able to optimally tune the admission and selection probabilities to achieve the maximum SU's service rate subject to stability conditions. Motivated by this, we relax the assumption of both Q_B and Q_{sp} having finite lengths. More specifically, we focus on two different energy constrained systems, namely, finite battery queue with infinite relay queue and finite relay queue with infinite battery queue, in the following two subsections.

A. Finite battery queue with infinite relay queue

Under this setting, we assume that Q_B remains with finite length M , but Q_{sp} becomes an infinite queue. Note that the admission and selection probabilities (a_i and b_i for $i = 0, \dots, M$) become only dependent on the state of Q_B . It is worth noting that the stability conditions for Q_p and Q_s , given by (2) and (3), will reduce to the following expressions

$$\lambda_p < f_{pd} + (1 - f_{pd}) f_{ps} \sum_{i=0}^M a_i \pi_i^B, \quad (10)$$

$$\lambda_s < \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \left(\sum_{i=1}^M b_i \pi_i^B + \delta b_0 \pi_0^B\right), \quad (11)$$

respectively, where π_i^B is the steady state probability that Q_B has i energy packets at a given time slot. By applying Loyens' theorem, the stability condition for Q_{sp} can be derived as follows. A packet is buffered at Q_{sp} if Q_p is not empty which happens with probability $\frac{\lambda_p}{\mu_p}$. In addition, the packet is not successfully decoded by the destination which happens with probability $1 - f_{pd}$, whereas it is successfully decoded by the SU which happens with probability f_{ps} and is admitted to Q_{sp} which has a probability of a_i , $i = 0, 1, \dots, M$. Thus, λ_{sp} is given by

$$\lambda_{sp} = \frac{\lambda_p}{\mu_p} (1 - f_{pd}) f_{ps} \sum_{i=0}^M a_i \pi_i^B. \quad (12)$$

On the other hand, a packet leaves Q_{sp} if Q_p is empty which happens with probability $1 - \frac{\lambda_p}{\mu_p}$, Q_B is not empty or empty but there is an energy packet arrival, Q_{sp} is selected for transmission which happens with probability $1 - b_i$, $i = 0, 1, \dots, M$, and the packet is successfully decoded at the destination which has a probability of f_{sd} . Therefore, μ_{sp} and the stability condition for Q_{sp} are given respectively by

$$\mu_{sp} = \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \left(\sum_{i=1}^M (1 - b_i) \pi_i^B + \delta (1 - b_0) \pi_0^B\right), \quad (13)$$

$$\lambda_{sp} < \mu_{sp}. \quad (14)$$

With the purpose of fully characterizing the stability region, we shall now calculate the steady state probabilities of Q_B (π_i^B for $i = 0, \dots, M$). Q_B can be modeled as a discrete time M|M|1|M. The MC is shown in Fig. 3 where state i denotes that the number of packets in Q_B is i . Let λ_i^B and μ_i^B denote the probability of moving from state i to state $i + 1$ and the probability of moving from state i to state $i - 1$, respectively. λ_i^B is the probability that Q_p is not empty and an energy packet arrives at Q_B . On the other hand, μ_i^B is the probability that Q_p is empty and there is no an arrival energy packet. Thus, using the balance equations, the steady state probabilities of Q_B are given by

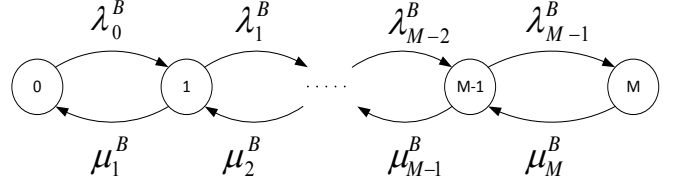


Fig. 3. Discrete time MC model for Q_B in finite battery queue with infinite relay queue system.

$$\pi_{i+1}^B = \frac{\delta \frac{\lambda_p}{\mu_p}}{\left(1 - \frac{\lambda_p}{\mu_p}\right) (1 - \delta)} \pi_i^B, \quad (15)$$

where $i = 0, 1, \dots, M - 1$. Applying the normalization condition

$$\sum_{i=0}^M \pi_i^B = 1, \quad (16)$$

along with (15), the steady state distribution of Q_B can be completely characterized.

B. Finite relay queue with infinite battery queue

Under this setting, we assume that Q_{sp} remains with finite length N , but Q_B becomes an infinite queue. Although the relay finiteness effects are studied in [5], it was implicitly assumed that the system has no energy limitation, i.e., the SU always has energy packets to transmit whenever it has the opportunity to access the channel. On the contrary, in this subsection, we focus on the more interesting practical scenario of having a limited-energy system. The energy limitation is characterized through the constraint that the energy arrival rate at Q_B is strictly less than its service rate. Thus, the number of energy packets inside Q_B will never grow to infinity and there is always a non-zero probability of having an empty Q_B .

In this scenario, the admission and selection probabilities (a_j and b_j for $j = 0, \dots, N$) become only dependent on the state of (Q_{sp}). We set $a_N = 0$ to prevent Q_{sp} from admitting any overheard PU's packet when it is full, and $b_0 = 1$ to prevent allocating any transmission time slots for Q_{sp} when it is empty. The stability conditions for Q_p and Q_s , given by (2) and (3), will reduce to the following expressions

$$\lambda_p < f_{pd} + (1 - f_{pd}) f_{ps} \sum_{j=0}^N a_j \pi_j^{sp}, \quad (17)$$

$$\lambda_s < \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \frac{\delta}{1 - \frac{\lambda_p}{\mu_p}} \sum_{j=0}^N b_j \pi_j^{sp}, \quad (18)$$

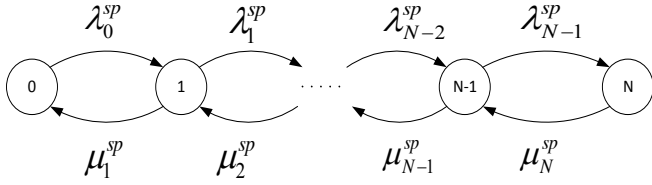


Fig. 4. Discrete time MC model for Q_{sp} in finite relay queue with infinite battery queue system.

respectively, where π_j^{sp} is the steady state probability that Q_{sp} has j packets at a given time slot. Note that the fraction $\delta / \left(1 - \frac{\lambda_p}{\mu_p}\right)$ in (18) represents the probability that Q_B is capable of supporting the transmission of the SU's packet, and it is the sum of the two probabilities: the probability of having non empty Q_B and the probability of having an empty Q_B but there is an energy packet arrival. Finally, the energy limitation constraint is given by

$$\delta \frac{\lambda_p}{\mu_p} < \left(1 - \frac{\lambda_p}{\mu_p}\right) (1 - \delta). \quad (19)$$

Next, we shall now calculate the steady state distribution of Q_{sp} . Q_{sp} can be modeled as a discrete time M|M|1|N. The MC is shown in Fig. 4 where state j denotes that the number of packets in Q_{sp} is j . Let λ_j^{sp} and μ_j^{sp} denote the probability of moving from state j to state $j+1$ and the probability of moving from state j to state $j-1$, respectively. λ_j^{sp} is the probability that Q_p is not empty, the packet is not successfully decoded by the destination, whereas it is successfully decoded by the SU and is admitted to Q_{sp} . On the other hand, μ_j^{sp} is the probability that Q_p is empty, Q_B is capable of supporting the transmission of the SU's packet as discussed in (18), Q_{sp} is selected for transmission and the packet is successfully decoded at the destination. Thus, using the balance equations, the steady state probabilities of Q_{sp} are given by

$$\pi_{j+1}^{sp} = \frac{\lambda_p f_{ps} (1 - f_{pd}) a_j}{f_{sd} \delta (1 - b_{j+1})} \pi_j^{sp}, \quad (20)$$

where $j = 0, 1, \dots, N-1$. Applying the normalization condition

$$\sum_{i=0}^N \pi_i^{sp} = 1, \quad (21)$$

along with (20), the steady state distribution of Q_{sp} can be completely characterized. In the next section, we formulate the stable throughput region optimization problems for the two systems studied in this section and discuss the solution for them.

IV. SU'S THROUGHPUT MAXIMIZATION PROBLEM

In this section, our objective is to investigate the optimal admission and selection probabilities to achieve the maximum SU's service rate for both considered energy constrained systems, namely, finite battery queue with infinite relay queue and finite relay queue with infinite battery queue. The maximum SU's service rates are investigated subject to the stable throughput regions characterized in Section III.

A. Finite battery queue with infinite relay queue

The SU's service rate maximization problem for finite battery queue with infinite relay queue system can be formulated as

$$\begin{aligned} \mathbf{P1} : \quad & \max_{a_i, b_i, \pi_i^B, \mu_p} \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \left(\sum_{i=1}^M b_i \pi_i^B + \delta b_0 \pi_0^B\right) \\ \text{s.t.} \quad & \lambda_p < \mu_p, \\ & \mu_p = f_{pd} + (1 - f_{pd}) f_{ps} \sum_{i=0}^M a_i \pi_i^B, \\ & \lambda_{sp} < \mu_{sp}, \\ & 0 \leq \pi_i^B, a_i, b_i \leq 1, \quad i = 0, \dots, M, \\ & (15), (16), \end{aligned} \quad (22)$$

where λ_{sp} and μ_{sp} are given by (12) and (13), respectively.

It is worth nothing that **P1** is a non-convex optimization problem. However, we exploit the problem structure in order to transform it into a linear program. More specifically, by defining the new variables

$$x_i = a_i \pi_i^B, y_i = b_i \pi_i^B, \quad i = 0, \dots, M, \quad (23)$$

P1 reduces into a linear program for a given μ_p as follows. First, we have the following constraints on the new defined variables

$$0 \leq x_i, y_i \leq \pi_i^B, \quad i = 0, \dots, M. \quad (24)$$

Second, we can rewrite the constraint in (14) as

$$\sum_{i=0}^M x_i < \frac{f_{sd} (\mu_p - \lambda_p)}{\lambda_p f_{ps} (1 - f_{pd})} \left(\sum_{i=1}^M (\pi_i^B - y_i) + \delta (\pi_0^B - y_0)\right). \quad (25)$$

Finally, by substituting with the new defined variables into the objective function and the remaining constraints, **P1** turns out to be a linear program for a given μ_p and can be expressed as follows

$$\begin{aligned} \mathbf{P1}^* : \quad & \max_{x_i, y_i, \pi_i^B} \left(1 - \frac{\lambda_p}{\mu_p}\right) f_{sd} \left(\sum_{i=1}^M y_i + \delta y_0\right) \\ \text{s.t.} \quad & \mu_p = f_{pd} + (1 - f_{pd}) f_{ps} \sum_{i=0}^M x_i, \\ & 0 \leq \pi_i^B \leq 1, \quad i = 0, \dots, M, \\ & (15), (16), (24), (25). \end{aligned} \quad (26)$$

From (10), the feasible values of μ_p over which the linear program runs are given by $\max(\lambda_p, f_{pd}) \leq \mu_p \leq f_{pd} + (1 - f_{pd}) f_{ps}$.

B. Finite relay queue with infinite battery queue

The SU's service rate maximization problem for finite relay queue with infinite battery queue system can be formulated as

$$\begin{aligned}
\mathbf{P2} : \quad & \max_{a_j, b_j, \pi_j^{sp}, \mu_p} \delta f_{sd} \sum_{j=0}^N b_j \pi_j^{sp} \\
\text{s.t.} \quad & \lambda_p < \mu_p, \\
& \mu_p = f_{pd} + (1 - f_{pd}) f_{ps} \sum_{j=0}^N a_j \pi_j^{sp}, \\
& \delta \frac{\lambda_p}{\mu_p} < \left(1 - \frac{\lambda_p}{\mu_p}\right) (1 - \delta), \\
& a_N = 0, b_0 = 1, \\
& 0 \leq a_j, b_j, \pi_j^{sp} \leq 1, j = 0, \dots, N, \\
& (20), (21).
\end{aligned} \tag{27}$$

By inspecting **P2**, we can easily see that it is a non-convex optimization problem. However, similar to **P1**, **P2**'s structure can be exploited to transform it into a linear program. By defining the new variables

$$x_j = a_j \pi_j^{sp}, y_j = b_j \pi_j^{sp}, j = 0, \dots, N, \tag{28}$$

P1 reduces into a linear program for a given μ_p as follows. First, we have the following constraints on the new defined variables

$$0 \leq x_j, y_j \leq \pi_j^{sp}, j = 0, \dots, N. \tag{29}$$

Second, we can rewrite the constraint in (20) as

$$\pi_{j+1}^{sp} - y_{j+1} = \frac{\lambda_p f_{ps} (1 - f_{pd})}{\mu_p f_{sd} \delta} x_j, j = 0, \dots, N - 1. \tag{30}$$

Finally, by substituting with the new defined variables into the objective function and the remaining constraints, **P2** turns out to be a linear program for a given μ_p and can be expressed as follows

$$\begin{aligned}
\mathbf{P2}^* : \quad & \max_{x_j, y_j, \pi_j^{sp}} \delta f_{sd} \sum_{j=0}^N y_j \\
\text{s.t.} \quad & \mu_p = f_{pd} + (1 - f_{pd}) f_{ps} \sum_{j=0}^N x_j, \\
& x_N = 0, y_0 = \pi_0^{sp}, \\
& 0 \leq \pi_j^{sp} \leq 1, j = 0, \dots, N, \\
& (21), (29), (30).
\end{aligned} \tag{31}$$

From (17) and (19), the feasible values of μ_p over which the linear program runs are given by

$$\max\left(\frac{\lambda_p}{1 - \delta}, f_{pd}\right) \leq \mu_p \leq f_{pd} + (1 - f_{pd}) f_{ps}. \tag{32}$$

V. NUMERICAL RESULTS

In this section, we provide numerical results showing the merits of the formulated optimization problems and the associated trade-offs. Motivated by the convexity of the proposed linear programs, we use standard optimization tools, e.g., CVX [10], to obtain the optimal solution. If not otherwise stated, we use the following parameters $f_{pd} = 0.3$, $f_{ps} = 0.4$, $f_{sd} = 0.8$ and $\delta = 0.5$. Our objective is to compare the performance of our proposed cooperative cognitive radio network systems with limited energy sources with that of the baseline cooperative cognitive radio networks with non limited energy sources introduced in [4], [5].

In Fig. 5, Our objective is to show the effect of the arrival rate of the harvested energy packets at the SU on the stable throughput region for finite battery queue with infinite relay queue system (**P1**). Towards this objective, we fix $M = 10$ and plot the stable throughput region for different values of δ ($\delta = 0.3, 0.5, 0.7$ and 0.9). It is observed that as the average arrival rate of the harvested energy packets increases, the throughput region expands. This is attributed to the fact that as the average arrival rate of harvested energy packets increases, the likelihood that Q_B is empty decreases. This, in turn, manages the SU to achieve larger service rate (μ_s) for a given arrival data packets rate of the PU (λ_p). It is further observed that the stable throughput region obtained by **P1** approaches to that of [4] with non limited energy sources as δ increases.

Fig. 6 compares the achievable stable throughput region of the baseline cooperative cognitive radio [4] with that of **P1** for different values of battery queue length ($L_B = 1, 3, 10$ and 150). It is observed that the stable throughput region obtained by **P1** expands as M increases. Increasing M results in having a lower probability for both Q_B is empty and SU's harvested energy packets losses. Therefore, the service rate of the SU increases with M . However, even for large values of Q_B length ($M = 150$), the maximum achievable service rate by **P1** is half that of the baseline system with no energy limitations for $\lambda_p = 0$. This, in turn, highlights the impact of the probabilistic arrival of energy packets at the SU ($\delta = 0.5$).

In Fig. 7, we compare the stable throughput region achieved by finite relay queue with infinite battery queue system (**P2**), for different values of δ and $N = 10$, with the baseline system with infinite length Q_{sp} and no energy limitations [4], and the system studied in [5] with finite length Q_{sp} and no energy limitations. It is observed that the practical energy limitation constraint in **P2** greatly influences the achievable throughput region, compared to the scenario of no energy limitations [5]. More specifically, from (32), the energy limitation constraint restricts **P2** feasibility to be only feasible for $\lambda_p < (1 - \delta) (f_{pd} + (1 - f_{pd}) f_{ps})$. This, in turn, leads to a trade-off between achieving high SU's service rate for small values of λ_p and wide range of λ_p values over which **P2** is feasible (as a function of δ).

Fig. 8 compares the achievable stable throughput region of **P2** with that of [4], [5] for different values of Q_{sp} length

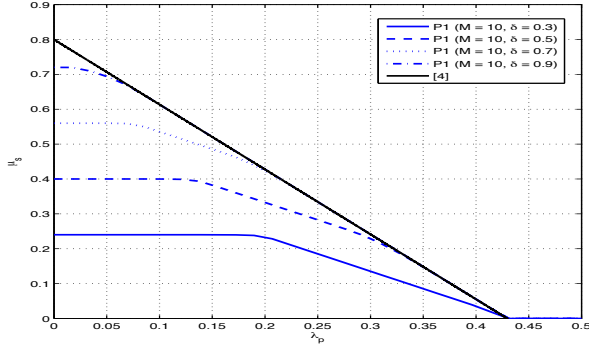


Fig. 5. The stable throughput region of finite battery queue with infinite relay queue system for different values of average arrival energy packets per time slot (δ).

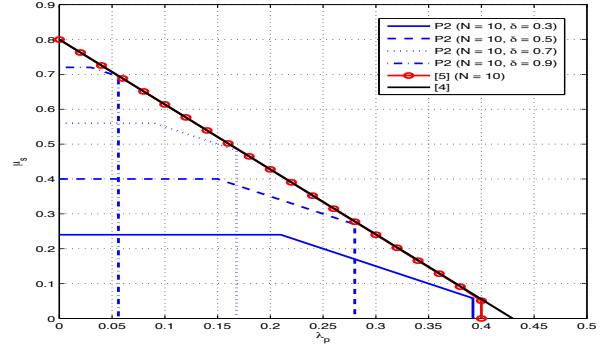


Fig. 7. The stable throughput region of finite relay queue with infinite battery queue system for different values of average arrival energy packets per time slot (δ).

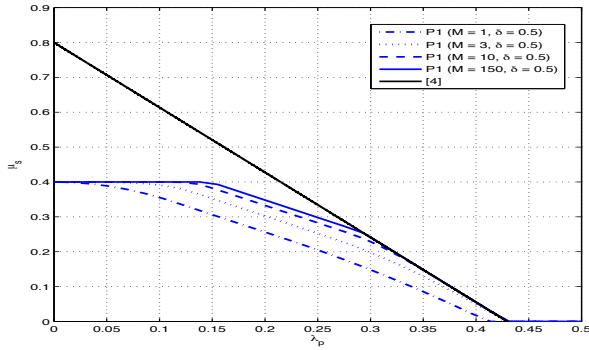


Fig. 6. The stable throughput region of finite battery queue with infinite relay queue system for different values of Q_B length.

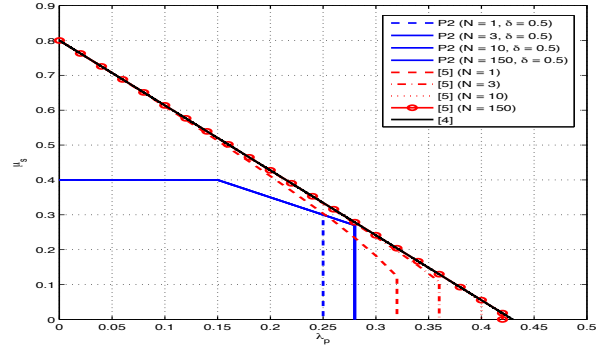


Fig. 8. The stable throughput region of finite relay queue with infinite battery queue system for different values of Q_{sp} length.

($L_{sp} = 1, 3, 10$ and 150). Despite the increase of Q_{sp} length, we observe that the stable throughput regions achieved by $N = 3$, $N = 10$ and $N = 150$ are identical. This happens due to the forced range of λ_p by the energy limitation constraint over which **P2** is feasible ($\lambda_p < 0.29$). In other words, the energy limitation constraint prevents the stable throughput region's expansion caused by increasing Q_{sp} length.

VI. CONCLUSION

In this paper, we studied cooperative cognitive radio network where the secondary user is equipped with finite length relaying queue as well as finite length battery queue. Motivated by the complexity of fully characterizing the stable throughput region, we relaxed the proposed system model and proposed two energy constrained systems. The stable throughput regions were characterized for our proposed systems. We formulated the stable throughput region optimization problem for each proposed system and showed how to solve it. Finally, we compared the achievable throughput region by each proposed system with that of the baseline system with unlimited energy sources and infinite queues.

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