Jointly Optimal Power and Rate Allocation for Layered Broadcast Over Amplify-and-Forward Relay Channels

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***Abstract*—We investigate the optimal power and rate allocation for multilayer transmission using the broadcast approach over a fading amplify-and-forward relay channel. The source uses multilayer source coding with successive refinement where the layers are transmitted using superposition coding at the source with optimal rate and power allocation. The destination applies successive interference cancellation after optimally combining the direct and relayed signals. The optimization objective is to maximize the expected user satisfaction which is usually defined by a differentiable concave increasing utility function of the total decoded rate. We propose a simple approximation for the end-to-end channel quality. This approximation is used to apply the power and rate allocation algorithm, which has a linear complexity with respect to the number of source layers. We provide many numerical examples to show the prospected gains of using the relay on the expected utility for different channel conditions.**

I. Introduction

We investigate the application of multilayer transmission using the broadcast approach [1]–[7] in the context of relay-assisted networks [8]. Our initial contribution in this topic was by considering decode-and-forward (DF) relays [9], while in this paper, we are interested in the amplify-and-forward (AF) relay scenario. We have investigated the AF scenario for the multilayer transmission recently in [10], where the rates of the source layers were fixed and predetermined. In particular, we examine the extension of the optimization framework for jointly optimal rate and power allocation presented in [7] to the relay channel case.

The design parameters that are subject to optimization in the broadcast approach are the allocated rates and power ratios of the different source layers. This is a rigorous problem that has been solved in [6], [7], where the optimization objective was defined to be a utility maximization problem. This problem formulation can fit different applications like maximizing the expected rate or minimizing the expected distortion. The interesting contribution of [6], [7] is that they provided generic algorithms to solve the optimization problem for any number of source layers and for any concave increasing utility function and for any channel statistical model that fits some conditions. Furthermore, their algorithms have linear computation complexity with respect to the number of layers. Also, the case of infinite number of layers was considered in [7] providing an upper bound for the performance. However, it was shown that this upper bound can be approached for relatively small number of layers.

We show in this paper that, unlike the DF relay case [9], the application of the algorithm presented in [7] to solve the optimization problem assuming AF relaying is feasible. This means that we can solve the joint rate and power allocation problem for any number of source layers while maintaining a linear computation complexity with respect to the number of source layers. On the other hand, in the DF case the solution was obtained using numerical random search methods and for two source layers only [9]. The extension into more than two layers in the DF relay case becomes prohibitively more complex. Notice that the expected utility function in our problem is a function of the channel statistics of the three links in the channel (i.e. source-destination, source-relay and relay-destination). So, we need to analytically characterize the end-to-end channel statistics in terms of the statistics of the three links of the channel model in order to be able to apply the algorithm presented in [7]. This is the main bottleneck in our problem. However, we propose a simple and useful approximation of the end-to-end channel quality given that all three links in our channel model are Rayleigh faded. Furthermore, we provide several numerical examples to show the joint optimal rate and power allocation for two different utility functions and to demonstrate the gains of relaying over the case when the relay is not utilized.

II. System Model and Problem Formulation

A. System Model and Transmission Scheme

We consider a system that consists of three nodes: source, destination and relay. We assume that the source signal is Gaussian and it is encoded into independent M layers, $\mathbf{L} = [L_1, L_2, \cdots, L_M]$, with rates $\mathbf{R} = [R_1, R_2, \cdots, R_M]$, and power ratios $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots \alpha_M]$ of the total source power P_s, and with each layer successively refining the information from the lower layers. Therefore, the source transmits layer

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L_i with a power $P_i = \alpha_i P_s$. The relay is half-duplex and applies amplify-and-forward strategy (AF) [8]. Therefore, the transmission is carried over two consecutive time slots of equal duration and bandwidth. The source broadcasts the layers to the relay and the destination using superposition coding in the first time slot. In the second time slot, the relay forwards the signal that was received from the source after amplifying it. The power of the relay is denoted by P_r. Notice that the power ratios of the source layers at the relay preserve the same ratios like the source node since the relay just amplifies the layers without decoding and regenerating them.

Two copies of the layers are received at the destination in the two time slots. The destination utilizes both copies in order to decode the source information up to the number of layers that can be decoded reliably based on the end-to-end instantaneous channel quality. The layers are decoded with successive interference cancellation (SIC). Thus, for the destination to decode layer L_i, it must be able to decode all "higher priority" layers first (i.e., all L_j where $j < i$).

We assume that the three nodes are equipped with a single antenna. The instantaneous Signal-to-Noise-Ratio (SNR) over the three links of the relay channel are denoted by γ_{sr}, γ_{sd} and γ_{rd} for the source-relay, source-destination, and relay-destination links, respectively. We assume that the source and the relay transmit using constant power. Furthermore, we assume that the channel gains, and consequently the SNRs, stay constant for the duration of one transmission block, which consists of two consecutive time slots. However, γ_{sr}, γ_{sd} and γ_{rd} vary from one channel block to another randomly. Furthermore, we assume that the source and the relay do not know the instantaneous values of the SNRs while transmitting.

In this work, we assume that the variation (i.e. fading) of the channels' gain is Rayleigh distributed. $\bar{\gamma}$ denotes the average SNR for the direct source-destination link and m_1 and m_2 denote the ratios between the average SNR of the source-relay and the relay-destination links to the source-destination link, respectively. We assume that $\bar{\gamma}$, m_1 and m_2 are known at the source node which utilizes its knowledge of the statistical channel qualities of the three links in the optimization of the power and rate allocation.

B. End-to-End Channel Condition

The two copies of the layers y_{sd} and y_{rd} received from the source and the relay in the two time slots, respectively, are combined at the destination using maximum ratio combining (MRC). Therefore, the "combined" signal can be given as

$$y_c = a y_{sd} + b y_{rd}, \tag{1}$$

where a and b are the combining ratios, and

$$y_{sd} = h_{sd}\Sigma_{i=1}^{M} L_i + n_{sd},\ y_{sr} = h_{sr}\Sigma_{i=1}^{M} L_i + n_{sr}, \tag{2a}$$
$$y_{rd} = h_{rd} A_r y_{sr} + n_{rd}, \tag{2b}$$

where h_{sd}, h_{sr} and h_{rd} are the independent channel gains, n_{sd}, n_{sr} and n_{rd} are the independent noise levels with variance N_o for the three links, and A_r is the amplifying gain at the relay node that is a function of the power constraint at the relay P_r. Hence,

$$A_r = \sqrt{\frac{P_r}{|h_{sr}|^2 P_s + N_o}}. \tag{3}$$

It can be shown that the signal to noise ratio of the combined signal with SIC for layer L_i can be easily written as

$$\text{SNR}_c^{(L_i)} = \frac{|a h_{sd} + b h_{rd} h_{sr} A_r|^2 \alpha_i P_s}{N_o\left(|a|^2 + |b|^2 + |b h_{rd} A_r|^2\right) + |a h_{sd} + b h_{rd} h_{sr} A_r|^2 \sum_{m>i}^{M} \alpha_m P_s}. \tag{4}$$

In order to get the MRC, we need to find the combining ratios a and b that will maximize $\text{SNR}_c^{(L_i)}$. We omit the optimization of the MRC for brevity. The resulting maximum SNR value for the layer L_i, denoted by $\text{SNR}_{\text{MRC}}^{(L_i)}$, is given by

$$\text{SNR}_{\text{MRC}}^{(L_i)} = \frac{\alpha_i}{\frac{1}{\gamma} + \sum_{m>i}^{M} \alpha_m},\ \gamma = \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1}, \tag{5}$$

where γ denotes the end-to-end SNR (i.e. the SNR at the destination after combining the direct and relayed signals optimally).

In order for the destination to decode and make use of layer L_i, it must be able to decode this layer as well as all the previous layers. Therefore, the value of γ must satisfy

$$R_j \le \frac{1}{2}\log\left(1 + \frac{\alpha_j}{\frac{1}{\gamma} + \sum_{m>j}^{M} \alpha_m}\right) \quad \forall j \le i. \tag{6}$$

This can be written as

$$\begin{aligned} \gamma \ge \bar{\gamma}_i &= \max\{\gamma_1, \gamma_2, \ldots, \gamma_i\} \\ &= \max\left\{\bar{\gamma}_{i-1}, \frac{1}{\frac{\alpha_i}{2^{2R_i}-1} - \sum_{m>i}^{M} \alpha_m}\right\}, \end{aligned} \tag{7}$$

where $\bar{\gamma}_i$, named as γ threshold, is the constraint on γ for the destination to be able to decode all the layers up to layer L_i, and γ_j is the minimum value for γ required to decode the layer L_j after correctly canceling all the previous layers, and can be written as

$$\gamma_j = \frac{1}{\frac{\alpha_j}{2^{2R_j}-1} - \sum_{m>j}^{M} \alpha_m}. \tag{8}$$

The destination only decodes the layers whose thresholds are below the instantaneous end-to-end channel condition γ.

C. Problem Formulation

Similar to [6], [7], we formulate the optimization problem as maximizing the expected user satisfaction that is defined by a utility function $U(\bar{R})$ of the total decoded rate $\bar{R}$ at the destination.

$$\max_{\alpha, R} \quad \int_0^\infty f_\gamma(\gamma) \quad U\left(\bar{R}(\gamma, \alpha, R)\right) d\gamma \tag{9a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \qquad \alpha_i \ge 0 \qquad \forall i, \tag{9b}$$

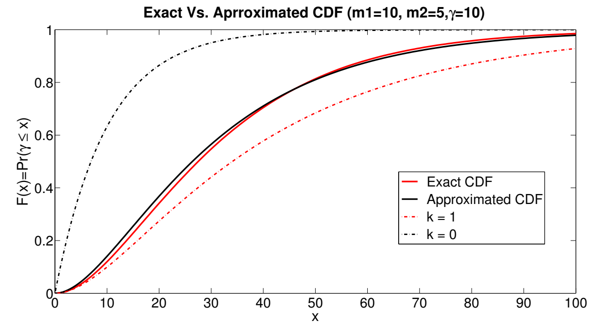

Fig. 1: The approximated CDF Vs. the true CDF for γ with $\bar{\gamma} = 10$, $m_1 = 10$, $m_2 = 5$, and $k = 0.675$.

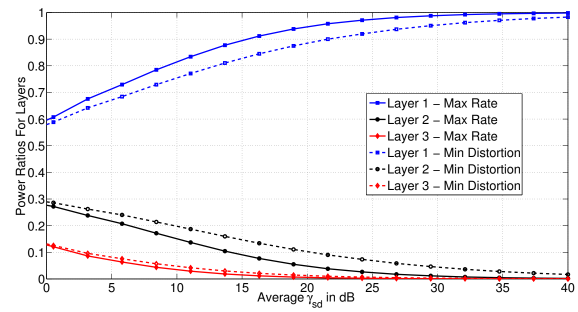

Fig. 2: The optimal power ratios of the layers versus $\bar{\gamma}$ for three layers transmitted over a Rayleigh fading AF relay channel with $(m_1, m_2) = (16, 16)$.

where $f_\gamma(\gamma)$ is the probability density function (PDF) of the end-to-end channel quality γ, $\bar{R}(\gamma, \alpha, R)$ is an indication that the total rate decoded successfully at a certain value of γ is a function of the power ratios $\alpha_i' s$ and the rates $R_i' s$ of the layers. As described in details in [6], [7], the problem in (9) can be equivalently written as

$$\max_{\alpha, R, \bar{\gamma}} \sum_{i=1}^{M} U(\bar{R}_i) \, \left(F_\gamma(\bar{\gamma}_{i+1}(\alpha, R)) - F_\gamma(\bar{\gamma}_i(\alpha, R))\right) \quad \text{(10a)}$$

$$\text{subject to} \quad \sum_{i=1}^{M} \alpha_i = 1, \qquad \alpha_i \geq 0 \qquad \forall i, \quad \text{(10b)}$$

$$\bar{\gamma}_i \geq \frac{1}{\frac{\alpha_i}{2^{2R_i}-1} - \sum_{m>i}^{M} \alpha_m} \qquad \forall i, \quad \text{(10c)}$$

$$\bar{\gamma}_M \geq \bar{\gamma}_{M-1} \geq \ldots \geq \bar{\gamma}_1 > 0, \quad \text{(10d)}$$

$$\bar{R}_M \geq \bar{R}_{M-1} \geq \ldots \geq \bar{R}_1 > 0, \quad \text{(10e)}$$

where $\bar{\gamma}_{M+1} = \infty$ for M number of layers, and $F_\gamma(\gamma)$ is the cumulative distribution function (CDF) of the end-to-end channel quality γ.

An efficient and optimal solution of this problem was described in [7]. The solution is based on a change of optimization variables step which enabled the application of a two-dimensional bisection search with linear computation complexity with respect to the total number of layers M. We can apply that algorithm to our problem as well. However, the missing step will be to obtain the PDF of the end-to-end channel quality γ. This is discussed in Section III.

III. End-To-End Channel Approximation

We aim in this Section to find the PDF (or equivalently CDF) for γ, given (5), in terms of the PDFs of γ_{sr}, γ_{sd} and γ_{rd}. The exact characterization of the CDF of γ is not straightforward. So, alternatively, we propose to use an approximation for it as follows. It can be easily shown that the value of γ can be bounded as

$$\gamma_{sd} < \gamma \leq \gamma_{sd} + \min\left(\gamma_{sr}, \gamma_{rd}\right). \quad (11)$$

So, intuitively, we can in general rewrite the definition of γ approximately as

$$\gamma \approx \gamma_{sd} + k \, \min\left(\gamma_{sr}, \gamma_{rd}\right), \quad (12)$$

where the appropriate value for k should be used $(0 < k \leq 1)$ such that the CDF of γ as defined in (12) becomes as close as possible to the exact CDF of γ as defined in (5). We have done this task for different values of $\bar{\gamma}$, m_1 and m_2 to get a close approximation for the CDF of γ (results of the best values of k are omitted due to space limitations).

Notice that for $m_1\bar{\gamma} < 0.5$ or $m_2\bar{\gamma} < 0.5$, we find that we can choose any value for $0 < k \leq 1$ and get very close approximation for γ. Therefore, we treated the case for $m_1\bar{\gamma} < 0.5$ as $m_1\bar{\gamma} = 0.5$ ($m_2\bar{\gamma} < 0.5$ as $m_2\bar{\gamma} = 0.5$).

We assume that the PDF of the channels follows an exponential distribution. Based on the proposed approximation formula, γ will be the sum of two independent exponential random variables, then we can easily write the CDF using the definition in (12) as

$$F_\gamma(\gamma) = 1 - \frac{\beta_1}{\beta_1 - \beta'} e^{-\gamma\beta'} + \frac{\beta'}{\beta_1 - \beta'} e^{-\gamma\beta_1}. \quad (13)$$

where $\beta' = \frac{\beta_2 + \beta_3}{k}$, $\beta_1 = \frac{1}{\bar{\gamma}}$, $\beta_2 = \frac{1}{m_1\bar{\gamma}}$, and $\beta_3 = \frac{1}{m_2\bar{\gamma}}$.

Figs. 1 shows the CDF for the approximated γ in (13) compared to the CDF of the exact γ as defined in (5), which is obtained numerically, for some values of $\bar{\gamma}$, m_1, and m_2. This figure demonstrates that the approximation given by (12) is appropriate.

IV. Numerical Results

Figs. 2 and 3 show the optimal power ratios and rates for a three layers case with $(m_1, m_2) = (16, 16)$, where the solid curves represent the case with the objective of maximizing the expected sum rate, and the dotted curves represent the case with the objective of minimizing the average distortion. Fig. 2 shows that the first layer for both cases of utility functions is given higher power allocation than the upper layers. This is because more protection should be given to the base layer. Also, we can see that as $\bar{\gamma}$ increases, the power ratio for the first layer increases. We can notice from Fig. 3 that Layer 1

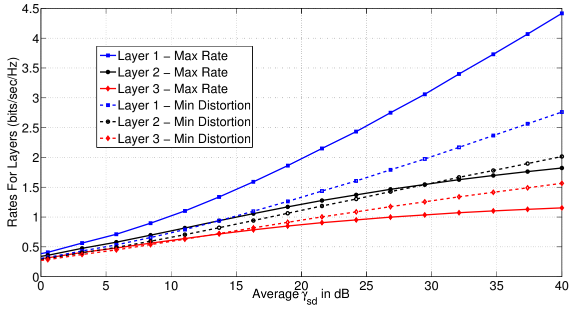

Fig. 3: The optimal rates of the layers versus $\bar{\gamma}$ for three layers transmitted over a Rayleigh fading AF relay channel with $(m_1, m_2) = (16, 16)$.

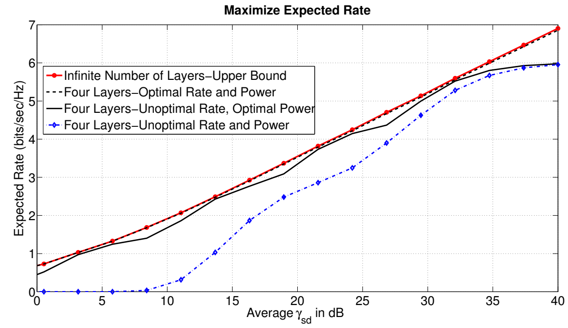

Fig. 5: The expected sum rate versus $\bar{\gamma}$ over a Rayleigh fading AF relay channel with $(m_1, m_2) = (16, 16)$ with the objective of maximizing the expected sum rate.

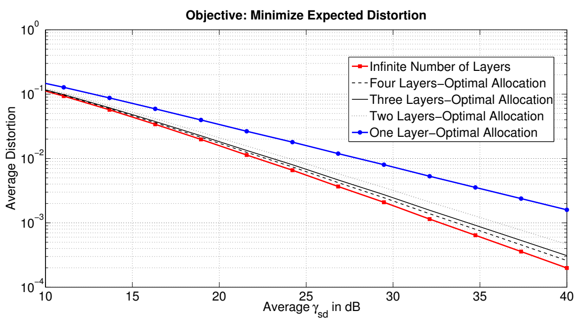

Fig. 4: The average distortion versus $\bar{\gamma}$ for various number of layers transmitted over a Rayleigh fading AF relay channel with $(m_1, m_2) = (16, 16)$ with the objective of minimizing the average distortion.

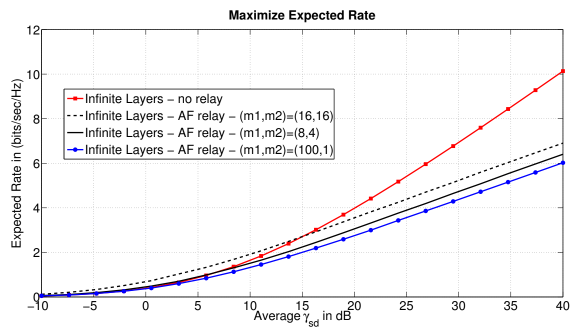

Fig. 6: The expected sum rate versus $\bar{\gamma}$ with infinite number of layers transmitted over a Rayleigh fading channel with and without using a relay with the objective of maximizing the expected sum rate.

is given lower rates, and Layer 3 is given higher rates for the case of minimizing the average distortion compared with the case of maximizing the expected sum rate.

Fig. 4 shows the effect of increasing the number of layers for the case of minimum average distortion. It is clear that we can get close to the lower bound of distortion by transmitting a relatively small number of layers.

Fig. 5 shows a comparison between the maximum expected rate with the optimal power and rate allocation, fixed power and rate allocation, and optimal power allocation with fixed sub-optimal equal rates. We can see that the jointly optimal power and rate allocation increases the expected sum rate compared with the other sub-optimal allocations.

In Fig. 6, we plot the maximum expected sum rate for different values of m_1 and m_2, which corresponds to different relay positions, and we consider also the case of direct transmission without the assistance of the relay. We can see that the maximum expected sum rate is greater than the no-relay case only for low values of $\bar{\gamma}$, and when $\bar{\gamma}$ increases above a certain level, the expected sum rate for the no-relay case will be greater than the relay-assisted channel case. This is because the gain, which is due to the enhancement in the end-to-end channel quality that is caused by using the relay, will be less than the multiplexing loss that is due to transmitting over two time slots.

V. Conclusion

In this paper, we have considered layered source coding using superposition coding at the transmitter with successive interference cancellation at the receiver. The transmission is relay-aided, and the relay applies amplify and forward strategy. The objective is to maximize the expected user satisfaction that is defined by a utility function of the total decoded rate at the destination. However, we needed to obtain the end-to-end channel statistics analytically. So, we have proposed a simple and appropriate approximation for the AF relay scenario. Several numerical examples were obtained for two different utility functions, which are maximizing the expected rate and minimizing the expected distortion of a Gaussian source. The numerical results demonstrate that relaying causes gain for the case of minimizing the expected distortion. However, it was shown that for high values of SNR and for the objective of maximizing the expected rate, the no-relay case shows better performance.

References

[1] W. Equitz and T. Cover, “Successive refinement of information,” *IEEE Transactions on Information Theory*, vol. 37, no. 3, pp. 269–275, Mar. 1991.

[2] S. Shamai, “A broadcast approach for the Gaussian slowly fading channel,” in *Proceedings IEEE International Symposium on Information Theory (ISIT)*, Ulm, Germany, Jul. 1997, p. 150.

[3] S. Shamai and A. Steiner, “A broadcast approach for a single-user slowly fading MIMO channel,” *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2617–2635, Oct. 2003.

[4] C. Tian, A. Steiner, S. Shamai, and S. Diggavi, “Successive refinement via broadcast: Optimizing expected distortion of a Gaussian source over a Gaussian fading channel,” *IEEE Transactions on Information Theory*, vol. 54, no. 7, pp. 2903–2918, Jul. 2008.

[5] C. Ng, D. Gunduz, A. Goldsmith, and E. Erkip, “Distortion minimization in Gaussian layered broadcast coding with successive refinement,” *IEEE Transactions on Information Theory*, vol. 55, no. 11, pp. 5074–5086, Nov. 2009.

[6] M. Shaqfeh, W. Mesbah, and H. Alnuweiri, “Utility maximization for layered transmission using the broadcast approach,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1228–1238, Mar. 2012.

[7] W. Mesbah, M. Shaqfeh, and H. Alnuweiri, “Jointly optimal rate and power allocation for multilayer transmission,” *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 834–845, Feb. 2014.

[8] J. N. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[9] M. A. Attia, M. Shaqfeh, K. Seddik, and H. Alnuweiri, “Power optimization for layered transmission over decode-and-forward relay channels,” in *10th International Wireless Communications and Mobile Computing Conference (IWCMC)*, Nicosia, Cyprus, Aug. 2014.

[10] ——, “Optimal power allocation for layered broadcast over amplify-and-forward relay channels,” in *2nd Global Conference on Signal and Information Processing (GlobalSIP)*, Atlanta, Georgia, USA, Dec. 2014.