Correspondence

Distributed Space-Frequency Coding for Cooperative Diversity Over Broadband Relay Channels With DF Relaying

A. Y. Al-nahari, K. G. Seddik, M. I. Dessouky, and F. E. Abd El-Samie

***Abstract*—Multipath fading is one of the main challenges in transmission over wireless broadband relay channels due to frequency selectivity, which may deteriorate the received signal. Exploiting the extra source of multipath diversity, aside from cooperative (user) diversity, is important when coping with these wireless channel limitations. In this correspondence, we propose a new distributed space-frequency code (SFC) for broadband fading relay channels that can exploit both the spatial and multipath diversities in a distributed fashion. An upper bound for pairwise error probability (PEP) is derived, and from this PEP bound, we show that the proposed code achieves a diversity of order NL, where N is the number of relay nodes, and L is the channel memory length. The decode-and-forward (DF) protocol and erroneous decoding at the relay nodes are considered. Moreover, it is shown that the proposed code structure achieves the maximum coding gain among the linearly coded systems over channels with uniform power delay profiles.**

***Index Terms*—Broadband relay channels, cooperative diversity, decode and forward (DF), space-frequency coding.**

I. Introduction

Diversity techniques are effective means for combating the detrimental effects of broadband wireless channel fading. It is well known that multiple-input–multiple-output (MIMO) systems can significantly improve the reliability of communication over fading channels using space-time coding [1]. However, it is difficult to equip small mobile units with more than one antenna with uncorrelated fading due to size and cost constraints. In such a case, transmit diversity can only be achieved through user cooperation, leading to what is known as cooperative diversity [2]–[4]. The seminal work in [4] introduced a variety of cooperative protocols using repetition coding. The most important among them are the amplify-and-forward (AF) protocol and the decode-and-forward (DF) protocol.

Distributed space-time coding (DSTC) was proposed to improve the spectral efficiency of cooperative systems [5], [6]. Most of the proposed DSTC schemes in the literature are suitable only for flat-fading channels. On the other hand, broadband relay channels suffer from severe frequency selectivity, which results in intersymbol interference (ISI). Orthogonal frequency-division multiplexing (OFDM) is considered as a solution to remove ISI. There is also another source of diversity: the multipath or frequency diversity, which can be exploited in the code design process. The first space-frequency code (SFC) designed for MIMO systems using OFDM was proposed in [7]. However, this SFC can only achieve space diversity gain, whereas the maximum diversity gain in frequency-selective MIMO channels is the product of the number of transmit antennas, the number of receive antennas, and the channel memory length [8], [9]. In [10] and [11], full-rate full-diversity SFCs were proposed. Recently, distributed space frequency coding (DSFC) schemes were proposed in [12]–[16] with DF signaling. In [12], a distributed SFC that combats synchronization errors was proposed. All relays were considered to correctly decode, which may not be always true, particularly under bad channel conditions between the source and relay nodes. In [13], a single-relay-assisted coded cooperative protocol for the OFDM multiple access system was proposed. No practical code design was proposed, and the authors proved that the protocol can achieve both cooperative and multipath diversities using outage analysis. The code in [14] was designed to achieve both spatial and multipath diversities, and the results indicated that the DF protocol gives better performance than the AF-based protocol. In [15], Wang *et al.* proposed a distributed bandwidth-efficient linear convolutive SFC with multiple frequency offsets. The code was designed for flat-fading channels.

In this correspondence, we propose a distributed SFC first introduced in [16] that achieves the maximum available spatial and multipath diversities for an arbitrary number of relay nodes and arbitrary channel memory lengths. We consider the use of the code with the DF protocol. We take into account the case of erroneous decoding at the relay nodes. We also show that the proposed code structure achieves the optimal coding gain among the linear coding schemes over channels with uniform power delay profile. Simulation results indicate that better performance is achieved by the proposed code, compared with the code proposed in [14] with no extra coding or decoding complexities.

The rest of this correspondence is organized as follows: In Section II, the system and channel models are presented. In Section III, the code constructions at both the source node and the relay nodes are presented. The performance analysis and the sufficient conditions for the proposed code to achieve full diversity are presented in Section IV. Section V presents the simulation results. Finally, Section VI gives concluding remarks.

II. System Model

Throughout this correspondence, the following notations will be adopted. Superscripts ^{T}, $^\mathcal{H}$, and * denote transpose, conjugate (Hermitian) transpose, and conjugate operations, respectively. $\|\mathbf{A}\|_F^2$ denotes the Frobenius norm of matrix $\mathbf{A}$. $\lfloor x \rfloor$ denotes the largest integer that is less than or equal to x. The notation $\mathbf{diag}\{a_1, a_2, \ldots, a_N\}$ denotes the $N \times N$ diagonal matrix with diagonal entries $a_1, a_2, \ldots, a_N$. $\det(\mathbf{A})$ and $\mathrm{rank}(\mathbf{A})$ stand for the determinant and rank of matrix $\mathbf{A}$, respectively. $\mathbf{I}_N$ is an $N \times N$ identity matrix. $|\mathcal{X}|$ denotes the cardinality of set $\mathcal{X}$.

We consider a two-hop relay network model with single-antenna nodes, as shown in Fig. 1. The channel between the source node and

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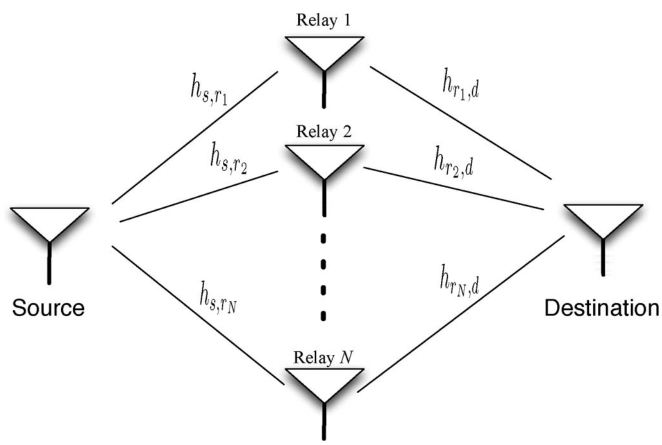

Fig. 1. System model.

the nth relay node is modeled as a multipath fading channel with L paths as

$$h_{s,r_n}(\tau) = \sum_{l=1}^{L} \alpha_{s,r_n}(l)\delta(\tau - \tau_l) \tag{1}$$

where τ_l is the delay of the lth path, $\delta(\cdot)$ is the Dirac delta function, and $\alpha_{s,r_n}(l)$ is the Rayleigh channel coefficient of the lth path modeled as zero-mean complex Gaussian random variable with variance σ_l^2 for all n, where we have assumed symmetry between the relay nodes for simplicity of presentation. The system is based on OFDM modulation with K subcarriers. From (1), the channel frequency response at the kth subcarrier between the source node and the nth relay node is given by

$$H_{s,r_n}(k) = \sum_{l=1}^{L} \alpha_{s,r_n}(l)e^{-j2\pi(k-1)\Delta f \tau_l}, \qquad k = 1, 2, \ldots, K \tag{2}$$

where $\Delta f = 1/T$ is the frequency separation between any two adjacent subcarriers, and T is the OFDM symbol duration.

The cooperative transmissions contain two phases: in the first phase, the source node broadcasts the information to the N relay nodes. The received signals in the frequency domain on the K subcarriers at the nth relay node are given by

$$\mathbf{y}_{s,r_n} = \sqrt{P_s}\mathbf{\Lambda}_{s,r_n}\mathbf{c} + \mathbf{n}_{s,r_n} \tag{3}$$

where P_s is the transmitted source node power, $\mathbf{\Lambda}_{s,r_n} = \mathbf{diag}\{H_{s,r_n}(1), H_{s,r_n}(2), \ldots, H_{s,r_n}(K)\}$, $\mathbf{c}$ is the $K \times 1$ transmitted codeword vector, and $\mathbf{n}_{s,r_n}$ denotes the additive white Gaussian noise (AWGN) on the subcarriers at the nth relay node with each element modeled as zero-mean circularly symmetric complex Gaussian random variable with variance $N_0/2$ per dimension.

In the second phase, the relays that have correctly decoded will arrange the received symbols to construct the code matrix. The transmitted $N \times K$ SFC matrix from the relay nodes is given by

$$\mathbf{C}_r = \begin{pmatrix} c(1,1) & c(1,2) & \cdots & c(1,K) \\ c(2,1) & c(2,2) & \cdots & c(2,K) \\ \vdots & \vdots & \ddots & \vdots \\ c(N,1) & c(N,2) & \cdots & c(N,K) \end{pmatrix} \tag{4}$$

where $c(n,k)$ is the symbol transmitted by the nth relay node on the kth subcarrier with the power constraint $\|\mathbf{C}_r\|_F^2 \le K$. The relays that have not correctly decoded will not transmit in the second phase, and the rows corresponding to these relays in (4) will be replaced by zeros. We assume perfect synchronization among all relay nodes [5], [14].

The received signal at the destination node on the kth subcarrier is given by

$$y_d(k) = \sqrt{P_r}\sum_{n=1}^{N} H_{r_n,d}(kc(n,k)\mathcal{I}_n + n_{r_n,d}(k) \tag{5}$$

where P_r is the relay node transmitted power, $H_{r_n,d}(k)$ is the frequency response of the channel between the nth relay node and the destination node on the kth subcarrier, $n_{r_n,d}(k)$ is the AWGN at the destination node on the kth subcarrier, and $\mathcal{I}_n$ is the state of the nth relay, depending on whether it has correctly decoded or not in the first phase. Define

$$\mathcal{I} = [\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]^T \tag{6}$$

as the state vector for the relay nodes. $\mathcal{I}_n$ is a Bernoulli random variable, which is equal to 0 or 1 with probabilities of SER and $1 - SER$, respectively, where SER denotes the symbol error rate (SER) at the nth relay node.[1]

III. Code Construction

In this section, a distributed SFC structure that achieves the maximum available diversity of order NL is proposed. Two coding stages are considered: 1) coding at the source node to achieve a diversity of order L at any relay node and 2) coding at the relay nodes to achieve a diversity of order NL at the destination node.

A. Code Construction at the Source Node

The aim of the coding at the source node is to guarantee a diversity of order L at any relay node. Let $M = \lfloor K/L \rfloor$. The vector of input source symbols $\mathbf{s}$ at the source node is of size $ML \times 1$. The elements of $\mathbf{s}$ are carved from a constellation alphabet $\mathcal{A}$ such as quadratic amplitude modulation or phase-shift keying (PSK) constellations. We propose the partitioning of the transmitted OFDM block into M subblocks, each of length L. The source symbols can be represented as $\mathbf{s} = [s_1 s_2 \ldots s_{ML}]^T = [\mathbf{m}_1^T \mathbf{m}_2^T \ldots \mathbf{m}_M^T]^T$. Then, each subblock $\mathbf{m}_i$, $i = 1, 2, \ldots, M$, is multiplied by a rotation matrix $\mathbf{\Theta}$ of dimensions $L \times L$, which will be defined in Section III-C, to produce a vector $\mathbf{q}_i = \mathbf{\Theta}\mathbf{m}_i$. Therefore, the transmitted codeword from the source node is given by

$$\mathbf{c} = \left[\mathbf{q}_1^T \mathbf{q}_2^T \cdots \mathbf{q}_M^T \mathbf{0}_{(K-ML)\times 1}^T\right]^T \tag{7}$$

where $\mathbf{q}_i = [q_{(i-1)L+1}\, q_{(i-1)L+2} \cdots q_{(i-1)L+L}]^T$, and $\mathbf{0}_{(K-ML)\times 1}$ is an all-zero column vector of size $(K - ML) \times 1$. Zero padding is used if K is not an integer multiple of L. The codeword in (7) will be broadcasted to all relays.

B. Code Construction at the Relay Nodes

In this section, the construction of the proposed code at the relay nodes is presented. All the relay nodes that correctly decoded in the first phase will transmit their parts of the codeword matrix in the second phase. Let $G = \lfloor K/NL \rfloor$. First, the source symbols are divided into G groups of vectors $\{\mathbf{s}_g\}_{g=0}^{G-1}$ of size $NL \times 1$ each. Then, each $\mathbf{s}_g$ is left multiplied by a rotation matrix $\mathbf{\Theta}$ of dimensions $NL \times NL$ to produce a vector $\mathbf{v}_g = \mathbf{\Theta}\mathbf{s}_g$ of dimensions $NL \times 1$.

[1]SER at any relay node will be the same due to our symmetry assumption, but our results can be easily extended to the asymmetric case.

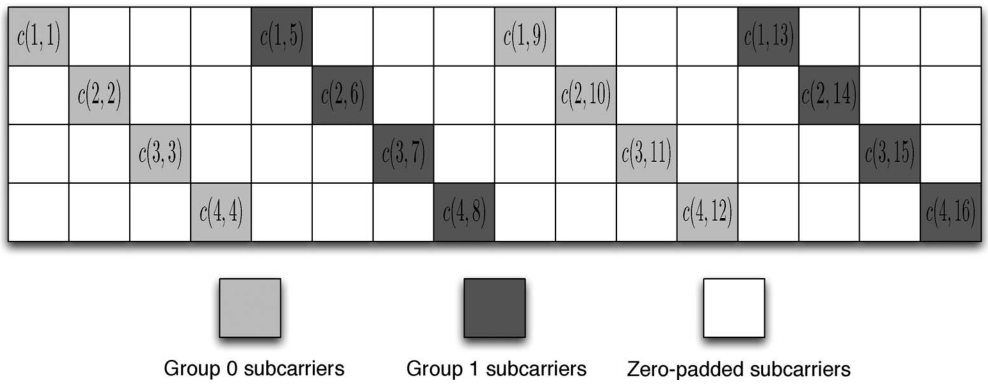

Fig. 2. Structure of code matrix $\mathbf{C}_r$ defined in (4) for a special case of two groups, four relays, two channel taps, and 16 subcarriers.

Then, the vector $\mathbf{v}_g$ is partitioned into L subvectors, each of size $N \times 1$; each subvector is used to create an $N \times N$ diagonal matrix $\boldsymbol{\mathcal{D}}_{\mathbf{s}_g,l} = \mathbf{diag}\{\boldsymbol{\theta}_{N(l-1)+1}^T \mathbf{s}_g, \ldots, \boldsymbol{\theta}_{Nl}^T \mathbf{s}_g\}$, for $l = 1, 2, \ldots, L$, where $\boldsymbol{\theta}_i^T$ denotes the ith row of $\boldsymbol{\Theta}$. These diagonal submatrices belong to the same group. Then, these submatrices are mapped, as in the example shown in Fig. 2, to constitute the final code matrix as

$$\mathbf{C}_r = \Big[\mathcal{D}_{\mathbf{s}_0,1}\mathcal{D}_{\mathbf{s}_1,1} \cdots \mathcal{D}_{\mathbf{s}_{G-1},1} \cdots \\ \mathcal{D}_{\mathbf{s}_0,L}\mathcal{D}_{\mathbf{s}_1,L} \cdots \mathcal{D}_{\mathbf{s}_{G-1},L}\mathbf{0}_{N\times(K-GNL)}\Big]. \quad (8)$$

Zero padding is used if K is not an integer multiple of NL.

C. Precoding Design

Here, we will consider the design of the rotation matrix $\boldsymbol{\Theta}$. The precoding at the relays requires mapping an NL subvector $\mathbf{s}_g$ by using a matrix $\boldsymbol{\Theta}$ of dimensions $NL \times NL$ to a new vector given by $\mathbf{v}_g = \boldsymbol{\Theta}\mathbf{s}_g$, $g = 0, 1, \ldots, G-1$. According to [17] and [18], for $NL = 2^z$, z is an integer ($z \geq 1$), and $\boldsymbol{\Theta}$ can be designed as

$$\boldsymbol{\Theta} = \frac{1}{\sqrt{NL}} \begin{pmatrix} 1 & \gamma_1 & \cdots & \gamma_1^{NL-1} \\ 1 & \gamma_2 & \cdots & \gamma_2^{NL-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \gamma_{NL} & \cdots & \gamma_{NL}^{NL-1} \end{pmatrix} \quad (9)$$

where $\gamma_t = e^{((4t-3)/(2NL))\pi}$, $t = 1, 2, \ldots, NL$. The main idea of using the rotation matrix $\boldsymbol{\Theta}$ is that the resulting rotated vectors have the following full-diversity property: For all $\mathbf{s}_g$ and $\tilde{\mathbf{s}}_g \in \mathcal{A}^{NL}$ and $\mathbf{s}_g \neq \tilde{\mathbf{s}}_g$, then $[\boldsymbol{\Theta}\mathbf{s}_g]_i \neq [\boldsymbol{\Theta}\tilde{\mathbf{s}}_g]_i$ for all $i = 1, 2, \ldots, NL$, where $[\cdot]_i$ denotes the ith element of a vector. This property means that one can uniquely detect $\mathbf{s}_g$ from its rotated vector $\mathbf{v}_g = \boldsymbol{\Theta}\mathbf{s}_g$ as long as at least one of the entries $[\mathbf{v}_g]_i$ is not severely attenuated during transmission.

In the proposed code, we exploit the following property of the matrix $\boldsymbol{\Theta}$ in the precoding at the relay nodes: for all distinct pairs $\{\mathbf{s}_g, \tilde{\mathbf{s}}_g\}$, the corresponding error vector $\mathbf{e}_g = \mathbf{v}_g - \tilde{\mathbf{v}}_g$ will have no zero elements. Therefore, if we generate $\tilde{\boldsymbol{\mathcal{D}}}_{\tilde{\mathbf{s}}_g,l}$, $l = 1, 2, \ldots, L$ from $\tilde{\mathbf{s}}_g$, then the L diagonal error matrices $\mathbf{A}_{(l-1)G+g+1} = \boldsymbol{\mathcal{D}}_{\mathbf{s}_g,l} - \tilde{\boldsymbol{\mathcal{D}}}_{\tilde{\mathbf{s}}_g,l}$, $l = 1, 2, \ldots, L$ will have all nonzero diagonal elements, and hence, they have full rank. Similarly, a matrix $\boldsymbol{\Theta}$ of dimension $L \times L$ is used for the precoding at the source node.

IV. Performance Analysis

In this section, the performance of the proposed code structure is evaluated. We adopt the design criteria presented in [8] and [9] for the MIMO systems. For the subsequent analysis, it is assumed that there is no spatial correlation between the different transmitters and channels with uniform power delay profiles.

A. Performance Analysis at the Relay Nodes

For two distinct transmitted codewords $\mathbf{c}$ and $\tilde{\mathbf{c}}$, the pairwise error probability (PEP) is upper bounded by [8]–[10]

$$\mathrm{PEP}(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) \leq \prod_{i=0}^{\mathrm{rank}(\mathbf{S})-1} \left[1 + \lambda_i(\mathbf{S})\frac{P_s}{4N_0}\right]^{-1} \quad (10)$$

where $\mathbf{S} = \boldsymbol{\Phi}(\mathbf{c}, \tilde{\mathbf{c}})\boldsymbol{\Phi}^{\mathcal{H}}(\mathbf{c}, \tilde{\mathbf{c}})$, and $\boldsymbol{\Phi}(\mathbf{c}, \tilde{\mathbf{c}}) = [(\mathbf{c} - \tilde{\mathbf{c}}) \quad \mathbf{D}(\mathbf{c} - \tilde{\mathbf{c}}) \quad \mathbf{D}^2(\mathbf{c} - \tilde{\mathbf{c}}) \cdots \mathbf{D}^{L-1}(\mathbf{c} - \tilde{\mathbf{c}})]$ is of dimension $K \times L$. $\mathbf{D} = \{\mathbf{diag}\{\exp(-j(2\pi k/K))\}_{k=0}^{K-1}\}$, and $\lambda_i(\mathbf{S})$ is the ith eigenvalue of matrix $\mathbf{S}$. $\mathbf{c}$ and $\tilde{\mathbf{c}}$ are two distinct codewords. The diversity gain that can be achieved is determined by the minimum rank of the matrix $\mathbf{S}$ over all the pairs of distinct codewords, which satisfies $\mathrm{rank}(\mathbf{S}) = \mathrm{rank}(\boldsymbol{\Phi}(\mathbf{c}, \tilde{\mathbf{c}})) = \mathrm{rank}(\boldsymbol{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}}))$. Matrix $\boldsymbol{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}})$ of dimension $L \times K$ can be written as

$$\boldsymbol{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}}) = \begin{pmatrix} (\mathbf{c} - \tilde{\mathbf{c}})^T \\ (\mathbf{c} - \tilde{\mathbf{c}})^T\mathbf{D} \\ (\mathbf{c} - \tilde{\mathbf{c}})^T\mathbf{D}^2 \\ \vdots \\ (\mathbf{c} - \tilde{\mathbf{c}})^T\mathbf{D}^{L-1} \end{pmatrix}. \quad (11)$$

We want to prove that matrix $\boldsymbol{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}})$ in (11) has full rank. For two distinct source vectors $\mathbf{s}$ and $\tilde{\mathbf{s}}$,there exists at least one vector $\mathbf{m}_i$ such that $\mathbf{m}_i \neq \tilde{\mathbf{m}}_i$ for some $i \in [1, 2, \ldots, M]$. Let this vector be $\mathbf{m}_1 \neq \tilde{\mathbf{m}}_1$. Define the difference vectors $\mathbf{b}_i = \mathbf{q}_i - \tilde{\mathbf{q}}_i$, $i = 1, 2, \ldots, M$. Note that, from the property of the matrix $\boldsymbol{\Theta}$, $\mathbf{b}_1$ has all its elements to be nonzero. Matrix $\mathbf{c} - \tilde{\mathbf{c}}$ can be written as

$$(\mathbf{c} - \tilde{\mathbf{c}})^T = \begin{bmatrix}\mathbf{b}_1^T & \mathbf{b}_2^T \cdots \mathbf{b}_M^T\end{bmatrix}. \quad (12)$$

If we divide matrix $\mathbf{D}$ into M diagonal submatrices $\{\mathbf{D}_i\}_{i=1}^M$, each of size $L \times L$, such that $\mathbf{D} = \mathbf{diag}\{\mathbf{D}_1, \mathbf{D}_2, \ldots, \mathbf{D}_M\}$, we can write

$$(\mathbf{c} - \tilde{\mathbf{c}})^T\mathbf{D}^i = \begin{bmatrix}\mathbf{b}_1^T\mathbf{D}_1^i & \mathbf{b}_2^T\mathbf{D}_2^i \cdots \mathbf{b}_M^T\mathbf{D}_M^i\end{bmatrix}. \quad (13)$$

From (12) and (13), we can rewrite (11) as

$$\boldsymbol{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}}) = \begin{pmatrix} \mathbf{b}_1^T & \mathbf{b}_2^T & \cdots & \mathbf{b}_M^T \\ \mathbf{b}_1^T\mathbf{D}_1^1 & \mathbf{b}_2^T\mathbf{D}_2^1 & \cdots & \mathbf{b}_M^T\mathbf{D}_M^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{b}_1^T\mathbf{D}_1^{L-1} & \mathbf{b}_2^T\mathbf{D}_2^{L-1} & \cdots & \mathbf{b}_M^T\mathbf{D}_M^{L-1} \end{pmatrix}. \quad (14)$$

If we consider an $L \times L$ submatrix from (14), we will have

$$\begin{pmatrix} \mathbf{b}_1^T \\ \mathbf{b}_1^T \mathbf{D}_1^1 \\ \vdots \\ \mathbf{b}_1^T \mathbf{D}_1^{L-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \psi_1 & \psi_2 & \cdots & \psi_L \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1^{L-1} & \psi_2^{L-1} & \cdots & \psi_L^{L-1} \end{pmatrix} \begin{pmatrix} b_1 & 0 & \cdots & 0 \\ 0 & b_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & b_L \end{pmatrix} \tag{15}$$

where $\mathbf{D}_1 = \{\mathbf{diag}\{\exp(-j(2\pi k/K))\}_{k=0}^{L-1}\}$, $\psi_i = \exp(-j2\pi(i-1)/K)$, $i = 1, \ldots, L$, and $\mathbf{b}_1 = [b_1\ b_2 \ldots b_L]^T$. The first matrix on the right-hand side of (15) is a Vandermonde matrix, where elements ψ_i are distinct. Therefore, this matrix is nonsingular (has nonzero determinant) [19]. In addition, $\mathbf{b}_1$ has nonzero elements; hence, the second matrix on the right-hand side of (15) is nonsingular. Therefore, the determinant of the submatrix on the left-hand side of (15) is nonzero; hence, it is of full rank L.[2] Therefore, the minimum rank of matrix $\mathbf{\Phi}^T(\mathbf{c}, \tilde{\mathbf{c}})$ is also L, because we can always find a submatrix of it that has a rank L. At high signal-to-noise ratio (SNR) values, the probability of error at any relay node can be calculated as

$$\begin{aligned} SER &= \sum_{\mathbf{c} \in \mathcal{C}} \Pr(\mathbf{c}) \Pr\{\text{error}|\mathbf{c} \text{ was transmitted}\} \\ &\leq \sum_{\mathbf{c} \in \mathcal{C}} \sum_{\tilde{\mathbf{c}} \in \mathcal{C}, \tilde{\mathbf{c}} \neq \mathbf{c}} \Pr(\mathbf{c} \to \tilde{\mathbf{c}}) \\ &\leq \sum_{\mathbf{c} \in \mathcal{C}} \sum_{\tilde{\mathbf{c}} \in \mathcal{C}, \tilde{\mathbf{c}} \neq \mathbf{c}} \prod_{i=0}^{L-1} [\lambda_i(\mathbf{S}) SNR]^{-1} \leq C\, SNR^{-L} \end{aligned} \tag{16}$$

where $\mathcal{C}$ is the set of all possible codewords, and C is a constant that does not depend on the SNR defined as $SNR = P_s/N_0$ at high SNR values.

B. Performance Analysis at the Destination Node

In this section, we evaluate the performance of the proposed distributed SFC at the destination node. For two distinct source codewords $\mathbf{c}$ and $\tilde{\mathbf{c}}$, the conditional PEP for a given $\boldsymbol{\mathcal{I}}$ is upper bounded by [8]–[10]

$$\text{PEP}(\mathbf{c} \to \tilde{\mathbf{c}}|\mathcal{I}) \leq \prod_{i=0}^{\text{rank}(\mathbf{S}_r(\mathcal{I}))-1} \left[1 + \lambda_i\left(\mathbf{S}_r(\mathcal{I})\right) \frac{P_r}{4N_0}\right]^{-1} \tag{17}$$

where $\mathbf{S}_r(\boldsymbol{\mathcal{I}}) = \mathbf{G}_{\boldsymbol{\mathcal{I}}}(\mathbf{C}_r, \tilde{\mathbf{C}}_r)\ \mathbf{G}_{\boldsymbol{\mathcal{I}}}^{\mathcal{H}}\ (\mathbf{C}_r, \tilde{\mathbf{C}}_r)$, $\mathbf{C}_r$ and $\tilde{\mathbf{C}}_r$ are two possible transmitted codewords from the relay nodes, λ_i $(\mathbf{S}_r(\boldsymbol{\mathcal{I}})$ is the ith eigenvalue of matrix $\mathbf{S}_r(\boldsymbol{\mathcal{I}})$, and $\mathbf{G}_{\boldsymbol{\mathcal{I}}}(\mathbf{C}_r, \tilde{\mathbf{C}}_r) = [(\mathbf{C}_r - \tilde{\mathbf{C}}_r)^T \mathbf{diag}(\boldsymbol{\mathcal{I}}) \quad \mathbf{D}(\mathbf{C}_r - \tilde{\mathbf{C}}_r)^T \mathbf{diag}(\boldsymbol{\mathcal{I}}) \cdots \mathbf{D}^{L-1}(\mathbf{C}_r - \tilde{\mathbf{C}}_r)^T \mathbf{diag}(\boldsymbol{\mathcal{I}})]$. It is clear from (17) that the diversity order is given by $\text{rank}(\mathbf{S}_r(\boldsymbol{\mathcal{I}})) = \text{rank}(\mathbf{G}_{\boldsymbol{\mathcal{I}}}(\mathbf{C}_r, \tilde{\mathbf{C}}_r))$ over all the pairs $\{\mathbf{C}_r, \tilde{\mathbf{C}}_r\}$. Matrix $\mathbf{G}_{\boldsymbol{\mathcal{I}}}^T(\mathbf{C}_r, \tilde{\mathbf{C}}_r)$, of dimensions $NL \times K$, can be written as

$$\mathbf{G}_{\mathcal{I}}^T\left(\mathbf{C}_r, \tilde{\mathbf{C}}_r\right) = \begin{pmatrix} \mathbf{diag}(\mathcal{I})(\mathbf{C}_r - \tilde{\mathbf{C}}_r) \\ \mathbf{diag}(\mathcal{I})(\mathbf{C}_r - \tilde{\mathbf{C}}_r)\mathbf{D} \\ \vdots \\ \mathbf{diag}(\mathcal{I})(\mathbf{C}_r - \tilde{\mathbf{C}}_r)\mathbf{D}^{L-1} \end{pmatrix}. \tag{18}$$

For any two distinct codewords $\mathbf{C}_r$ and $\tilde{\mathbf{C}}_r$, there exists at least one vector $\mathbf{v}_g \neq \tilde{\mathbf{v}}_g$ for some g. Let, without loss of generality, $\mathbf{v}_0 \neq \tilde{\mathbf{v}}_0$. Note that each of the L matrices $\{\mathbf{A}_{(l-1)G+1}\}_{l=1}^{L}$ will be of full rank. Let us partition the diagonal matrix $\mathbf{D}$ into GL diagonal submatrices $\{\bar{\mathbf{D}}_i\}_{i=1}^{GL}$, each of dimensions $N \times N$ such that $\mathbf{D} = \mathbf{diag}\{\bar{\mathbf{D}}_1, \bar{\mathbf{D}}_2, \ldots, \bar{\mathbf{D}}_{GL}\}$. Following a similar procedure to that used to obtain (15), we can find a submatrix of dimensions $NL \times NL$ from (18) that will equal

$$\begin{pmatrix} \mathbf{I}_N & \mathbf{I}_N & \cdots & \mathbf{I}_N \\ \bar{\mathbf{D}}_1 & \bar{\mathbf{D}}_{G+1} & \cdots & \bar{\mathbf{D}}_{(L-1)G+1} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{D}}_1^{L-1} & \bar{\mathbf{D}}_{G+1}^{L-1} & \cdots & \bar{\mathbf{D}}_{(L-1)G+1}^{L-1} \end{pmatrix} \times \begin{pmatrix} \mathbf{diag}(\mathcal{I})\mathbf{A}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{diag}(\mathcal{I})\mathbf{A}_{G+1} & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \mathbf{diag}(\mathcal{I})\mathbf{A}_{(L-1)G+1} \end{pmatrix}. \tag{19}$$

The first matrix in (19) is a Vandermonde matrix composed of diagonal submatrices. It is easy to see that these diagonal submatrices are distinct, so that the determinant of the first matrix is not a zero [19, p. 29]. As before, in the case in which all the relays correctly decode, the second matrix in (19) is of full rank NL, and therefore, we can say that the minimum rank of matrix $\mathbf{G}_{\mathcal{I}}^T(\mathbf{C}_r, \tilde{\mathbf{C}}_r)$ is NL. However, the second matrix in (19) will be rank deficient if some relays have failed to correctly decode in the first phase.

Theorem 1: Under the assumptions and codes structures presented in Sections II–IV, of no spatial correlation between the different transmitters and uniform power delay profile, the proposed distributed SFC achieves a diversity of order NL.

Proof: Let $n_{\mathcal{I}}$ denotes the number of the relays that have correctly decoded in the first phase. Then, it is easy to see that the minimum rank of $\mathbf{G}_{\mathcal{I}}^T(\mathbf{C}_r, \tilde{\mathbf{C}}_r)$ will be $n_{\mathcal{I}}L$. Note that $n_{\mathcal{I}}$ is a binomial random variable. Considering (16), the destination PEP can be calculated as

$$\begin{aligned} &\text{PEP}(\mathbf{c} \to \tilde{\mathbf{c}}) \\ &\quad = \sum_{\mathcal{I}} \Pr(\mathcal{I}) \text{PEP}(\mathbf{c} \to \tilde{\mathbf{c}}|\mathcal{I}) \\ &\quad = \sum_{k=0}^{N} \Pr(n_{\mathcal{I}} = k) \sum_{\{\mathcal{I}:\ n_{\mathcal{I}} = k\}} \text{PEP}(\mathbf{c} \to \tilde{\mathbf{c}}|\mathcal{I}) \\ &\quad \leq \sum_{k=0}^{N} \binom{N}{k} (1 - SER)^k SER^{N-k} \\ &\qquad \times \sum_{\{\mathcal{I}: n_{\mathcal{I}} = k\}} \prod_{i=0}^{\text{rank}(\mathbf{S}_r(\mathcal{I}))-1} \left[1 + \lambda_i\left(\mathbf{S}_r(\mathcal{I})\right) \frac{P_r}{4N_0}\right]^{-1}. \end{aligned} \tag{20}$$

Considering the high-SNR regime, $(1 - SER)$ can be approximated by 1. Substituting from (16) into (20), we get

$$\text{PEP}(\mathbf{c} \to \tilde{\mathbf{c}}) \leq C_1 \cdot SNR^{-NL} \tag{21}$$

where C_1 is a constant that does not depend on SNR. This completes the proof. ■

C. Coding Gain

The coding gain of the proposed code when the code achieves full diversity can be evaluated from (17) when all the relay nodes correctly

[2] For the square matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, if $\mathbf{A} = \mathbf{BC}$, then $\det(\mathbf{A}) = \det(\mathbf{B}) \det(\mathbf{C})$.

decode. Therefore, from (17), based on the subcarrier grouping [20], the coding gain is given by

$$G_c^g = \min_{\forall \mathbf{C}_r^g \neq \tilde{\mathbf{C}}_r^g} \left[\prod_{i=0}^{\mathrm{rank}(\mathbf{S}_r^g)-1} \lambda_i\left(\mathbf{S}_r^g\right) \right]^{\frac{1}{NL}} \tag{22}$$

where $\mathbf{S}_r^g = \mathbf{G}_g \mathbf{G}_g^{\mathcal{H}}$ has dimensions of $NL \times NL$. $\lambda_i(\mathbf{S}_r^g)$ is the ith eigenvalue of matrix $\mathbf{S}_r^g$. $\mathbf{G}_g = [(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g)^T \; \mathbf{D}_g(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g)^T \cdots \mathbf{D}_g^{L-1}(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g)^T]$, where $\mathbf{C}_r^g$ denotes the gth group of the transmitted codeword $\mathbf{C}_r$ in (8), $\mathbf{D}_g = \mathbf{diag}\{\bar{\mathbf{D}}_{g+1} \; \bar{\mathbf{D}}_{G+g+1} \cdots \bar{\mathbf{D}}_{(L-1)G+g+1}\}$, and $\bar{\mathbf{D}}_i$ is defined in Section IV-B. The coding gain in (22) can be represented as

$$G_c^g = \min_{\forall \mathbf{C}_r^g \neq \tilde{\mathbf{C}}_r^g} \left[\det\left(\mathbf{G}_g \mathbf{G}_g^{\mathcal{H}}\right)\right]^{\frac{1}{NL}}. \tag{23}$$

Note that we have (24), shown at the bottom of the page. Using the definition of $\mathbf{A}_i$ from Section III-C, it can be shown that (25), shown at the bottom of the page, holds. Note that matrices $\mathbf{G}_g$, $\mathbf{Y}$, and $\mathbf{Z}$ are square. Therefore, we get

$$\begin{aligned} \det\left(\mathbf{G}_g \mathbf{G}_g^{\mathcal{H}}\right) &= \det\left(\mathbf{G}_g^{\mathcal{H}} \mathbf{G}_g\right) \\ &= \det(\mathbf{Y}\mathbf{Z}\mathbf{Y}^{\mathcal{H}}) \\ &= \det(\mathbf{Y}\mathbf{Y}^{\mathcal{H}}) \det(\mathbf{Z}). \end{aligned} \tag{26}$$

From (25), we have (27), shown at the bottom of the page. In obtaining the first equality of (27), we use the property $\bar{\mathbf{D}}_i^{-1} = \bar{\mathbf{D}}_i^{\mathcal{H}}$, and the final result comes from the fact that $\bar{\mathbf{D}}_{iG+g+1} = \exp(j(2\pi i/L))\bar{\mathbf{D}}_{g+1}$ and $\sum_{i=0}^{L-1} \exp(j(2\pi i l/L))$, $0 < l \leq L-1$. Therefore, $\det \mathbf{Y}\mathbf{Y}^{\mathcal{H}} = L^{NL}$. In addition, it is easy to show that $\det \mathbf{Z} = \prod_{i=1}^{NL} |\mathbf{v}_g(i) - \tilde{\mathbf{v}}_g(i)|^2$. Incorporating these results into (26), the coding gain in (23) can be obtained as

$$G_c^g = L \times \min_{\forall \mathbf{C}_r \neq \tilde{\mathbf{C}}_r} \left[\prod_{i=1}^{NL} |\mathbf{v}_g(i) - \tilde{\mathbf{v}}_g(i)|^2 \right]^{\frac{1}{NL}}. \tag{28}$$

According to [20], the coding gain brought by linear constellation precoding (LCP) is $\xi_{\mathrm{LCP}} = \min_{\forall \mathbf{C}_r \neq \tilde{\mathbf{C}}_r} \prod_{i=1}^{NL} |\mathbf{v}_g(i) - \tilde{\mathbf{v}}_g(i)|^2 = (\Delta_{\min}^2/\beta^2)^{NL}$, where $\Delta_{\min}$ is the minimum distance among constellation points and $\beta^2 = NL$ when NL is a power of two or an Euler number. Therefore, (28) can be given as

$$G_c^g = L \frac{\Delta_{\min}^2}{\beta^2} = \frac{\Delta_{\min}^2}{N}. \tag{29}$$

Interestingly, the obtained coding gain in (29) is the same as the maximum coding gain $G_{c,\max} = \det(\mathbf{R}_h)^{1/(NL)}(\Delta_{\min}^2/N)$, which is obtained in [21], where $\mathbf{R}_h$ is the channel correlation matrix. Under our assumption of no spatial correlation and uniform power delay profile, correlation matrix $\mathbf{R}_h$ is an identity matrix that is also

$$\mathbf{G}_g^{\mathcal{H}} \mathbf{G}_g = \begin{pmatrix} \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T & \cdots & \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \mathbf{D}_g^{L-1} \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T \\ \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \mathbf{D}_g^{\mathcal{H}} \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T & \cdots & \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \mathbf{D}_g^{L-2} \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T \\ \vdots & \vdots & \vdots \\ \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \left(\mathbf{D}_g^{\mathcal{H}}\right)^{L-1} \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T & \cdots & \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^* \left(\mathbf{C}_r^g - \tilde{\mathbf{C}}_r^g\right)^T \end{pmatrix} \tag{24}$$

$$\begin{aligned} \mathbf{G}_g^{\mathcal{H}} \mathbf{G}_g = & \underbrace{\begin{pmatrix} \mathbf{I}_N & \mathbf{I}_N & \cdots & \mathbf{I}_N \\ \bar{\mathbf{D}}_{g+1}^{\mathcal{H}} & \bar{\mathbf{D}}_{G+g+1}^{\mathcal{H}} & \cdots & \bar{\mathbf{D}}_{(L-1)G+g+1}^{\mathcal{H}} \\ \vdots & \vdots & \cdots & \vdots \\ \left(\bar{\mathbf{D}}_{g+1}^{\mathcal{H}}\right)^{L-1} & \left(\bar{\mathbf{D}}_{G+g+1}^{\mathcal{H}}\right)^{L-1} & \cdots & \left(\bar{\mathbf{D}}_{(L-1)G+g+1}^{\mathcal{H}}\right)^{L-1} \end{pmatrix}}_{\mathbf{Y}} \\ & \times \underbrace{\begin{pmatrix} \mathbf{A}_{g+1}^* \mathbf{A}_{g+1}^T & & \\ & \ddots & \\ & & \mathbf{A}_{(L-1)G+g+1}^* \mathbf{A}_{(L-1)G+g+1}^T \end{pmatrix}}_{\mathbf{Z}} \underbrace{\begin{pmatrix} \mathbf{I}_N & \bar{\mathbf{D}}_{g+1} & \cdots & \bar{\mathbf{D}}_{g+1}^{L-1} \\ \mathbf{I}_N & \bar{\mathbf{D}}_{G+g+1} & \cdots & \bar{\mathbf{D}}_{G+g+1}^{L-1} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{I}_N & \bar{\mathbf{D}}_{(L-1)G+g+1} & \cdots & \bar{\mathbf{D}}_{(L-1)G+g+1}^{L-1} \end{pmatrix}}_{\mathbf{Y}^{\mathcal{H}}} \end{aligned} \tag{25}$$

$$\mathbf{Y}\mathbf{Y}^{\mathcal{H}} = \begin{pmatrix} L\mathbf{I}_N & \sum_{i=0}^{L-1} \bar{\mathbf{D}}_{iG+g+1} & \cdots & \sum_{i=0}^{L-1} \bar{\mathbf{D}}_{iG+g+1}^{L-1} \\ \sum_{i=0}^{L-1} \bar{\mathbf{D}}_{iG+g+1}^{-1} & L\mathbf{I}_N & \cdots & \sum_{i=0}^{L-1} \bar{\mathbf{D}}_{iG+g+1}^{L-2} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{L-1} \bar{\mathbf{D}}_{iG+g+1}^{-(L-1)} & \cdots & & L\mathbf{I}_N \end{pmatrix} = L \times \mathbf{I}_{NL}. \tag{27}$$

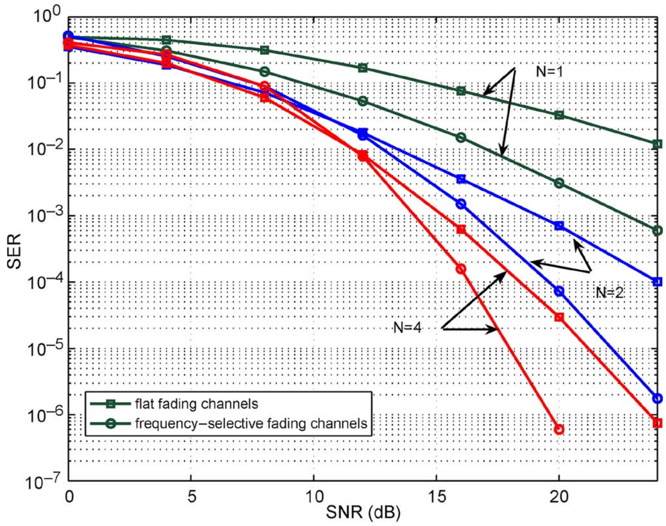

Fig. 3. SER versus SNR over both frequency-selective fading channels and flat-fading channels with one, two, and four relays. BPSK modulation is used. The delay vector [0, 0.5] μs is used for both the source-to-relay and relay-to-destination channels.

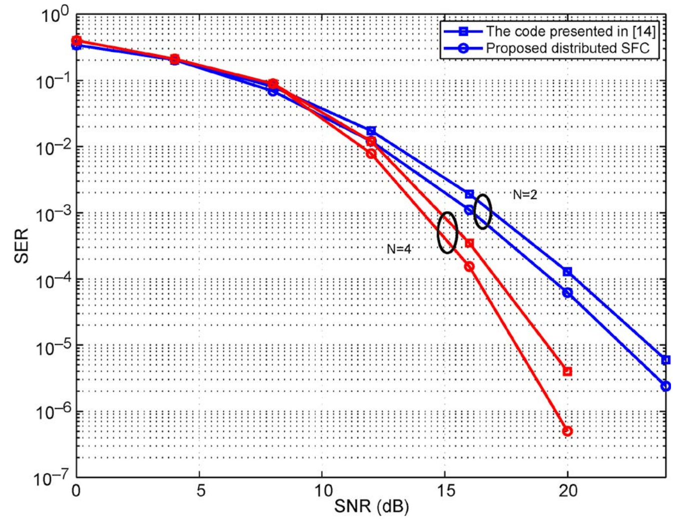

Fig. 4. Performance comparison between the proposed distributed SFC and the code proposed in [14] with different numbers of relay nodes and BPSK modulation.

the assumption considered in [8]–[10] for simplicity of presentation. Therefore, our proposed code structure achieves the maximum coding gain among the linearly precoded systems.

V. Simulation Results

In this section, we test the effectiveness of the proposed code via computer simulations. The simulated OFDM system has 64 subcarriers, and the system bandwidth is 20 MHz with a sampling time of 5×10^{-8} s. The carrier frequency spacing Δf is 312.5 kHz. The power of the system is equally divided between the source node and all the relay nodes. The SNR used in the simulations is defined as $SNR = (P_s + P_r)/N_0$, where $P_s = P_r$. Binary PSK (BPSK) modulation is used. A two-ray channel model is considered in the simulation. The two rays are of equal power, and the two taps have a total power of 1. The path delay vector is [0, 0.5] μs for all the source–relay $(S \to R_i)$ and the relay–destination $(R_i \to D)$ channels. We have assumed that the relays are able to detect the errors. This can be achieved through the use of error detecting codes such as the cyclic redundancy check codes [22], as also assumed in [14] and [23]. The relay nodes are assumed to be located at the midpoint between the source and the destination.

First, we simulated the cases of one, two, and four relays. Fig. 3 shows the variation of the SER with SNR for the proposed code. Note that both channels ($S \to R_i$ and $R_i \to D$) are either frequency selective or flat fading. It can be seen that the performance over the frequency-selective channel is better than that over the flat-fading channel. The best diversity is obtained when $N = 4$, which achieves a diversity of $4 \times 2 = 8$. It can also be seen from Fig. 3 that the diversity gain obtained in the case of $N = 1$ in the frequency-selective channel is the same as that in the case of $N = 2$ with the flat-fading channel, because the maximum diversity order is NL, as proved in the theoretical analysis. Similarly, the slope of the curve in the case of $N = 2$ with the frequency-selective channel is the same as that for the case of $N = 4$ with the flat-fading channel, which means that both cases achieve a diversity of order 4.

Then, the performance of the proposed code has been compared with the code proposed in [14], considering the two-tap channel model. Two and four relays are considered. As shown in Fig. 4, the proposed code gives a better performance. The proposed code structure separates the subcarriers used at any relay node and results in less correlation among the channel attenuations on the subcarriers used by a certain relay node. It was proven in the previous section that the proposed code structure achieves the maximum coding gain, i.e., our subcarrier assignment is optimal. It should be noted that the coding gain of the proposed code in [14] was derived in [11], and the obtained result is not the maximum coding gain. Some methods were investigated to maximize the coding gain under the condition that the power delay profile is known at the transmitter side. This indicates that the proposed scheme outperforms that proposed in [14], as illustrated in Fig. 4.

In the proposed scheme, no error correcting codes were used in the simulation. It was shown in [24] that, by combining linear precoding with convolutional error-control coding, the diversity order becomes equal to the minimum Hamming distance of the error-control code multiplied by the precoder size at the expense of increasing the decoding complexity. Combining error-correction coding with the proposed SFC could be an interesting future research topic.

VI. Conclusion

This paper has presented a distributed SFC for broadband wireless relay networks with DF signaling at the relay nodes. It has been demonstrated through theoretical analysis and computer simulations that the proposed code can exploit both the spatial and the multipath diversities. Sufficient conditions for the proposed code to achieve the maximum diversity and coding gains have been derived. Two stages of coding have been performed at the source and relay nodes to obtain the maximum diversity of order. The performance comparison with existing distributed SFCs has demonstrated the superiority of the proposed code.

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Impact of Detection Uncertainties on the Performance of a Spectrum-Sharing Cognitive Radio With Soft Sensing

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***Abstract*—We investigate the impact of detection uncertainties in the sensing information on the power-allocation policy that achieves the maximum capacity offered by a cognitive radio (CR) in a spectrum-sharing system. It is assumed that the transmit power of the secondary user can be adjusted based on soft-sensing information pertaining to the activity of the primary user in the secondary transmission region. In particular, considering an imperfect soft-sensing mechanism at the secondary system, we obtain the optimal power transmission policy in terms of false alarm and detection probabilities and under constraints on the average interference power at the primary receiver. Furthermore, we present a quantized sensing mechanism that considers only restricted levels of the sensing observations. Finally, we illustrate our analysis through numerical results and comparisons and explore the impact of imperfect spectrum sensing information on the performance of CR systems.**

***Index Terms*—Cognitive radio (CR), ergodic capacity, false alarm and detection probabilities, resource allocation.**

I. Introduction

The reason to use sensing information in cognitive radio (CR) spectrum-sharing systems is to better adapt the transmission resources of the secondary user (SU) communications and, of course, to control the amount of interference caused to the primary system of the spectrum band [1]. Through sensing, a CR detects the portions of the spectrum that are available for the cognitive user (SU) at a specific location or time. Using a sensing detector at the secondary transmitter (ST), the SU gets the ability to optimize its transmission power to maximize the channel capacity, while adhering to the interference limitations set by the primary user (PU).

It is important to note that if the SU fails to detect the PU's activity in the spectrum, harmful interference might occur. To prevent this, two issues must be considered. First, the SU must control its transmit power such that a relatively low amount of interference affects the primary's communication [2], [3]. This can be addressed by implementing a power transmission policy that adaptively changes the transmission parameters based on the soft-sensing information (SSI) about the PU's activity in the shared spectrum band, as studied in [1] and [4]. Second, the detection mechanism must be able to determine the activity of the PU with sufficient certitude. In this regard, appropriate parameters need to be set, such as the number of sensing samples.

In general, the performance of detection techniques is investigated in terms of the probability of detection and probability of false alarm [5]. Details about the performance of different detection techniques is available in the open literature (see, e.g., [6] and [7] and references therein). Hence, in this paper, we consider that the said estimation about the PU's activity, which is calculated at the sensing detector,

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