Multi-Resolution Multicasting Using Grassmannian Codes and Space Shift Keying

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***Abstract*—In this paper, we develop a novel layered coding scheme for the multiple-input multiple-output multicast channel. In this scheme information is encoded in two layers, a base low-resolution (LR) layer and refining high-resolution (HR) one. The LR layer is encoded using Grassmannian noncoherent codes and the HR layer is encoded in the indices of the active transmitter antennas using the so-called Space Shift Keying (SSK) modulation. An efficient algorithm is proposed to optimize the HR codebook. The LR information can be detected noncoherently without invoking any channel state information (CSI), whereas the HR information must be detected coherently and thus requires accurate CSI. Hence, receivers with perfect CSI can decode both the LR and HR information, whereas those with no CSI can only decode the LR information. For receivers with accurate CSI, we propose a computationally efficient two-step detector and we show that the noncoherent detector performance is not affected by the transmission of the incremental HR information encoded in the transmit antenna indices.**

I. Introduction

The capacity of point-to-point multiple-input multiple-output (MIMO) communication systems operating in a richly scattered Rayleigh fading environment is usually manifold of its single-input single-output (SISO) counterpart [1]. However, achieving this capacity depends, among other factors, on the relative mobility of the transmitter and the receiver. For example, high mobility, which is expected to be a dominating feature of future wireless systems can render the acquisition of reliable channel estimates rather difficult. To achieve efficient communication for such systems, requires in-depth understanding of the fundamental limits of noncoherent MIMO communication systems in which neither the receiver nor the transmitter has access to channel state information (CSI). Towards that end, the structure of capacity achieving signals was derived in [2], and the actual capacity was derived in [3] for systems operating at high signal-to-noise ratios (SNRs). It was shown in [3] that achieving this capacity is equivalent to packing spheres on the compact Grassmann manifold. This finding instigated several attempts to design Grassmannian codes to facilitate efficient communication over the noncoherent MIMO channel, e.g., [4]–[7].

Apart from Grassmannian signalling, a coherent MIMO communication scheme called Spatial Modulation (SM) was proposed in [8] to take advantage of the richly scattered propagation environment to convey information. The philosophy of SM is to encode information in the amplitude and phase of the transmitted symbol, and the particular index of the antenna used at the transmitter. Later in [9], a scheme known as *Space Shift Keying* (SSK) was proposed to use the index of the active antenna as the only means of transmitting information. The performance of SSK was shown to be close to that of SM, but requires less computational complexity. SSK was generalized in [10] to relax the requirements on the number of antennas used at the transmitter. In generalized SSK (GSSK), only M_A out of M antennas are used for transmission, yielding $\binom{M}{M_A}$ possible combinations, i.e. constellation points. Although SSK and GSSK utilize multiple transmit antennas, they do not provide transmit diversity [10] unless combined with space-time block codes (STBCs) [11].

In this paper, we propose a novel encoding scheme for the multi-resolution multicast channel [12]. Unlike previous works, e.g., [11], [13]–[15], which combined GSSK and STBCs in a single layer to increase the spectral efficiency, the objective of the proposed scheme is to multicast information encoded in two layers for two distinct classes of receivers. Different from layered architectures based on signal-to-noise ratio [16], [17], we characterize receivers by their ability to acquire reliable CSI. The first class comprises receivers that are incapable of obtaining reliable CSI due to their channel conditions, e.g. high mobility, or hardware constraints, e.g. IoT receivers, [18] whereas the second class comprises receivers that have access to perfect CSI. We combine noncoherent Grassmannian codes with GSSK to encode information in two layers: a basic low-resolution (LR) layer which can be detected noncoherently, and thus available to both classes of receivers, and an incremental high-resolution (HR) layer which must be detected coherently and thereby only available to receivers with accurate CSI. The LR information is encoded in the subspace spanned by the transmitted Grassmannian codeword matrix, whereas the HR information is encoded in the indices of the antennas used for transmission during the signaling interval. This type of multi-layer multicasting can find application in mobile TVs, where different receivers are able to encode multimedia streams with different rates/qualities depending on their channel conditions.

Our contributions can be summarized as follows. Different from [12], which combined Grassmannian signaling with unitary codes, we propose a two-layer scheme that combines Grassmannian signaling with SSK to enable simultaneous transmission of HR coherent and LR noncoherent information. We propose an efficient algorithm to optimize the HR codebook that decouples the original problem into smaller, more manageable problems without compromising performance. We

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also provide the theoretical justification of the performance advantage of the proposed construction over [12]. We show that the transmission of HR information is transparent to the transmission of the LR information and requires no additional power. Furthermore, we propose a computationally-efficient two-step detector that is significantly less complex than exhaustive-search. Finally, we present simulations results to corroborate our claims.

II. System Model

We consider a MIMO multicast system. The transmitter has M antennas of which only M_A are active at any time, and the i-th receiver has N_i antennas. The channel is assumed to be a quasi-static Rayleigh flat fading, and the noise is additive white Gaussian. The system can be modelled as:

$$\mathbf{Y}_i = \mathbf{U}\mathbf{A}\mathbf{H}_i + \sqrt{\frac{M_A}{\rho T}}\mathbf{W}_i, \quad i \in \mathcal{N}_C \cup \mathcal{N}_{NC}, \tag{1}$$

where $\mathbf{Y}_i$ is the $T \times N_i$ received matrix at the i-th receiver. The transmitted matrix $\mathbf{X} = \mathbf{U}\mathbf{A}$ is a $T \times M$ matrix, where $\mathbf{U}$ is the $T \times M_A$ matrix containing the LR information and $\mathbf{A}$ is the $M_A \times M$ antenna selection matrix containing the HR information. The matrix $\mathbf{H}_i$ represents the $M \times N_i$ channel matrix between the transmitter and the i-th receiver and $\mathbf{W}_i$ denotes the $T \times N_i$ noise matrix at the i-th receiver. The sets $\mathcal{N}_C$ and $\mathcal{N}_{NC}$ denote the set of coherent and noncoherent receivers, respectively. The entries of the channel and noise matrices are independent, and identically distributed, circularly symmetric, complex Gaussian random variables with zero means and unit variances. The channel matrix entries are assumed to remain constant for the transmission duration, T, and then change independently to a new realization. Throughout the rest of the paper, it is assumed that $N_i \geq M_A$, $\forall i$. For notational convenience the receiver index i will be dropped.

We consider two classes of receivers: 1) those that have perfect CSI, and are thus able to perform coherent detection to retrieve both the LR information encoded in $\mathbf{U}$ and the incremental HR information encoded in $\mathbf{A}$; and 2) those that do not have any CSI and can only perform noncoherent detection to retrieve the LR information in $\mathbf{U}$.

The LR information is encoded in the subspace spanned by the matrix $\mathbf{U}$ which represents a single point on the Grassmann manifold, whereas the incremental HR information is encoded in the indices of the M_A active antennas used to transmit the matrix $\mathbf{U}$. The rows of the antenna selection matrix $\mathbf{A}$ are phase shifted unit vectors, as will be described later, specifying which antennas are active during the signaling period T and ensuring maximal separation between transmitted matrices. The construction of the matrices $\mathbf{U}$, $\mathbf{A}$ and the role of phase shifting will be discussed in Section III.

It can be readily verified that encoding the HR information in the selection matrix $\mathbf{A}$ is completely "transparent" to the noncoherent layer. In particular, we can write the $M_A \times N_i$ equivalent channel matrix "seen" by the LR codeword $\mathbf{U}$ as $\mathbf{H}_{eq} = \mathbf{A}\mathbf{H}_i$, and for any realization of $\mathbf{A}$, the statistics of the equivalent channel matrix remains the same, as if no spatial modulation has been applied.

Remark 1: In [12], the equivalent channel matrix $\mathbf{H}'_{eq} = \mathbf{Q}\mathbf{H}$, where $\mathbf{Q}$ is a square unitary matrix that is used for encoding incremental information . Although both $\mathbf{H}'_{eq}$ in [12] and $\mathbf{H}_{eq}$ herein have the same statistical Gaussian distribution, the error performance of the system proposed herein is significantly superior to that in [12]. This is because decoding $\mathbf{Q}$ relies solely on the rotation of the channel matrix, whereas decoding $\mathbf{H}_i$ relies on a completely different realization of the channel matrix. □

III. Code Structure

Let the sets $\mathcal{U}$, $\mathcal{A}$ and $\mathcal{X}$ denote the LR constellation, the HR constellation, and the composite constellation, respectively. The transmitted matrix $\mathbf{X} \in \mathcal{X}$ is the product of the LR encoding matrix $\mathbf{U} \in \mathcal{U}$, which represents a point on the Grassmann manifold, and the HR encoding matrix $\mathbf{A} \in \mathcal{A}$, which represents the antenna selection operation. The construction of LR and HR constellations is discussed next.

A. LR Layer (noncoherent) Code Construction

To achieve the high SNR capacity of the noncoherent layer, the matrix $\mathbf{U}$ is drawn from a constellation $\mathcal{U}$ of unitary matrices representing M_A-dimensional isotropically distributed (i.d.) subspaces of $\mathbb{C}^T$, $\mathbb{G}_{M_A}(\mathbb{C}^T)$, provided that $T \geq N_i + M_A$ and $M_A \leq \lfloor T/2 \rfloor$, $\forall i$ [3]; these conditions are assumed to be satisfied throughout. One way to generate such a constellation was provided in [6], wherein the elements of $\mathcal{U}$ were generated by solving the following program [19]:

$$\begin{aligned} &\min_{\{\mathbf{U}_k\}_{k=1}^{|\mathcal{U}|}} \max_{1 \leq i,j \leq |\mathcal{U}|} \operatorname{Tr}(\mathbf{\Sigma}_{ij}) \\ &\text{subject to} \quad \mathbf{U}_k \in \mathbb{G}_{M_A}(\mathbb{C}^T), \quad k = 1, 2, \ldots, |\mathcal{U}|, \end{aligned} \tag{2}$$

where, for any two matrices $\mathbf{U}_i$ and $\mathbf{U}_j$, $\mathbf{\Sigma}_{ij}$ is the diagonal matrix containing the singular values of $\mathbf{U}_i^H \mathbf{U}_j$ [19].

B. HR Layer (Coherent) Code Construction

Incremental HR information is encoded in the matrix $\mathbf{A}$ which represents the indices of the M_A antennas active during the signaling interval, T. Each realization of $\mathbf{A}$ represents a particular choice of M_A antennas out of the $\binom{M}{M_A}$ possible combinations. Such an $\mathbf{A}$ will take the form:

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{e}_{k_1} e^{-j\theta_{k_1}} & \mathbf{e}_{k_2} e^{-j\theta_{k_2}} & \cdots & \mathbf{e}_{k_{M_A}} e^{-j\theta_{k_{M_A}}} \end{bmatrix}^{\mathcal{H}}, \tag{3}$$

where $(\cdot)^{\mathcal{H}}$ denotes the Hermitian transpose operation and $\mathbf{e}_i$ denotes the i-th column of the $M \times M$ identity matrix $\mathbf{I}$.

For a given Grassmannian constellation, the rotation angles $\theta_{k_1}, \cdots, \theta_{k_{M_A}}$ are optimized to ensure maximal transmit diversity of M_A and to maximize the minimum Frobenius distance between any two codewords $\mathbf{X}_i$ and $\mathbf{X}_j$, denoted by $\|\mathbf{X}_i - \mathbf{X}_j\|_F$, $\mathbf{X}_i, \mathbf{X}_j \in \mathcal{X}$, $i \neq j$. The use of rotation angles to enhance diversity and error performance has been considered in [11], [13]–[15] and we will later show their impact on the performance of the proposed system. Define the minimum distance of a subset of constellation points $\mathcal{X}_i$ by

$$d_{\min}(\mathcal{X}_i) = \min_{\substack{\mathbf{X}_j, \mathbf{X}_k \in \mathcal{X}_i \\ j \neq k}} \|\mathbf{X}_j - \mathbf{X}_k\|_F. \tag{4}$$

We propose the following efficient technique to generate $\mathcal{A}$.

1) Given the total number of transmit antennas M and the number of active antennas M_A, find the cardinality of $\mathcal{A}$, i.e., $|\mathcal{A}|$, as the largest integer that is a power of 2 and at most equal to $\binom{M}{M_A}$.
2) Choose any $|\mathcal{A}|$ antenna combinations from the possible $\binom{M}{M_A}$ combinations.
3) Group all points that share an antenna into subsets, $\{\mathcal{A}_i\}_{i=1}^{M}$, such that $\mathcal{A}_i$ comprises the codewords that use the i-th antenna. The number of subsets is equal to the number of available antennas, M.
4) For each subset, $\mathcal{A}_i$, construct the corresponding composite subset $\mathcal{X}_i = \{\mathbf{U}\mathbf{A} | \mathbf{U} \in \mathcal{U}, \mathbf{A} \in \mathcal{A}_i\}$.
5) For each composite subset, $\mathcal{X}_i$, find the vector of rotation angles for the i-th antenna, $\boldsymbol{\theta}_{\text{ant. i}}$, that maximizes the minimum pairwise distance of $\mathcal{X}_i$ such that

$$\boldsymbol{\theta}^*_{\text{ant. i}} = \arg\max_{\boldsymbol{\theta}_{\text{ant. i}}} d_{\min}\left(\mathcal{X}_i\right). \tag{5}$$

Now, we have all the rotation angles we need for the entire constellation, $\mathcal{A}$.

Using the proposed construction, it can be readily verified that the combined spectral efficiency of both layers is given by $\eta = \frac{1}{T}\left(\log_2|\mathcal{A}| + \log_2|\mathcal{U}|\right)$ bits/s/Hz.[1]

Next, we provide an example to illustrate the proposed construction.

Example 1: Consider a system with a total of $M = 4$ transmit antennas of which only $M_A = 2$ can be active at any given time, and a channel coherence interval of $T = 4$. Hence, the dimension of the LR Grassmannian codewords $\mathbf{U} = [\mathbf{u}_1\, \mathbf{u}_2]$ is 4×2, and the dimension of the HR codewords $\mathbf{A}$ is 2×4. Suppose that $|\mathcal{A}| = 4$, i.e., $\mathbf{A}$ can take one of four different realizations. These realizations represent four distinct active antenna indices out of the possible $\binom{4}{2} = 6$ and convey two HR information bits per signaling interval T. The four HR codewords in $\mathcal{A}$ can be chosen as follows:

$$\mathcal{A} = \left\{ \begin{bmatrix} e^{j\theta_1} & 0 & 0 & 0 \\ 0 & e^{j\theta_2} & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & e^{j\theta_3} & 0 \\ 0 & 0 & 0 & e^{j\theta_4} \end{bmatrix}, \begin{bmatrix} 0 & e^{j\theta_5} & 0 & 0 \\ 0 & 0 & e^{j\theta_6} & 0 \end{bmatrix}, \begin{bmatrix} e^{j\theta_7} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{j\theta_8} \end{bmatrix} \right\}. \tag{6}$$

Then, we can construct the four subsets, $\{\mathcal{A}_i\}_{i=1}^{4}$, one for each antenna as

$$\begin{aligned}
\mathcal{A}_1 &= \left\{ \begin{bmatrix} e^{j\theta_1} & 0 & 0 & 0 \\ 0 & e^{j\theta_2} & 0 & 0 \end{bmatrix}, \begin{bmatrix} e^{j\theta_7} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{j\theta_8} \end{bmatrix} \right\}, \\
\mathcal{A}_2 &= \left\{ \begin{bmatrix} e^{j\theta_1} & 0 & 0 & 0 \\ 0 & e^{j\theta_2} & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & e^{j\theta_5} & 0 & 0 \\ 0 & 0 & e^{j\theta_6} & 0 \end{bmatrix} \right\}, \\
\mathcal{A}_3 &= \left\{ \begin{bmatrix} 0 & 0 & e^{j\theta_3} & 0 \\ 0 & 0 & 0 & e^{j\theta_4} \end{bmatrix}, \begin{bmatrix} 0 & e^{j\theta_5} & 0 & 0 \\ 0 & 0 & e^{j\theta_6} & 0 \end{bmatrix} \right\}, \\
\mathcal{A}_4 &= \left\{ \begin{bmatrix} 0 & 0 & e^{j\theta_3} & 0 \\ 0 & 0 & 0 & e^{j\theta_4} \end{bmatrix}, \begin{bmatrix} e^{j\theta_7} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{j\theta_8} \end{bmatrix} \right\}.
\end{aligned} \tag{7}$$

[1]Although our main goal is multi-resolution multicasting and not increasing spectral efficiency, the proposed scheme can be thought of as increasing the spectral efficiency of the non-coherent scheme by superimposing "transparent" coherent information.

Consequently, the composite subsets $\{\mathcal{X}_i\}_{i=1}^{4}$ are given by

$$\begin{aligned}
\mathcal{X}_1 = \{ &\left[\mathbf{u}_1 e^{j\theta_1} \quad \mathbf{u}_2 e^{j\theta_2} \quad \mathbf{0}_{4\times 2}\right], \\
&\left[\mathbf{u}_1 e^{j\theta_7} \quad \mathbf{0}_{4\times 2} \quad \mathbf{u}_2 e^{j\theta_8}\right] \Big| [\mathbf{u}_1 \mathbf{u}_2] \in \mathcal{U}\}, \\
\mathcal{X}_2 = \{ &\left[\mathbf{u}_1 e^{j\theta_1} \quad \mathbf{u}_2 e^{j\theta_2} \quad \mathbf{0}_{4\times 2}\right], \\
&\left[\mathbf{0}_{4\times 1}\ \mathbf{u}_1 e^{j\theta_5} \quad \mathbf{u}_2 e^{j\theta_6} \quad \mathbf{0}_{4\times 1}\right] \Big| [\mathbf{u}_1 \mathbf{u}_2] \in \mathcal{U}\}, \\
\mathcal{X}_3 = \{ &\left[\mathbf{0}_{4\times 2} \quad \mathbf{u}_1 e^{j\theta_3} \quad \mathbf{u}_2 e^{j\theta_4}\right], \\
&\left[\mathbf{0}_{4\times 1}\ \mathbf{u}_1 e^{j\theta_5} \quad \mathbf{u}_2 e^{j\theta_6} \quad \mathbf{0}_{4\times 1}\right] \Big| [\mathbf{u}_1 \mathbf{u}_2] \in \mathcal{U}\}, \\
\mathcal{X}_4 = \{ &\left[\mathbf{0}_{4\times 2} \quad \mathbf{u}_1 e^{j\theta_3} \quad \mathbf{u}_2 e^{j\theta_4}\right], \\
&\left[\mathbf{u}_1 e^{j\theta_7} \quad \mathbf{0}_{4\times 2} \quad \mathbf{u}_2 e^{j\theta_8}\right] \Big| [\mathbf{u}_1 \mathbf{u}_2] \in \mathcal{U}\}.
\end{aligned} \tag{8}$$

Hence, the rotation angles for each antenna can be readily computed from

$$\begin{aligned}
\boldsymbol{\theta}^*_{\text{ant. 1}} &= \begin{bmatrix} \theta_1 \\ \theta_7 \end{bmatrix} = \arg\max_{\boldsymbol{\theta}_{\text{ant. 1}}}\ d_{\min}\left(\mathcal{X}_1\right), \\
\boldsymbol{\theta}^*_{\text{ant. 2}} &= \begin{bmatrix} \theta_2 \\ \theta_5 \end{bmatrix} = \arg\max_{\boldsymbol{\theta}_{\text{ant. 2}}}\ d_{\min}\left(\mathcal{X}_2\right), \\
\boldsymbol{\theta}^*_{\text{ant. 3}} &= \begin{bmatrix} \theta_3 \\ \theta_6 \end{bmatrix} = \arg\max_{\boldsymbol{\theta}_{\text{ant. 3}}}\ d_{\min}\left(\mathcal{X}_3\right), \\
\boldsymbol{\theta}^*_{\text{ant. 4}} &= \begin{bmatrix} \theta_4 \\ \theta_8 \end{bmatrix} = \arg\max_{\boldsymbol{\theta}_{\text{ant. 4}}}\ d_{\min}\left(\mathcal{X}_4\right).
\end{aligned} \tag{9}$$

□

Note that the proposed construction reduces the complexity of finding the optimal rotation angles in two ways. First, it avoids computing the distances between non-overlapping codewords, since those distances are not affected by the rotation angles. Second, it decomposes the original problem into M smaller problems by decoupling rotation angles that do not need to be jointly optimized. In particular, instead of solving one optimization problem of size $M_A\,|\mathcal{A}|$, M problems of size $\frac{M_A}{M}\,|\mathcal{A}| - 1$ are solved (first angle in each subset can be discarded). This is a substantial reduction in complexity without any compromise in performance.

To gain further insight into the role of rotation angles, consider the two constellation points $\mathbf{X}_1$ and $\mathbf{X}_2$ in Example 1, where $\mathbf{X}_1 = \left[\mathbf{u}_1 e^{j\theta_1}\ \mathbf{u}_2 e^{j\theta_2}\ \mathbf{0}_{4\times 2}\right]$ and $\mathbf{X}_2 = \left[\mathbf{u}_1 e^{j\theta_7}\ \mathbf{0}_{4\times 2}\ \mathbf{u}_2 e^{j\theta_8}\right]$. Both $\mathbf{X}_1$ and $\mathbf{X}_2$ will transmit $\mathbf{u}_1$ from the first antenna. Hence, without the rotation angles, the data transmitted from antenna 1 cannot be used to differentiate between these two codewords, and this can be readily shown to incur a loss in the diversity order of the system.

IV. Detectors

In this section, we present three detectors for the two classes of receiver. First, we present the optimal noncoherent detector and show that its performance is unaffected by the HR layer code; then, we present the optimal coherent detector that decodes the LR and HR jointly. Finally, we present a suboptimal, but computationally efficient, two-step detector for coherent layer.

A. The Optimal noncoherent Detector

As discussed in Section II, the equivalent channel matrix "seen" by the noncoherent layer has standard i.i.d. complex circularly-symmetric Gaussian entries. In this case, the optimal maximum likelihood (ML) detector when the channel is unknown to the receiver is given by [2]

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} p\left(\mathbf{Y}|\mathbf{U}\right) = \arg\max_{\mathbf{U}} \mathrm{Tr}(\mathbf{Y}^{\mathcal{H}}\mathbf{U}\mathbf{U}^{\mathcal{H}}\mathbf{Y}). \quad (10)$$

Using this detector, the technique in [4] can be used to show that the pairwise error probability (PEP) between two codewords $\mathbf{U}_i$ and $\mathbf{U}_j$ can be upper bounded by

$$P\left(\mathbf{U}_i \rightarrow \mathbf{U}_j\right) \leq \frac{1}{2}\prod_{m=1}^{M_A}\left[1 + \frac{\left(\rho T/M_A\right)^2\left(1-d_m^2\right)}{4\left(1+\rho T/M_A\right)}\right]^{-N}, \quad (11)$$

where $1 \geq d_1 \geq \cdots \geq d_{M_A} \geq 0$ are the singular values of the $M_A \times M_A$ matrix $\mathbf{U}_j^{\mathcal{H}}\mathbf{U}_i$. Two observations can be drawn from (11). First, since the expressions in (10) and (11) are independent of $\mathbf{A}$, it can be concluded that the performance of the noncoherent detector is unaffected by the incremental HR information. Second, the expression in (11) implies that a diversity order of $M_A N$ is achieved by the noncoherent layer if the underlying LR constellation $\mathcal{U}$ achieves full diversity. In other words, the HR information does not compromise the diversity order of the system.

B. Optimal (Joint) One-Step Coherent Detector

The optimal detector when the channel matrix $\mathbf{H}$ is known at the receiver is the minimum distance detector given by:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}\in\mathcal{X}} \|\mathbf{Y} - \mathbf{X}\mathbf{H}\|_F^2. \quad (12)$$

The pairwise error probability (PEP) of this detector can be found to be upper bounded by [1]

$$P\left(\mathbf{X}_i \rightarrow \mathbf{X}_j\right) \leq \frac{1}{2}\prod_{m=1}^{M_A}\left[1 + \frac{\rho T}{4M_A}\delta_m^2\right]^{-N}, \quad (13)$$

where $2 \geq \delta_1 \geq \cdots \geq \delta_{M_A} \geq 0$ are the singular values of the rank M_A matrix $\left(\mathbf{X}_j - \mathbf{X}_i\right)/\sqrt{T}$. Using (12), one can draw insight into the role of the rotation angles, $\boldsymbol{\theta}$. Without applying these rotation angles and for codewords transmitting the same vector from the same antenna, we will have a one-dimension rank loss of the matrix $\left(\mathbf{X}_j - \mathbf{X}_i\right)/\sqrt{T}$, which means that there will be a diversity loss; each identical column will reduce the number of non-zero singular value and hence the rank of the difference matrix by one. Now, using rotation angles, we can guarantee that the matrix $\left(\mathbf{X}_j - \mathbf{X}_i\right)/\sqrt{T}$ is always of rank M_A for any pair of transmitted codewords irrespective of which antennas are active and which $\mathbf{U}$ is transmitted. Hence, the rotation angles ensure that the joint detector achieves a diversity order of $M_A N$.

Unfortunately, this detector requires searching over $|\mathcal{U}|\,|\mathcal{A}|$ matrices, which is could be computationally prohibitive for large constellations.

C. Two-Step Suboptimal Coherent Detector

To reduce detection complexity, we propose a sequential two-step detector, which detects the LR information first followed by the incremental HR information. In the first step, the optimal noncoherent detector in (10) is used to detect the Grassmannian codeword in $\mathbf{U}$. The detected matrix $\hat{\mathbf{U}}$ is

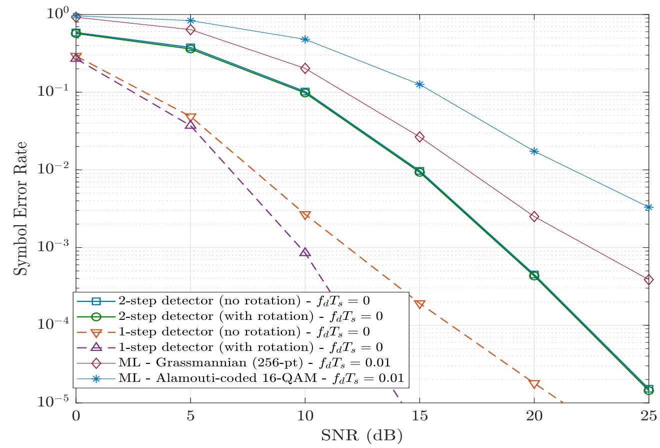

Fig. 1. Performance of one-step and two-step detectors: 256-point LR constellation constructed on $\mathbb{G}_{4,2}$ and 4-point HR spatial constellation. $\theta_5 = \theta_6 = \theta_7 = \theta_8 = \frac{\pi}{2}$. f_dT_s refers to normalized Doppler spread.

assumed to be correct, and is fed to the second step. In the second step, the ML detector is used to detect $\mathbf{A}$:

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \|\mathbf{Y} - \hat{\mathbf{U}}\mathbf{A}\mathbf{H}\|_F^2. \quad (14)$$

The two-step detector requires only $|\mathcal{U}| + |\mathcal{A}|$ metric computations, which is significantly less than the $|\mathcal{U}|\,|\mathcal{A}|$ computations required by the optimal joint detector. However, the proposed detector suffers from performance degradation due to error propagation and the fact that the noncoherent ML detector used in the first step does not exploit the available CSI. Despite its inherent suboptimality, this receiver can be readily shown to achieve full diversity, i.e., NM_A.

V. Simulation Results

In this section, we present simulation results of the proposed system. In all simulations, $M_A = N = 2$, $T = 4$, the HR layer codes are designed using the approach in Section III-B, MATLAB Global Optimization Toolbox is used to optimize the rotation angles, and the LR Grassmannian constellations are designed using the direct method discussed in Section III-A.

In Fig. 1, we compare the symbol error rate of the coherent layer for two-step and one-step detectors, with and without rotation. The gain of the rotation is not immediately clear in the two-step detector. This is because the size of the LR constellation is much larger than the spatial HR constellation, and most errors occur because the first step fed an incorrect $\hat{\mathbf{U}}$ to the second step. We also show the performance of Grassmannian codes and Alamouti-coded 16-QAM (with training) using the fading model from [20] with normalized Doppler spread equal to 0.01. An important observation from Fig. 1 is that the optimized rotation angles in $\mathbf{A}$ provide a significant performance advantage in the case of one-step detection.

To investigate the effect of optimized rotation angles on the performance of the two-step detector, the constellation size of the LR layer was reduced to two. Fig. 2 shows the performance of the two-step detector in that case. The vital role of the rotation angles is evident, and a gain of almost 7 dB can be observed at a symbol error rate of 10^{-4}.

Finally, in Fig. 3, we compare the performance of the proposed HR layer code against the HR layer code in [12]. In [12], the LR layer information is encoded in the subspace spanned

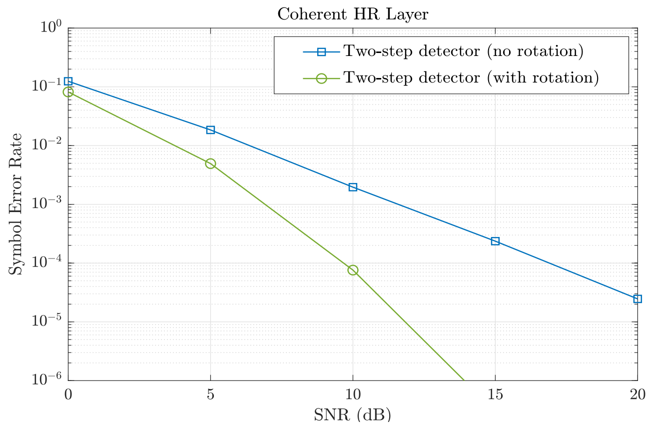

Fig. 2. Performance of the two-step coherent detector: 2-point LR constellation constructed on $\mathbb{G}_{4,2}$ and 4-point HR spatial constellation. $\theta_5 = \theta_6 = \theta_7 = \theta_8 = \frac{\pi}{2}$.

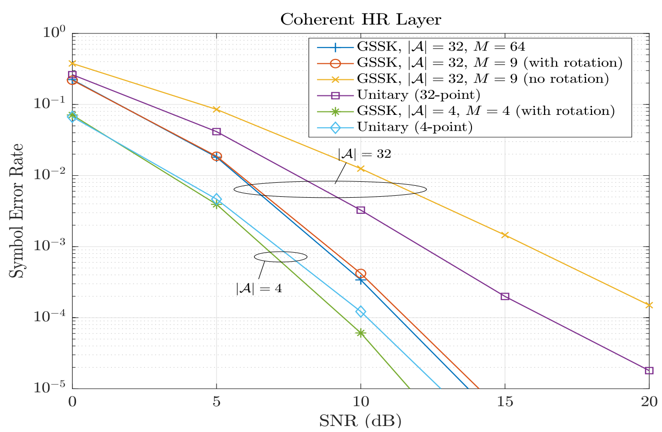

Fig. 3. Comparison with the unitary codes in [12]: 2-point LR constellation constructed on $\mathbb{G}_{4,2}$, 4-points and 32-points HR constellations. The two-step decoder was used for all curves.

by the transmitted codeword, but the HR layer information is encoded in the particular basis of the subspace. Square unitary matrices are used to rotate the subspace basis, and are designed by direct optimization on the unitary group $\mathbb{U}_M$. In agreement with the observation made in Remark 1, simulation results confirm that the proposed GSSK constellations outperform the unitary constellation proposed in [12] at the cost of having more antennas at the transmitter. For performance comparison, Fig. 3 shows that, for 4-point constellations, the proposed GSSK constellation outperform the corresponding constellation in [12] by more than 1 dB at a symbol error rate of 10^{-5}, while for 32-point constellations, the advantage of the GSSK constellation over its unitary counterpart in [12] is about 5 dB at a symbol error rate of 10^{-4}. Note that for the case of $M = 64$ antennas there is no need for rotation as the 32 GSSK constellation is implemented using two distinct transmit antennas for each constellation points with no overlaps. For the case of $M = 9$, the rotation angles are optimized using the technique in Section III-B. From the above results it can be seen that in all cases the use of rotation angles yields significant performance benefits.

VI. Conclusion

We proposed a new multi-resolution space-time signaling scheme for the MIMO multicast channel. This scheme encodes information in two layers: LR information is encoded using a Grassmannian noncoherent code that could be decoded without invoking CSI at the receiver, while HR incremental information is encoded in the indices of the transmit antennas using GSSK. We proposed an efficient algorithm to optimize the HR constellation. We showed that the HR layer is transparent to the underlying LR layer, and we proposed a low complexity two-step detector for the HR layer information. Numerical simulations suggest that the error performance of the proposed scheme is superior to that of previously proposed schemes.

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