# Opportunistic Relaying with Partial CSI and Dynamic Resource Allocation

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***Abstract*—In this paper, we consider the problem of minimizing the source node average power required to achieve a certain transmission rate in the existence of a relay node. Some works have considered this problem with the assumption of perfect channel state information (CSI) at the source node. We consider more practical scenarios where the source does not know the channel between the relay and the destination, but receives one-bit feedback on the state of that channel and where a maximum power constraint exists at each transmitting node. We consider two relaying protocols, namely, the Multi-Hop (MH) and the Opportunistic Decode and Forward (ODF) protocols. We derive closed form expressions for the average power required under each protocol and compare their performances with the system that assumes perfect relay-destination channel knowledge at the source node. We find that the performance is close, indicating that one-bit feedback is very useful. We also derive an upper bound on the average power required by the MH protocol, and derive the outage probability expressions for the two protocols.**

***Index Terms*—Dynamic Resource Allocation, Relaying**

## I. Introduction

Cooperative communication is a growing area of research and there is a plethora of work on the achievable diversity and/or multiplexing gains [1], [2]. The importance of user co-operation has emerged due to size and complexity limitations in the small mobile devices which result in limiting the number of antennas. User cooperation depends on the existence of a relay between a source and a destination which will forward the transmitted message from the source to the destination using some relaying protocol as Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF), etc. The relay could actually be another source, i.e., different sources act as relays at different times. There are some works on heterogeneous relaying in which different relay nodes use different relaying protocols as in [3], [4] and [5].

Different opportunistic relaying protocols that have been considered in [6] but all of these protocols require partial CSIT (only amplitudes) of all the channels at the source node. In this paper, we consider a more practical scenario for these protocols in which the channel between the relay and the destination is not known at the source node. The source node has partial knowledge about the relay-destination channel through a one-bit feedback. The second difference between [6] and this paper is that here we introduce a maximum power constraint for both the source and the relay. In this paper, we analyze the performance of the system with one-bit feedback from the relay to the source informing it about the status of the relay-destination channel. Consequently, we may compute the average power required to achieve a target rate.

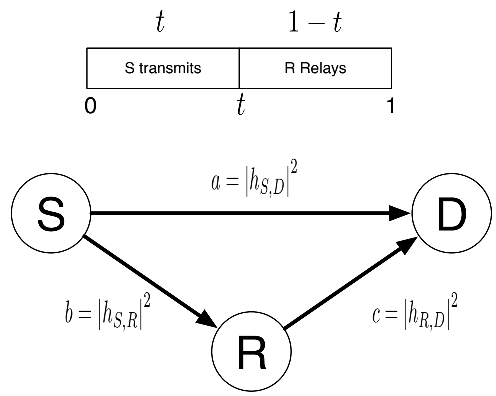


Fig. 1. The network topology

We consider two opportunistic relaying protocols, namely, the MH and the ODF protocols. Our target in this paper is to minimize the average power required by any of the two protocols to achieve a target transmission rate from the source node. The power allocation functions and the decision regions for the MH and ODF protocols are given in [6], [7] under slightly modified assumptions.

## II. System Model

We consider a single source node (S), a destination node (D) and a relay node (R). The channel gains $h_{S,D}$, $h_{S,R}$ and $h_{R,D}$ denote the source-destination, source-relay and relay-destination channels, respectively, and are modeled as circularly symmetric complex Gaussian random variables. Let $a = |h_{S,D}|^2$, $b = |h_{S,R}|^2$ and $c = |h_{R,D}|^2$ as shown in Fig. 1. The instantaneous channels $a$, $b$ and $c$ are independent exponential random variables with means $\frac{1}{\lambda_a}$, $\frac{1}{\lambda_b}$ and $\frac{1}{\lambda_c}$, respectively. The time is divided into two slots of lengths $t$ and $1 - t$ and $0 \le t \le 1$. In the first slot, $t$, the source transmits to the relay and the destination. In the second slot, $1 - t$, the relay helps the source to convey its message to the destination using either the multi-hop (MH) protocol or the opportunistic decode and forward (ODF).

In the MH protocol, the system will choose between the direct transmission mode and the two-hop mode in which the source will transmit the message to the relay node and the relay node will forward the message to the destination node based on the DF protocol. In the ODF protocol, the system will choose between direct transmission mode and DF based mode in which the relay forwards the message and the destination will combine the signals received from the source and the relay nodes.

## III. Protocols Performance Analysis

Using the power allocation functions in [6] for the MH protocol, we can write the thresholds of the channels for this protocol as follows.

$$a_{th} = \frac{2^R-1}{P_S^{max}},\ b_{th} = \frac{2^{2R}-1}{P_S^{max}} \text{ and } c_{th} = \frac{2^{2R}-1}{P_R^{max}}, \tag{1}$$

where $R$ is the target source node rate, $P_S^{max}$ is the source node maximum power and $P_R^{max}$ is the relay node maximum power. As a result we cannot transmit over any channel if its values was below its threshold value because this will require higher power than the maximum allowable power for the transmitting node. Algorithm 1 presents the transmission mode selection for the MH protocol (a similar algorithm will apply to the ODF protocol as discussed in [6] but with different thresholds due to different power allocation function). In Algorithm 1, $P_{inst}^{DT} = \frac{2^R-1}{a}$ denotes the required instantaneous power for the direct transmission mode and $P_{avg}^{MH}$ denotes the average power required for the MH transmission mode (note that we do not know the required instantaneous power for the MH mode since the source node does not have perfect knowledge of $c$ and only has one-bit feedback to indicate whether $c$ is above or below the threshold $c_{th}$).

*Lemma 3.1:* The average power required by the MH protocol to achieve a certain rate $R$ according to Algorithm 1 is given by

$$\begin{aligned}E(P) &= -(2^R-1)\lambda_a \mathbf{Ei}(-\lambda_a a_{th})(1-e^{-b_{th}\lambda_b - c_{th}\lambda_c}) \\ &- \frac{2^{2R}-1}{2}\left(\lambda_b \mathbf{Ei}(-\lambda_b b_{th})e^{-\lambda_c c_{th}} + \lambda_c \mathbf{Ei}(-\lambda_c c_{th})e^{-\lambda_b b_{th}}\right) \\ &- \beta_1 - \beta_2,\end{aligned} \tag{2}$$

where
$\beta_1 = (2^R-1)\lambda_a\lambda_b e^{-\lambda_c c_{th}} \int_{b_{th}}^{\infty} e^{-\lambda_b b}\mathbf{Ei}(-\lambda_a \max(a_{th}, \frac{b}{k_1+k_2 b}))\, db$,
$\beta_2 = \frac{2^{2R}-1}{2}\lambda_b\lambda_c \int_{b_{th}}^{\infty} e^{-\lambda_b b} e^{-\lambda_a \frac{b}{k_1+k_2 b}} \left(\frac{e^{-\lambda_c c_{th}}}{\lambda_c b} - \mathbf{Ei}(-\lambda_c c_{th})\right) db$,
and $\mathbf{Ei}(x)$ is the exponential integral function defined as $\mathbf{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$.

*Proof:* From Algorithm 1 we can define the regions $A_1, A_2, A_3$ and $A_4$ as

$$\begin{aligned} A_1 &= \{(a,b,c) | a \geq a_{th} \text{ and } c < c_{th}\} \\ A_2 &= \{(a,b,c) | a \geq a_{th} \text{ and } b < b_{th} \text{ and } c \geq c_{th}\} \\ A_3 &= \{(a,b,c) | a < a_{th} \text{ and } b \geq b_{th} \text{ and } c \geq c_{th}\} \\ A_4 &= \{(a,b,c) | a \geq a_{th} \text{ and } b \geq b_{th} \text{ and } c \geq c_{th}\}. \end{aligned} \tag{3}$$

**Algorithm 1** Calculate $\min P_{avg}$

**if** $c > c_{th}$ **then**
  **if** $a > a_{th}$ **then**
    **if** $b > b_{th}$ **then**
      $P_{req} = \min(P_{inst}^{DT},\ P_{avg}^{MH})$
    **else**
      $P_{req} = P_{inst}^{DT}$
    **end if**
  **else**
    **if** $b > b_{th}$ **then**
      $P_{req} = P^{MH}$
    **else**
      $P_{req} = 0$
    **end if**
  **end if**
**else**
  **if** $a > a_{th}$ **then**
    $P_{req} = P^{DT}$
  **else**
    $P_{req} = 0$
  **end if**
**end if**

The average power is given by

$$\begin{aligned} E(P) = &E(P^{DT}|s \in A_1)\Pr(s \in A_1) + E(P^{DT}|s \in A_2)\Pr(s \in A_2) \\ &+ E(P^{MH}|s \in A_3)\Pr(s \in A_3) \\ &+ E(\min(P^{DT}, P^{MH})|s \in A_4)\Pr(s \in A_4), \end{aligned} \tag{4}$$

where $s$ denotes some realization of the channel gains. The first term in (4) is given by

$$\begin{aligned} &E(P^{DT}|s \in A_1)\Pr(s \in A_1) \\ &= \int_0^{c_{th}}\int_{a_{th}}^{\infty} \lambda_a\lambda_c \frac{2^R-1}{a} e^{-\lambda_a a} e^{-\lambda_c c}\, da\, dc \\ &= -(2^R-1)\lambda_a \left(1-e^{-\lambda_c c_{th}}\right)\mathbf{Ei}(-\lambda_a a_{th}), \end{aligned} \tag{5}$$

while the second term can be written as

$$\begin{aligned} &E(P^{DT}|s \in A_2)\Pr(s \in A_2) \\ &= \lambda_a\lambda_b\lambda_c(2^R-1)\int_0^{b_{th}}\int_{a_{th}}^{\infty}\int_{c_{th}}^{\infty} \frac{e^{-\lambda_a a}}{a} e^{-\lambda_b b} e^{-\lambda_c c}\, dc\, db\, da \\ &= -(2^R-1)\lambda_a\left(1-e^{-\lambda_b b_{th}}\right)e^{-\lambda_c c_{th}}\mathbf{Ei}(-\lambda_a a_{th}); \end{aligned} \tag{6}$$

the third term can be written as

$$\begin{aligned} &E(P^{MH}|s \in A_3)\Pr(s \in A_3) = \\ &\frac{2^{2R}-1}{2}\lambda_a\lambda_b\lambda_c \int_{c_{th}}^{\infty}\int_{b_{th}}^{\infty}\int_0^{a_{th}} \left(\frac{1}{b}+\frac{1}{c}\right) e^{-\lambda_a a} e^{-\lambda_b b} e^{-\lambda_c c}\, da\, db\, dc \\ &= -\frac{2^{2R}-1}{2}\left(\lambda_c \mathbf{Ei}(-\lambda_c c_{th})e^{-\lambda_b b_{th}} + \lambda_b \mathbf{Ei}(-\lambda_b b_{th})e^{-\lambda_c c_{th}}\right) \\ &\times \left(1-e^{\lambda_a a_{th}}\right). \end{aligned} \tag{7}$$

In order to find the last term in (4) we will divide $A_4$ into two regions; in the first one the DT mode gives lower instantaneous power than the MH mode and the opposite happens in the second region. So we will use the DT mode in $A_4$ only if

$$P_{inst}^{DT} < P_{avg}^{MH}, \tag{8}$$

where $P_{inst}^{DT} = \frac{2^R-1}{a}$ and $P_{avg}^{MH} = \frac{2^{2R}-1}{2}\int_{c_{th}}^{\infty}\left(\frac{1}{b}+\frac{1}{c}\right)\lambda_c e^{-\lambda_c c}\,dc = \frac{2^{2R}-1}{2}\left(\frac{e^{-\lambda_c c_{th}}}{b} - \lambda_c \mathbf{Ei}(-\lambda_c c_{th})\right)$. Here we have used the average power for the MH mode instead of the instantaneous power because $c$ is unknown at the source node. After some mathematical manipulations, the condition in (8) can be written as

$$a > \frac{b}{k_1 + k_2 b}, \tag{9}$$

where $k_1 = e^{-\lambda_c c_{th}}\frac{2^{2R}-1}{2(2^R-1)}$ and $k_2 = -\lambda_c\frac{2^{2R}-1}{2(2^R-1)}\mathbf{Ei}(-\lambda_c c_{th})$. We can write the average power in the region where DT mode is used as

$$\begin{aligned}
&E(P^{DT}|s \in A_4 \text{ and } a > \frac{b}{k_1+k_2b})\Pr(s \in A_4 \text{ and } a > \frac{b}{k_1+k_2b}) \\
&= \int_{b_{th}}^{\infty}\int_{\max(a_{th},\frac{b}{k_1+k_2b})}^{\infty}\int_{c_{th}}^{\infty}\frac{2^R-1}{a}\lambda_a\lambda_b\lambda_c e^{-\lambda_a a}e^{-\lambda_b b}e^{-\lambda_c c}\,da\,db\,dc \\
&= -(2^R-1)\lambda_a\lambda_b e^{-c_{th}\lambda_c}\int_{b_{th}}^{\infty} e^{-\lambda_b b}\mathbf{Ei}(-\lambda_a \max(a_{th},\frac{b}{k_1+k_2b}))\,db
\end{aligned} \tag{10}$$

On the other hand, we can write the average power in the region where MH is used as

$$\begin{aligned}
&E(P^{MH}|s \in A_4 \text{ and } a < \frac{b}{k_1+k_2b})\Pr(s \in A_4 \text{ and } a < \frac{b}{k_1+k_2b}) \\
&= \lambda_a\lambda_b\lambda_c\frac{2^{2R}-1}{2}\int_{b_{th}}^{\infty}\int_{a_{th}}^{\frac{b}{k_1+k_2b}}\int_{c_{th}}^{\infty}\left(\frac{1}{b}+\frac{1}{c}\right)e^{-\lambda_a a}e^{-\lambda_b b}e^{-\lambda_c c}\,dc\,da\,db \\
&= -\frac{2^{2R}-1}{2}e^{-\lambda_a a_{th}}\left(\lambda_b\mathbf{Ei}(-\lambda_b b_{th})e^{-\lambda_c c_{th}} + \lambda_c\mathbf{Ei}(-\lambda_c c_{th})e^{-\lambda_b b_{th}}\right) \\
&- \frac{2^{2R}-1}{2}\lambda_b\lambda_c\int_{b_{th}}^{\infty} e^{-\lambda_b b}e^{-\lambda_a\frac{b}{k_1+k_2b}}\left(\frac{e^{-\lambda_c c_{th}}}{\lambda_c b} - \mathbf{Ei}(-\lambda_c c_{th})\right)db.
\end{aligned} \tag{11}$$

Substituting with (5), (6), (7), (10) and (11) in (4), we get Lemma 3.1. ■

*Lemma 3.2:* When $a_{th} > \frac{1}{k_2} > \frac{b}{k_1+k_2b}$ we can write the average power as

$$\begin{aligned}
E(P) &= -(2^R-1)\lambda_a\mathbf{Ei}(-\lambda_a a_{th}) - \frac{2^{2R}-1}{2}(1-e^{-\lambda_a a_{th}}) \\
&\times \left(\lambda_b\mathbf{Ei}(-\lambda_b b_{th})e^{-\lambda_c c_{th}} + \lambda_c\mathbf{Ei}(-\lambda_c c_{th})e^{-\lambda_b b_{th}}\right).
\end{aligned} \tag{12}$$

*Proof:* when $a_{th} > \frac{1}{k_2} > \frac{b}{k_1+k_2b}$, the DT mode will always be used in region $A_4$. In this case, (10) can be written as

$$\begin{aligned}
&E(P^{DT}|s \in A_4 \text{and} a > \frac{b}{k_1+k_2b})\Pr(s \in A_4 \text{and} a > \frac{b}{k_1+k_2b}) \\
&= -(2^R-1)\lambda_a e^{-\lambda_c c_{th}}\mathbf{Ei}(-\lambda_a a_{th}).
\end{aligned} \tag{13}$$

The average power in Lemma 3.2 can be obtained by substituting with (5), (6), (7), and (13) in (4). ■

*Lemma 3.3:* The average power required to achieve a certain rate $R$ for the MH protocol can be upper bounded as

$$\begin{aligned}
&E(P) < -(2^R-1)\lambda_a\mathbf{Ei}(-\lambda_a a_{th})(1-e^{-\lambda_b b_{th}}e^{-\lambda_c c_{th}}) - \\
&\frac{2^{2R}-1}{2}(1-e^{-\frac{\lambda_a}{k_2}})\left(\lambda_b\mathbf{Ei}(-\lambda_b b_{th})e^{-\lambda_c c_{th}} + \lambda_c\mathbf{Ei}(-\lambda_c c_{th})e^{-\lambda_b b_{th}}\right) \\
&- (2^R-1)\lambda_a Ei\Big(-\lambda_a\max\left(a_{th},\frac{b_{th}}{k_1+k_2b_{th}}\right)\Big)e^{-\lambda_b b_{th}}e^{-\lambda_c c_{th}}.
\end{aligned} \tag{14}$$

The proof of Lemma 3.3 is omitted due to space limitations.

Algorithm 1 can be also used for the ODF protocol by replacing $P_{avg}^{MH}$ with $P_{avg}^{ODF}$ and $P^{MH}$ with $P^{ODF}$ where

$$P^{ODF} = t\frac{2^{\frac{R}{t}}-1}{b} + \frac{(1-t)}{c}\left[2^{\frac{R}{1-t}}\left(1+\frac{a}{b}\left(2^{\frac{R}{t}}-1\right)\right)^{\frac{t}{t-1}} - 1\right], \tag{15}$$

and

$$\begin{aligned}
P_{avg}^{ODF} &= \int_{c_{th}}^{\infty} P^{ODF}\lambda_c e^{-\lambda_c c}\,dc \\
&= t\frac{2^{\frac{R}{t}}-1}{b}e^{-\lambda_c c_{th}} - \\
&\lambda_c(1-t)\left[2^{\frac{R}{1-t}}\left(1+\frac{a}{b}\left(2^{\frac{R}{t}}-1\right)\right)^{\frac{t}{t-1}} - 1\right]\mathbf{Ei}(-\lambda_c c_{th}).
\end{aligned} \tag{16}$$

It is worth noting that for the ODF protocol to be tractable we have used the same thresholds obtained from the power allocation function of the MH protocol.

## IV. Protocols Outage Probabilities

*Lemma 4.1:* The outage probability for the Algorithm 1 is given by

$$P_{out} = \left(1-e^{-\lambda_a a_{th}}\right)\left(1-e^{-\lambda_b b_{th}}e^{-\lambda_c c_{th}}\right). \tag{17}$$

Proof is omitted since this Lemma can be proved in a straightforward manner.

Note that the last outage probability expression is valid for both the MH and the ODF protocols because we have used the same thresholds for both protocols which will result in the same decision regions.

## V. Simulation Results

Here we assumed the channel average $\frac{1}{\lambda_i} = \frac{1}{d_i^{\gamma}}$, where $d_i$ is the distance between the transmitter and the receiver and $\gamma$ is the path loss exponent. Throughout the simulation section we assume that the relay is located between the source and the destination on the straight line connecting them. We assume that the distance between the source and the destination is normalized to 1, the distance between the source and the

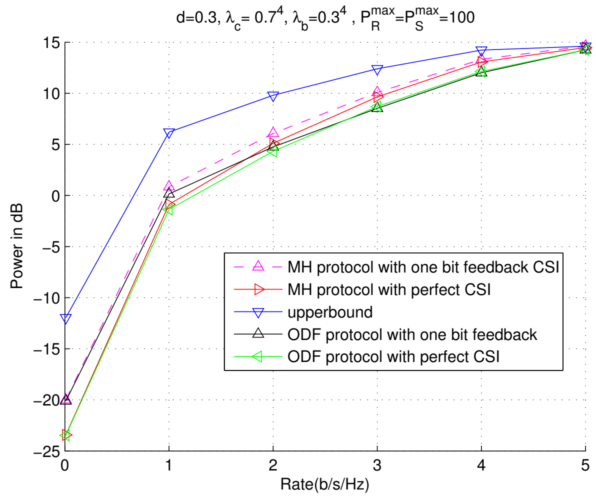


Fig. 2. The average power in case of $P_R^{max} = P_S^{max} = 100$ units

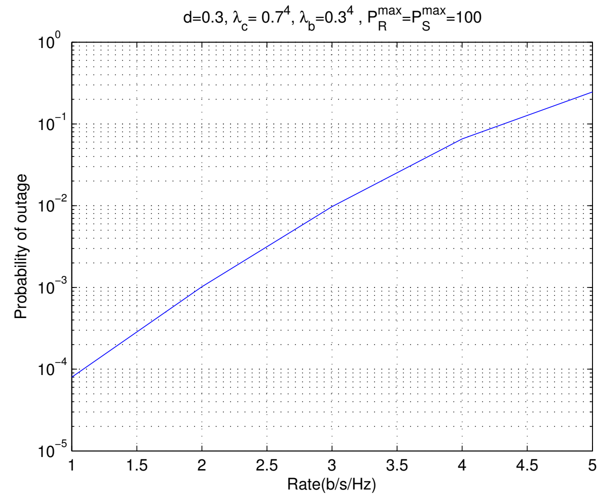


Fig. 3. The outage probability in case of $P_R^{max} = P_S^{max} = 100$ units

relay equals $d$ while the distance between the relay and the destination equals $1 - d$.

The average power for each protocol can be shown in Fig. 2. We can notice that the power required for the ODF with one-bit feedback is sometimes lower than the power required by the ODF with perfect knowledge of CSI and that is because we choose the time $t$ to minimize the average power and not the instantaneous power so this not guaranteed to result in the minimum average power (the minimum average power will result if we optimize $t$ with every new channel realization). Fig. 3 shows the outage probability for the presented algorithm. We can see that the one-bit feedback has very small effect on the average power requirement for a target rate.

We investigate the relay location effect in Fig. 4. It worth noting that the best relay location is in the middle between the source and the destination but in this case the knowledge of $c$ at the source is important; on the other hand, when the relay exists in the vicinity of the source or the destination the knowledge of $c$ is not of that importance and a one-bit feedback suffices.

## VI. Conclusion

In this paper, we have considered the problem of minimizing the average power required to achieve a certain rate at the source node with the help of a relay node. We have considered two opportunistic relaying protocols, namely, the MH protocol and the ODF protocol. We have considered a more practical scenario where each node has a maximum power constraint as well as limited channel state information knowledge at the source node in terms of a one-bit feedback on the relay-destination channel status. We have shown that this limited channel knowledge results in small degradation on the required average power of the system compared to the perfect CSI case.

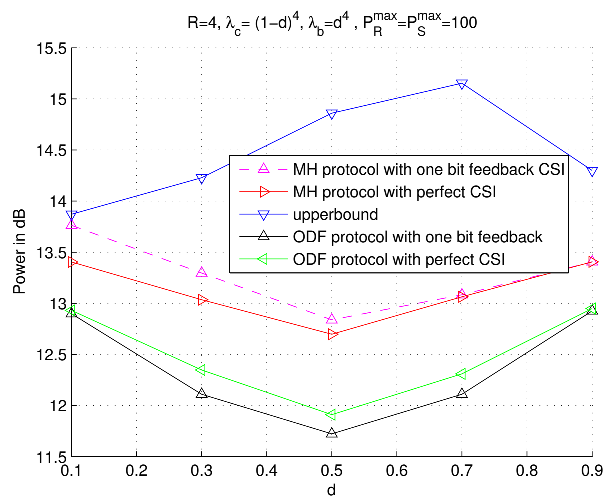


Fig. 4. The effect of the relay location

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