

# Cooperation in Multi-User Wireless Powered Communication Networks

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**Abstract**—Energy harvesting has been gaining a lot of attention in the past decade due to its ability to provide a -virtually- endless energy supply. Nodes in a Wireless Powered Communication Networks (WPCN) depend, totally or partially, on the energy harvested from the Central Node (CN) which has a constant power supply. This work addresses a solution to the problem of lack of fairness in the distribution of energy broadcast to nodes from the CN. The solution presented here depends on cooperation between nodes, in which nodes that have harvested more energy can help other nodes in their transmission to achieve fairness. The main objective is to achieve a maximized common throughput by selecting the best relay node assuming Amplify-and-Forward relaying. An optimization problem is formulated to allocate time and energy resources for nodes' transmissions and relaying. The formulated optimization problem is proved to be convex, which allows for efficient solution calculation. Simulation results show the improved performance of our proposed cooperation and relay selection algorithms as compared to the non-cooperative scenario.

## I. INTRODUCTION

One of the main challenges facing wireless communication networks is energy limitation constraints. For instance, Wireless Sensor Networks (WSNs) are formed by distributing a relatively large number of sensors to collect data about a certain phenomenon. Sensors are power supplied by batteries, which cause restrictions on transmission and processing. Furthermore, in some cases, these batteries are almost impossible to replace (as in battlefield surveillance or as in sensors implanted in human bodies) [1]. Harvesting energy from wireless transmission is a promising research direction towards an unlimited power supply to communication networks. There are two main approaches in literature concerning energy harvesting through wireless energy transfer [2]; the first is called Simultaneous Wireless Information and Power Transfer (SWIPT). In this approach, Wireless Energy Transfer (WET) and Wireless Information Transmission (WIT) occur simultaneously [3] where energy and information are transmitted together in the same signal. The second approach is called Wireless Powered Communication Network (WPCN) in which WIT occurs in the Uplink (UL) using the previously achieved WET in the Downlink (DL) [4].

In this paper, the WPCN scenario is considered. A WPCN, as in Fig. 1, consists of one Central Node (CN) that broadcasts energy wirelessly to the distributed nodes around it in the DL (this is the WET phase). In the UL, users use the energy

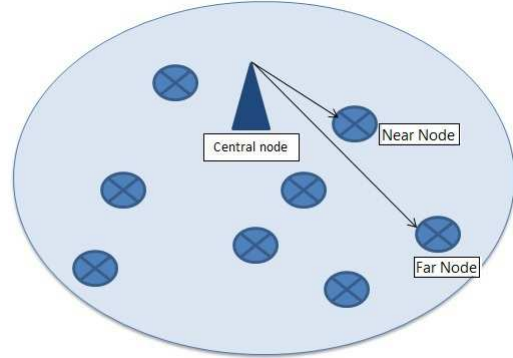


Fig. 1. WPCN network suffering from double near-far problem.

harvested in the DL to transmit their independent information to the CN through Time Division Multiple Access (TDMA) (this is the WIT phase). The main issue in this adopted scenario is the so-called “doubly near-far problem” [5]. Far users receive less energy than near users in the WET phase while they need more energy in the WIT phase. An effective way to overcome this problem is to consider user cooperation. Far nodes, which receive relatively less energy in the DL, select relays from the near nodes to help them transmit their information in their the UL transmission aiming to achieve fairness and a guaranteed maximized minimum (max-min) throughput in the network. This work answers the question of “which is the best relay to select and how does this relay allocate resources (time and energy) for its own transmission and for relaying?”.

The main contribution of this work is to propose a relay selection algorithm that takes into consideration the scenarios with limited channel state information. The relay is assumed to use the amplify-and-forward relaying scheme to simplify the required processing at the relay (not to waste energy on the required processing at the relay node). An optimization problem is formulated for the time and energy allocation of all nodes in the network to maximize the minimum throughput to achieve fairness among the nodes. The problem is proved to be convex which allows for efficient solution calculation using any standard convex optimization toolbox. Two scenarios are considered in this work, based on the amount of system

information available while selecting the relay nodes. The first one is assuming knowledge of locations of all nodes and Channel Status Information (CSI) of all links. This scenario serves as an upper bound system rather than a real situation. The second scenario is assuming that inter-node links are unknown (which is a more practical case).

The rest of the paper is arranged as follows: Related work in literature is summarized in Section II. The system model is presented in Section III. The proposed approach is described in Sections IV and V. Simulation parameters and results of the performance evaluation are presented in Section VI. The paper is concluded in Section VII.

## II. RELATED WORK

The novelty of this work lies in the fusion between WPCNs, user cooperation and relay selection. WPCN has been thoroughly studied in literature. In [6], Zungeru et al. propose a practical approach for RF energy harvesting and management of the harvested energy for wireless sensor networks. Also, in [7], the operation of a sensor network under the energy transfer technology is investigated. The scenario considered is a mobile charging vehicle periodically traveling inside a sensor network and charging each sensor battery wirelessly. Moreover, Lee et al. in [8], use the energy harvesting concept and extend it to the classic cognitive radio (CR) network model. They propose a novel method for nodes to harvest ambient RF energy from transmissions by nearby active transmitters, while opportunistically accessing the spectrum licensed to the primary network.

The ‘‘doubly-near-far’’ problem has been investigated also in literature. To solve this problem, multiple directions were proposed. Ju and Zhang, in [4] carried out the time and power allocation using the common throughput objective function instead of sum throughput in order to achieve fairness. In [9] and [10], beamforming (a multiple antenna AP) has been used. It should be pointed out that the system model in this paper considers a single antenna access point for which beamforming is not applicable.

Additionally, user cooperation was used in literature to solve the fairness issue in WPCN as in [5] which considers a two-user network. User cooperation can also be used to enhance the performance, mitigate channel effects and increase coverage as in [11] and [12].

The objective of this work is to select the best relay and allocate time and energy for nodes to achieve a maximized common throughput in a multi-user WPCN using either total or partial knowledge of Channel Status Information (CSI) and investigating the effect of this selection on the performance of the network.

## III. SYSTEM MODEL

This paper considers a model close to that in [5] with slight modifications. Consider a WPCN with WET in the DL and WIT in the UL. The network consists of one Central Node (CN) and  $K$  users (e.g., sensors) operating over the same frequency. Nodes are all equipped with one antenna. The CN has a constant energy supply while nodes depend totally on the

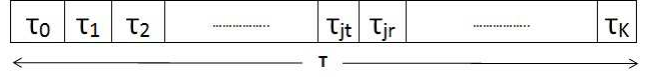


Fig. 2. Transmission time block. Node  $j$  is considered as a relay node.

energy harvested from WET. Relay selection occurs at the CN since it is the only node that has no energy constraints, hence no processing constraints. Also, the CN has more information about the channel state information than any other node in the network. We assume block-based transmissions over quasi-static flat-fading channels, where channel power gains are assumed to remain constant during each block transmission time, denoted by  $T$  (which is normalized to 1 without loss of generality), but can vary independently from one block to another. The locations of nodes are assumed to be stationary and known to the CN. Nodes are randomly distributed (following a uniform probability density function) around the CN in a circular area with radius  $D$ . As mentioned above, the selected relays adopt the amplify-and-forward scheme rather than the Decode-and-Forward scheme to simplify the required processing at the relay node and not to waste the already limited relay node energy on processing the relayed signal.

The following notations are considered:

- $\bar{R}$  is the common throughput to be maximized.
- $\tau_0$  is the time slot of DL energy transfer.
- $\tau_i$  is the time slot for UL information transmission of node  $i$  ( $i = 1, 2, \dots, K$ ).
- For relayed transmissions,  $\tau_i$  is divided into two portions; the first part is assigned for the node to transmit its own data. The second part is assigned for relaying some other node transmission using amplify-and-forward relaying. An illustration of the transmission time block is shown in Fig. 2 in which node  $j$  is considered a relay node.  $\tau_{jt}$  is the time used by node  $j$  to transmit its own data.  $\tau_{jr}$  is the time used for relaying some other node’s data. Note that if node  $j$  acts as a relay for node  $i$ , then we assume that  $\tau_{jr} = \tau_i$  for the amplify-and-forward relaying scheme to work properly (i.e., the signal received during the  $\tau_i$  transmission interval of node  $i$  is amplified and retransmitted by node  $j$  during the  $\tau_{jr} = \tau_i$  relaying interval).
- $R_i$  is the individual achievable throughput of a single link between node  $i$  and the CN.

$$R_i = A \log_2(1 + B \frac{\tau_0}{\tau_i}) \quad (1)$$

where, for direct links:  $A = \tau_i$  and  $B = \frac{P_0 h_{io}^2}{\sigma^2}$ , while for relayed links  $A = \tau_{ir}$  and  $B = \frac{P_0 h_{io} h_{jo} h_{ij}}{\sigma^2 (h_{jo} + 1)}$  [13].

- $P_0$  is the fixed DL power.
- $h_{io}$  is the channel gain between source  $i$  and destination (Central Node  $o$ ). Channel reciprocity is assumed.
- $h_{jo}$  is the channel between the relay  $j$  and the destination  $o$ .

- $h_{ij}$  is the channel between source  $i$  and relay  $j$ .
- $\sigma^2$  is the noise power.
- We assume that *all* energy harvested in DL is used for UL transmission in each cycle.
- The amount of energy harvested at each node in downlink can be expressed as:

$$E_i = P_0 h_{io} \tau_0 \quad (2)$$

- At any relay node, the energy used to forward the relayed information is denoted by  $E_{jr}$  and the energy used to transmit its own information is denoted by  $E_{jt}$ . The values of  $E_{jr}$  and  $E_{jt}$  are determined by solving an optimization problem that will be explained in Section V.

The main steps of the presented algorithm are “Relay Selection” and “Time and Power Allocation” to maximize a common throughput. The next two sections explain the above two steps in detail.

#### IV. RELAY SELECTION

The main objective of this work is to achieve fairness which can be necessary in many applications of the WPCN such as wireless sensor networks, where all the sensors may need to periodically send their sensing data to the CN with the same rate. That is why some nodes will allocate a portion of their time slot and harvested energy to relay data for other nodes.

The proposed scheme depends on two main steps. The first step is the relay selection process which is described in this section in detail. The second is the time and power allocation for each node to reach a maximized common throughput. This step is explained thoroughly in the next section.

In this work, we assume that the relay selection and the time and power allocation are all done at the Central Node (CN); two scenarios are considered in this regard. The first scenario assumes full CSI knowledge at the CN, including the CSI between the different nodes in the network, which might be difficult to attain in practice. However, this scenario will provide an upper bound for the achievable common throughput. The second scenario, which is the more practical scenario, assumes partial CSI knowledge at the CN, where the node-CN channel gains are known while the node-node channel gains are unknown.

- 1) **Assuming Full CSI Knowledge at CN:** Since the adopted relaying scheme is the amplify-and-forward, then the best relay, in this scenario, is the one with the maximum harmonic mean of the Source-Relay channel gain ( $h_{ij}$ ) and the Relay-CN channel gain ( $h_{jo}$ ) [14], [15], [16]. Hence, the relay  $r_i$  for node  $i$  can be selected as:

$$r_i = \arg \max_{j \in \{1, 2, \dots, K\}, j \neq i} H(h_{ij}, h_{jo}) \quad (3)$$

where  $H(x, y) = \frac{2xy}{x+y}$ .

- 2) **Assuming Partial CSI Knowledge at CN:** In this case, the channel gains harmonic mean mentioned in the previous scenario can not be calculated due to the lack of knowledge of the Node-Relay channel gain. In this case, our

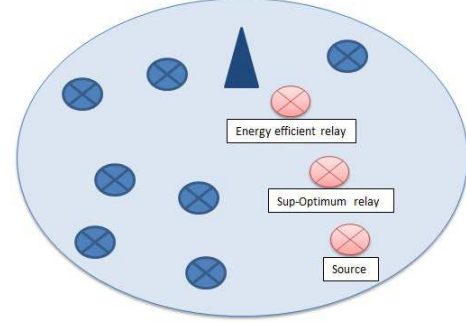


Fig. 3. Sub-optimum and Energy Efficient Relays.

selection for the relay node will be based on the node that lies in the middle between the source and the CN (we select the node that is closest to the middle to be the relay). This selection approach will be referred to as the “sub-optimum relay” later on.

Another approach to consider maximum usage of the limited energy resource is to let the nearest node to the CN (which will harvest the largest amount of energy) be the relay for the furthest node. However, in the amplify-and-forward relaying scheme, the quality of the relayed copy will be limited by the worst channel of the source-relay and relay-CN channels. This scenario will be referred to as the “energy efficient relay” later on. Both scenarios are illustrated in Fig. 3.

#### Proposed Procedure for Relay Selection:

The procedure of relay selection can be summarized as:

- 1) Start with the worst-case node which is the node at the furthest location from the CN and/or the least channel power gain. This step guarantees some degree of fairness.
- 2) Select the group of candidate relays depending on their locations relative to the source and CN. The candidate relay list contains any node that is closer to the CN from the source and within a certain angle ( $70^\circ$ ) around the line connecting the source and the CN.
- 3) For the candidate relays:
  - a) If full CSI knowledge is assumed, the CN calculates the harmonic mean of the Node-Relay channel gain and the Relay-CN channel gain and chooses the relay node that results in the maximum harmonic mean.
  - b) If partial CSI knowledge is assumed, the CN selects the node whose location is close to the middle between the source and the CN.
  - c) In the other approach with partial CSI knowledge, the CN selects the nearest node to be the relay for the furthest node, the second nearest to the second furthest and so on.

An algorithm to illustrate the relay selection procedure for node  $i$  (starting from the furthest node to the CN) is shown in Algorithm 1.

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**Algorithm 1** Relay Selection Procedure

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1: procedure RELAYSELECTION
2:   for Each node  $i$  in the network starting from furthest node
    $i$ .
3:   do   ▷ Create a list of all nodes that are candidates to be the
         relay of node  $i$ .
4:     for Each node  $j$  other than  $i$  in the network do
5:       if  $Distance$  of node  $j$  <
          $Distance$  of node  $i$  & node  $j$  in the sector of Angle  $2 *
         \delta$  from CN to node  $i$  then
6:         Candidate Relay $_i$   $\leftarrow$  node  $j$ 
7:       end if
8:     end for
9:     Choose the best relay from the list of
     Candidate Relays.
10:    ▷ This loop examines all nodes in the "Candidate Relays" list
    to choose the best relay for node  $i$ .
11:    if Full CSI knowledge then
12:      Relay $_i$   $\leftarrow$   $\arg \max_{j \in \text{Candidate Relay}_i} H(h_{ij}, h_{jo})$ 
13:    else if Partial CSI knowledge and Sub –
    Optimum Relay is used then
14:      Relay $_i$   $\leftarrow$  node in the middle between CN and node
     $i$  from Candidate Relay $_i$ 
15:    else if Partial CSI knowledge and Energy –
    Efficient Relay is used then
16:      Relay $_i$   $\leftarrow$  Nearest available node to the CN from
    Candidate Relay $_i$ 
17:    end if
18:  end for
19: end procedure
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## V. TIME AND POWER ALLOCATION

After the relay selection process, the next step is to allocate the time and power portions for each node such that the common throughput of the network, denoted by  $\bar{R}$ , is maximized. We have the constraints that the sum of all time portions ( $\tau_i$ 's) is equal to the transmission block duration  $T$  (which is normalized to 1 for simplicity) and each  $\tau_i$  cannot exceed  $T$ . For any relay node, the sum of the energy portion used for relaying and that used for the transmission of its own data cannot exceed the harvested energy during the DL time interval.

This time and power allocation can be written as an optimization problem as follows:

$$\begin{aligned} & \max_{\bar{R}, \tau, \mathbf{E}} \quad \bar{R} \\ \text{subject to:} & \\ & R_i \geq \bar{R}, \\ & \sum_{i=0}^K \tau_i \leq 1, \\ & \tau_i \geq 0, \\ & \text{For relay nodes: } E_{jr} + E_{jt} \leq P_0 h_{jo} \tau_0, \end{aligned}$$

where  $\tau = [\tau_0 \ \tau_1 \ \tau_2 \ \dots \ \tau_K]$  is the time allocation vector and  $\mathbf{E}$  is the energy allocation vector. Note that  $E_{jr}$  and  $E_{jt}$  are the energies used by relay node  $j$  to relay some other node's data and to transmit its own data, respectively.

The last optimization problem is convex. The cost function is linear and all the constraints, except for the first one, are affine constraints. The LHS of the first constraint (the  $R_i$  expression) is the perspective of a concave function, which yields a concave function [17]. Being convex, the problem can be efficiently solved using any standard convex optimization toolbox. We have used the Matlab standard convex optimization toolbox based on the Interior Point Method.

It should be mentioned that for partial CSI knowledge, the  $R_i$  expression in (1) will be missing the  $h_{ij}$  term (as the optimization will be done at the CN which does not have access to this information as per the model assumptions). Therefore, in this case, the value of  $h_{ij}$  will be replaced by its expected value. As in [4], the channel power gains are modeled as:

$$h_{ij} = 10^{-3} \rho_i^2 D_{ij}^{-\alpha}, \quad i, j = 1, 2, \dots, K, \quad (4)$$

, where:

- $\alpha$  is the path loss exponent,
- $\rho_i$  is the Rayleigh distributed channel short-term fading,
- and  $D_{ij}$  is the distance between node  $i$  and node  $j$  which is estimated as the absolute difference between their distances from the CN  $D_i$  and  $D_j$ .

## VI. PERFORMANCE EVALUATION

To evaluate the performance of the presented scheme, three scenarios are considered:

- Relaying with full CSI knowledge.
- Relaying with partial CSI knowledge. In this case, both the sub-optimum relay selection and the energy efficient relay selection approaches are studied.
- No relaying at all.

Simulation Parameters are listed in Table I.

### A. Effect of Varying the Number of Nodes

The maximized common throughput is plotted vs the number of nodes in the network ( $K$ ) at Downlink Power of 100 dBm. The number of nodes,  $K$ , is varied from 2 nodes to 40 and the nodes are assumed to be uniformly distributed in a circular area around the central node with a radius of 100m. As will be shown later, as the number of nodes increases, the common throughput will certainly decrease since more nodes share the same time resources (i.e., less time will be allocated to each node as the number of nodes increases).

Fig. 4 shows the common throughput of the nodes for the different approaches as described above. It is clear in this figure, that relaying will always result in an improved common throughput as compared to the no relaying approach. Also, as expected, the relaying will full CSI resulted in the highest common throughput, which is an upper bound for the common rate of the other approaches.

Furthermore, it can be noticed that the "sub-optimum relay" scheme outperforms the "energy efficient relay" scheme from the maximum common throughput perspective. As explained above, the performance of the amplify-and-forward relaying scheme will be limited by the minimum of the source-relay

TABLE I  
SIMULATION PARAMETERS

Parameters	Definition	Values
$K$	Number of nodes	2:40
$P_0$	Power broadcast from CN	50-100 dBm
$D$	Radius of the circular area where nodes are located	100 m
$\alpha$	The path loss exponent	2

and relay-CN channel gains. Therefore, the best relay selection will be the one with comparable channel qualities to the source and the CN. However, in the energy efficient relay selection approach, the selection algorithm favors nodes near to the CN to act as relays; this will result in a bad “source-relay” channel that will limit the gains of the amplify-and-forward relaying scheme.

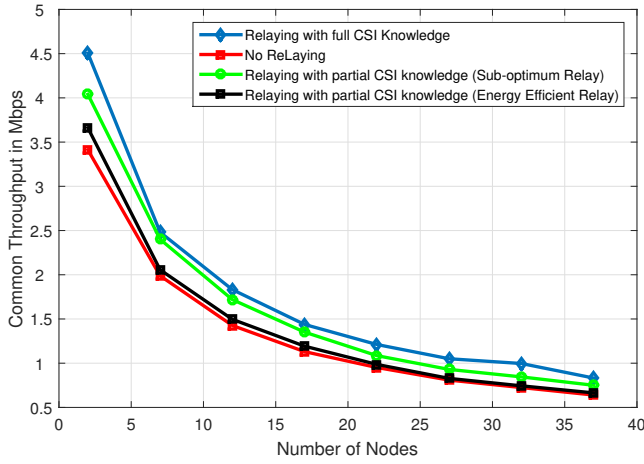


Fig. 4. Maximized Common Throughput in Mbps vs Number of Nodes ( $K$ )

### B. Effect of Varying the Downlink Power

In Fig. 5 the maximized common throughput is plotted against the DL transmitted power from the CN ( $P_0$ ) for 2 nodes in the network.  $P_0$  is varied from 50 dBm to 100 dBm. As the DL power increases, the common throughput increases as well since the throughput is a monotonic increasing function of the SNR.

In Fig. 5, it is clear that user cooperation has a positive role in maximizing the common throughput and achieving fairness in the network. And again, the “sub-optimum relay” selection approach outperforms the “energy efficient relay” selection approach.

### C. Comparing to Changing the Objective Function

As mentioned earlier, the objective function to be maximized is the common throughput  $\bar{R}$ . Here, it is required to investigate the difference between this work and the case in which the objective function to be maximized is the sum throughput  $\sum R_i$ . In this case, the optimization problem can

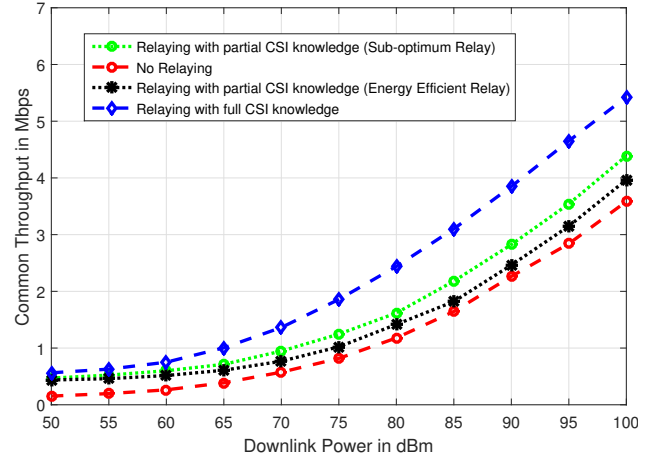


Fig. 5. Maximized Common Throughput in Mbps vs Broadcast Power ( $P_0$ )

be written as:

$$\max_{\tau, \mathbf{E}} \sum R_i$$

subject to:

$$\sum_{i=0}^K \tau_i \leq 1,$$

$$\tau_i \geq 0,$$

$$\text{For relay nodes: } E_{jr} + E_{jt} \leq P_0 h_{jo} \tau_0,$$

Comparison was carried out for 6 nodes in the network and DownLink power varying from 50 to 100 dBm assuming full CSI knowledge. Results are shown in Fig. 6 and Fig. 7.

As clear from Fig. 6, relaying results in a loss in the sum throughput. This is because the selection of relay nodes is done before time and power allocation. Also, relaying means assigning some of the “strong” nodes’ resources to serve the “weak” nodes which have less favorable channel condition. Clearly if the resources of the strong nodes are allocated to serve these strong nodes, that will in general result in a maximized sum throughput; however, maximizing the sum throughput does not provide any provisioning for fairness among the nodes and the nodes with less favorable channel conditions are expected to experience very low data rates. Therefore, as noticed from Fig. 7, relaying will certainly result in a better minimum throughput, i.e. better fairness, as some of the system resources are allocated to serve the weak nodes.

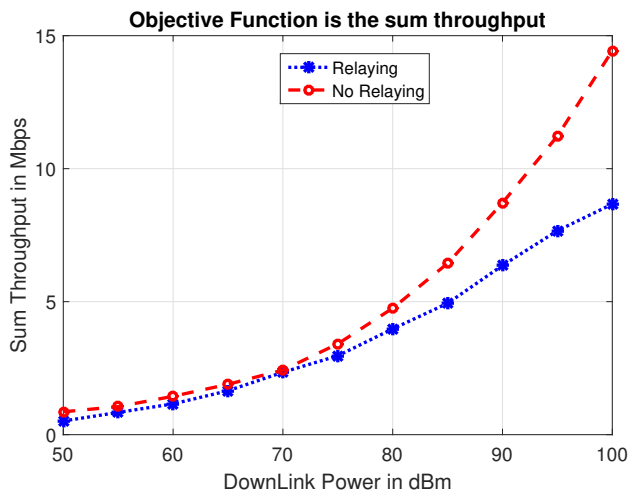


Fig. 6. Maximized Sum Throughput in Mbps vs Broadcast Power ( $P_0$ )

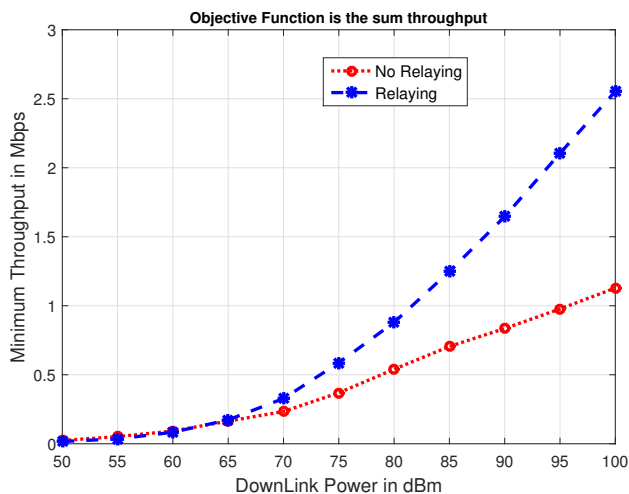


Fig. 7. Minimum Throughput in Mbps vs Broadcast Power ( $P_0$ ) when the sum throughput is maximized

## VII. CONCLUSION

In this paper, we have considered the use of the amplify-and-forward relaying scheme in Wireless Powered Communication Networks (WPCNs) to achieve fairness among the network nodes. We propose relay selection algorithms taking into account practical consideration and compare them to a benchmark algorithm that provides an upper bound for the achievable rate. We then formulate a minimum rate optimization problem for time and power allocation. The formulated problem is proved to be convex, and can be efficiently solved using any standard convex optimization toolbox. Our results show an improved performance of our proposed schemes as compared to the no relaying schemes. Also, our result show that selecting the relay node to be close to the midpoint between the source node and the central node will result in a good performance that is comparable in many cases to the benchmark upper bound.

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