On the ARQ Protocols over the Z-interference Channels: Diversity-Multiplexing-Delay Tradeoff

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***Abstract*—We characterize the achievable three-dimensional tradeoff between diversity, multiplexing, and delay of the single antenna Automatic Retransmission reQuest (ARQ) Z-interference channel. Non-cooperative and cooperative ARQ protocols are adopted under these assumptions. Considering no cooperation exists, we study the achievable tradeoff of the fixed-power split Han-Kobayashi (HK) approach. Interestingly, we demonstrate that if the second user transmits the common part only of its message in the event of its successful decoding and a decoding failure at the first user, communication is improved over that achieved by keeping or stopping the transmission of both the common and private messages. Under cooperation, two special cases of the HK are considered for static and dynamic decoders. The difference between the two decoders lies in the ability of the latter to dynamically choose which HK special-case decoding to apply. Cooperation is shown to dramatically increase the achievable first user diversity.**

I. Introduction

The diversity and multiplexing tradeoff (DMT) framework was initiated by Zheng and Tse [1] in standard Multi-Input Multi-Output (MIMO) channels. EL Gamal *et al.* extended Zheng and Tse's work using Automatic Retransmission reQuest (ARQ) in delay-limited single link MIMO channels [2]. The authors in [2] showed that the ARQ retransmission delay can be leveraged to enhance the reliability of the system at a negligible loss of the effective throughput rate. In addition, the authors in [3] considered cooperative schemes in ARQ relay networks. This work explores the achievable diversity, multiplexing, and delay tradeoff of the *outage limited* single antenna ARQ Z-interference channel (ZIC) [4] in the high signal-to-noise ratio (SNR) asymptote. In particular, we extend the tradeoff studied in [2] to the two user ARQ fading ZIC setting for *both* non-cooperative and cooperative scenarios.

This work first discusses a non-cooperative ARQ protocol using the fixed-power split Han-Kobayashi (HK) approach [5], [6]. We stipulate that the second user transmits only the common part of its message if it receives a positive acknowledgement (ACK) while a negative ACK (NACK) is received at the first transmitter. By considering two special cases of the HK splitting, a Common Message Only (CMO) scheme and a Treating Interference As Noise (TIAN) scheme (i.e. only a private message is sent from the second transmitter) [6], we show the superiority of our transmission policy over the other policies of continuing or stopping the transmission of both the common and private messages under the stated feedback states. We assume that the splitting parameters are determined according to the outage events at the end of the transmission block of the same information message at both users in order to optimize the achievable Diversity Gain Region (DGR) [6]. The channel state information (CSI) is assumed to be perfectly known at the receivers but is unknown at the transmitters. Therefore, we assume that the chosen splitting parameters remain fixed for fixed rates, interference level, and retransmission delay; the second transmitter can only continue or cease the transmission of its common or private message.

Next, we consider a cooperative ARQ scenario where the second transmitter assists the first one in relaying its message in the event of a NACK reception at the first transmitter. The cooperative protocol divides into static decoding and dynamic decoding. Under static decoding, we solve for the achievable tradeoff by tracing the maximum of using either the CMO or the TIAN schemes considering the relaying scenario. Under dynamic decoding, the decoder of the first user dynamically changes its decoding algorithm to either consider the CMO or the TIAN approach according to the channel conditions. Finally, we show the superiority of the dynamic decoding scheme over the static one.

We adopt a coherent delay-limited (or quasi-static) block fading channel model where the channel gains are assumed fixed over the transmission of the same information message. By doing this, we focus on the ARQ diversity advantage without exploiting temporal diversity.

II. System Model

We consider a two user single antenna Rayleigh fading ZIC model as shown in Fig. 1. Users always have information messages to send. Each user in our model employs an ARQ error control protocol with a maximum of L transmission rounds. The information message from each transmitter is encoded into a sequence of L vectors (blocks), $\{x_{i,l} \in \mathbb{C}^T : i = 1, 2 \text{ and } l = 1, \cdots, L\}$, where the transmission of each vector takes T channel uses. Each decoder is allowed to process its corresponding received signal over all l received blocks. Each receiver sends an ACK back to its corresponding transmitter when decoding is successful. A NACK is sent if decoding fails. The ARQ feedback channel is assumed to be error-free and of negligible delay.

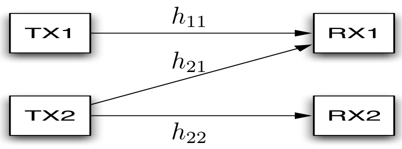

Fig. 1: The ZIC model.

We consider the following ARQ protocol. When both transmitters receive an ACK, they each proceed to send the first block of their next messages. If TX1 receives an ACK while TX2 receives a NACK, TX1 will cease its transmission until TX2 receives an ACK. When TX1 receives a NACK for its message, it begins the transmission of the next block of its current message; while the behavior of TX2 varies according to its feedback outcome and the used ARQ protocol as detailed in the next sections. The reason for differentiating between the case when TX1 receives an ACK while TX2 receives a NACK and the reverse case is that the first user message is not decoded at the second receiver but not vice versa. When the maximum number of protocol rounds L is reached, both transmitters start transmitting the first block of their next messages regardless of the feedback outcome. Error at each user occurs due to any of the following two events. Either L transmission rounds are reached and decoding fails or the decoder makes a decoding error at round $l \leq L$ and fails to detect it (undetected error event).

The received signal vectors are given by

$$\begin{aligned} \mathbf{y}_{1,l} &= \mu_{11} h_{11} \mathbf{x}_{1,l} + \mu_{21} h_{21} \mathbf{x}_{2,l} + \mathbf{n}_{1,l} \\ \mathbf{y}_{2,l} &= \mu_{22} h_{22} \mathbf{x}_{2,l} + \mathbf{n}_{2,l}, \end{aligned} \tag{1}$$

where $\{\mathbf{y}_{i,l}, \mathbf{n}_{i,l}\}$ denote the received vector and the noise vector at RXi, respectively. The noise vectors are modeled as complex Gaussian random vectors with zero mean, unit variance, and i.i.d. entries. They are also assumed to be temporally white. We use $\{h_{i,j} : \; i, j = 1, 2\}$ for the channel gain between transmitter i and receiver j. The channel gains are i.i.d. complex Gaussian random variables with zero mean and unit variance. They are assumed to remain constant over the L transmission rounds and change to new independent values with each new information message. We use a per-block power constraint such that $E\left[\frac{1}{T}||x_{i,l}||^2\right] \leq \rho$, i.e., the constraint on the *average* transmitted power in each transmission round of the ARQ protocol is the same. The parameter ρ takes on the meaning of *average* SNR per receiver antenna. We also parameterize the attenuation of transmit signal i at receiver j using the real-valued coefficients $\mu_{ij} > 0$. To simplify our results, we set $\mu_{11}^2 = \mu_{22}^2 = 1$ and $\mu_{21}^2 = \rho^{\beta-1}$. The parameter β represents the interference level, $\beta \geq 0$.

We consider a family of ARQ protocols that is based on a family of code pairs $\{C_1(\rho), C_2(\rho)\}$ with first block rates $R_1(\rho)$ and $R_2(\rho)$, respectively, and an overall block length TL. The individual error probability at RXi is $P_{e_i}(L,\rho)$ for $i = 1, 2$. For this family, the first block multiplexing gains r_i and the effective ARQ diversity gains $d_i(L)$ for L transmission rounds are defined as

$$r_i \triangleq \lim_{\rho\to\infty} \frac{R_i(\rho)}{\log\rho} \quad \text{and} \quad d_i(L) \triangleq -\lim_{\rho\to\infty} \frac{\log\{P_{e_i}(L,\rho)\}}{\log\rho}. \tag{2}$$

The long-term average throughputs of the ARQ protocol for TX1, $\eta_1(\rho)$, and for TX2, $\eta_2(\rho)$, are characterized as in [2], [7]. The effective ARQ multiplexing gains are defined as

$$r_{e_1} \triangleq \lim_{\rho\to\infty} \frac{\eta_1(\rho)}{\log(\rho)} \quad \text{and} \quad r_{e_2} \triangleq \lim_{\rho\to\infty} \frac{\eta_2(\rho)}{\log(\rho)}. \tag{3}$$

III. The Non-Cooperative ARQ Protocol

We consider here the use of the HK approach at TX2. Specifically, TX2 maintains a private message, with rate $S_2 = s_2 \log\rho$, and a common message, with rate $T_2 = t_2 \log\rho$. Hence, $r_2 = s_2 + t_2$, $s_2, t_2 \geq 0$, and $0 \leq r_i \leq 1$. At RX1, we consider a joint typical-set decoder applied to the message of TX1 and the common message of TX2. At RX2, joint-typical set detection is carried out for both the private and common messages of TX2. For TX2, we parameterize the ratio of the average private power to the total average power as

$$\alpha = \frac{1}{1+\rho^b} \quad \in [0,1], \qquad b \geq 0. \tag{4}$$

Thus, the transmitted powers of the common and private messages, in the high-ρ scale, can be written as[1]

$$P_{2,\text{private}} = \frac{\rho}{1+\rho^b} \quad \text{and} \quad P_{2,\text{common}} \doteq \rho. \tag{5}$$

When the two transmitters receive a NACK at round l, they both begin the transmission of the next block of their current messages. If TX1 receives a NACK while TX2 receives an ACK, we stipulate that TX2 stops sending its private message and keeps sending the common one until TX1 receives an ACK. We motivate this transmission policy by observing two special cases of the HK-splitting. The first special case is when TX2 uses the CMO scheme [6]. In this case, the best that TX2 can do when receiving an ACK while TX1 receives a NACK is to keep sending the same message until TX1 receives an ACK. This way, RX1 will accumulate more joint mutual information hence reducing the probability of the joint outage event at RX1 when joint decoding is performed.

The other special scenario is the TIAN scheme obtained from the HK approach by setting $b = 0$ and $t_2 = 0$ [6]. Under the TIAN scheme, we expect the diversity at RX1 to improve if TX2 ceases the transmission of its current message when receiving an ACK while TX1 receives a NACK since this provides for less interference. The HK scheme with generic splitting parameters lies midway between these two special schemes and it is for this reason that we stipulate the stopping of the private message when a NACK is received at RX1. Note that, the average transmitted power at TX1 or TX2 will not be affected by either continuing or stopping the transmission of the same message after receiving an ACK and until the other transmitter receives an ACK as the probabilities of such events are very small for the case of the high-ρ scale. The multiplexing rate of the both users will not be affected for the same reason [7]. *We can demonstrate the superiority of our transmission policy over other approaches which consider keeping or stopping the transmission of both the common and private messages of TX2 when it receives an ACK while TX1 receives a NACK. The rate regions of these approaches can be shown to be subsets of the rate region of our approach* [7].

[1] Throughout the work, we will use $\doteq$ to denote exponential equality, i.e., $f(z) \doteq z^b$ means that $\lim_{z\to\infty} \frac{\log f(z)}{\log z} = b$, $\dot{\leq}$ and $\dot{\geq}$ are defined similarly.

We demonstrated in [6] that the CMO scheme is a singular special case of the HK approach. So, we now state the achievable tradeoff of the non-cooperative ARQ protocol using the HK and the CMO approaches as they are distinct.

Theorem 1. *The Achievable diversity, multiplexing, and delay tradeoff of a two-user Rayleigh fading ZIC under the use of the non-cooperative ARQ protocol with a maximum of L transmission rounds for the HK approach and using our transmission policy is*

$$\begin{aligned}
&d_{1,\mathrm{HK}}(L) = \min_{i\in\{1,2,\cdots,L\}}\left\{\min\left\{\left[1-\frac{r_2}{i-1}\right]^+,\left[1-\frac{r_2-t_2}{i-1}-b\right]^+\right\}\right.\\
&\qquad\left.+\min\{d_{11,\mathrm{HK}}(L,i),d_{12,\mathrm{HK}}(L,i)\}\right\},\quad \textit{where,}\\
&d_{11,\mathrm{HK}}(L,i) = \max\left\{\left[1-\frac{r_1}{L-i}\right]^+,\left[1-\frac{r_1+i\,[\beta-b]^+}{L}\right]^+\right\}\\
&d_{12,\mathrm{HK}}(L,i) =\\
&\begin{cases}\left[1-\frac{(r_1+t_2)+i[\beta-b]^+}{L}\right]^+, & \textit{if } r_1+t_2\geq(L-i)\beta+ib>Lb\\ \left[1-\frac{(r_1+t_2)-ib}{L-i}\right]^+ + \left[\beta-\frac{(r_1+t_2)-ib}{L-i}\right]^+, & \textit{if } Lb<r_1+t_2<(L-i)\beta+ib\\ \left[1-\frac{r_1+t_2}{L}\right]^+ + \left[\beta-\frac{r_1+t_2}{L}\right]^+, & \textit{if } r_1+t_2\leq Lb,\end{cases}\\
&\textit{and,}\quad d_{2,\mathrm{HK}}(L) = \min\left\{\left[1-\frac{r_2}{L}\right]^+,\left[1-\frac{r_2-t_2}{L}-b\right]^+\right\}.
\end{aligned}\tag{6}$$

While the achievable tradeoff under the CMO scheme is

$$\begin{aligned}
&d_{1,\mathrm{CMO}}(L) = \min\left\{\left[1-\frac{r_1}{L}\right]^+,\left[1-\frac{r_1+r_2}{L}\right]^+ + \left[\beta-\frac{r_1+r_2}{L}\right]^+\right\}\\
&d_{2,\mathrm{CMO}}(L) = \left[1-\frac{r_2}{L}\right]^+.
\end{aligned}\tag{7}$$

Proof: Following the footsteps of [2], we can show that the individual error probabilities are exponentially equal to their respective outage probabilities for sufficiently large T. The use of joint typical-set decoding limits the probability of the undetected error event at any round $l \leq L$ to an arbitrarily small value. Following the same techniques in [1], the probability of decoding failure at round $l = L$ at either RX1 or RX2 is exponentially equal to the probability of the corresponding outage event at the end of the L transmission rounds. Thus, we have for $i = 1, 2$

$$P_{e_i}(L,\rho) \doteq \rho^{-d_{i,\mathrm{HK}}(L)} \doteq P_{\mathrm{out,i}}(L,\rho) \doteq \rho^{-d_{\mathrm{out,i}}(L)} \tag{8}$$

where $P_{\mathrm{out,i}}(L)$ is the individual outage probability at RXi and $d_{\mathrm{out,i}}(L)$ is the diversity gain associated with $P_{\mathrm{out,i}}(L)$.

We then derive the individual outage probabilities for the non-cooperative ARQ-ZIC system. When the accumulated mutual information over the consecutive rounds at RX1(RX2) is smaller than the first block rate $R_1(R_2)$, an outage occurs. It is sufficient, without loss of optimality, to assume that the input codewords are Gaussian distributed [2]. Thus, the mutual information is identical over the protocol rounds. Let us define $\overline{\mathcal{A}}_l$ and $\overline{\mathcal{B}}_l$ as the outage events at RX1 and RX2 at round l, respectively. For the HK scheme, the outage region at RX2 at round $l = L$ is given by

$$\begin{aligned}
\overline{\mathcal{B}}_L = \Big\{&h_{22} : L\log\left(1+|h_{22}|^2\rho\right) < R_2,\ \text{ or }\ L\log\left(1+|h_{22}|^2\rho\right) < T_2,\\
&\text{or }\ L\log\left(1+\frac{|h_{22}|^2\rho}{1+\rho^b}\right) < R_2 - T_2\Big\}.
\end{aligned}\tag{9}$$

The outage event $L\log\left(1+|h_{22}|^2\rho\right) < T_2$ is a subset of the outage event $L\log\left(1+|h_{22}|^2\rho\right) < R_2$ and can be eliminated. So, the outage probability at RX2 can be shown to be

$$P_{\mathrm{out},2}(L) \doteq \rho^{-\min\left\{\left[1-\frac{r_2}{L}\right]^+,\left[1-\frac{r_2-t_2}{L}-b\right]^+\right\}} \doteq \rho^{-d_{2,\mathrm{HK}}(L)}. \tag{10}$$

Thus, we have proved the result for $d_{2,\mathrm{HK}}(L)$ given in (6).

We define $\mathcal{C}_i$ as the event that TX2 receives an ACK at round i and receives a NACK at round $i-1$, [2] thus, $\mathcal{C}_i = \left\{\overline{\mathcal{B}}_{i-1}, \mathcal{B}_i\right\}$. The outage region at RX1 given $\mathcal{C}_i$ at round $l = L$ is given by

$$\begin{aligned}
\overline{\mathcal{A}}_L|\mathcal{C}_i = \Bigg\{&h_{11},h_{21} : i\log\left(1+\frac{|h_{11}|^2\rho}{1+\frac{|h_{21}|^2\rho^\beta}{1+\rho^b}}\right)\\
&+(L-i)\log\left(1+|h_{11}|^2\rho\right) < R_1 \quad\text{or}\quad i\log\left(1+\frac{|h_{11}|^2\rho+|h_{21}|^2\rho^\beta}{1+\frac{|h_{21}|^2\rho^\beta}{1+\rho^b}}\right)\\
&+(L-i)\log\left(1+|h_{11}|^2\rho+|h_{21}|^2\rho^\beta\right) < R_1+T_2\Bigg\}.
\end{aligned}\tag{11}$$

The outage probability at RX1 can be derived as follows,

$$P_{\mathrm{out},1}(L) = \sum_{i=1}^{L}\Pr(\overline{\mathcal{A}}_L|\mathcal{C}_i)\Pr(\mathcal{C}_i) \doteq \rho^{-d_{1,\mathrm{HK}}(L)}. \tag{12}$$

Using the outage events given in (11), we can show that [7]

$$\Pr(\overline{\mathcal{A}}_L|\mathcal{C}_i) \doteq \rho^{-\min\{d_{11,\mathrm{HK}}(L,i),d_{12,\mathrm{HK}}(L,i)\}}, \tag{13}$$

where, $d_{11,\mathrm{HK}}(L,i)$ and $d_{12,\mathrm{HK}}(L,i)$ are as given in (6).

The probability of the event $\mathcal{C}_i$ can be derived as follows,

$$\Pr(\mathcal{C}_i) = \Pr(\overline{\mathcal{B}}_{i-1})\Pr(\mathcal{B}_i|\overline{\mathcal{B}}_{i-1}) \doteq \Pr(\overline{\mathcal{B}}_{i-1}) \doteq \rho^{-d_{2,\mathrm{HK}}(i-1)}, \tag{14}$$

where, $\Pr(\mathcal{B}_i|\overline{\mathcal{B}}_{i-1}) \doteq 1$. Using (13) and (14) in (12), we get

$$P_{\mathrm{out},1}(L) \doteq \sum_{i=1}^{L}\rho^{-\left\{d_{2,\mathrm{HK}}(i-1)+\min\{d_{11,\mathrm{HK}}(L,i),d_{12,\mathrm{HK}}(L,i)\}\right\}}. \tag{15}$$

In the high-ρ scale, the minimum negative exponent dominates the previous summation. Thus, we proved the result for $d_{1,\mathrm{HK}}(L)$ given in (6).

As for the CMO scheme, the outage regions at RX1 and RX2 at round $l = L$ can be given as follows.

$$\begin{aligned}
\overline{\mathcal{A}}_L = \Big\{&h_{11},h_{21} : L\log\left(1+|h_{11}|^2\rho\right) < R_1\\
&\text{or }\ L\log\left(1+|h_{11}|^2\rho+|h_{21}|^2\rho^\beta\right) < R_1+R_2\Big\}
\end{aligned}\tag{16}$$

$$\overline{\mathcal{B}}_L = \left\{h_{22} : L\log\left(1+|h_{22}|^2\rho\right) < R_2\right\}.$$

Using these outage regions, the results for the CMO scheme given in (7) are deduced.

Following similar steps as in [2], the effective multiplexing gains r_{e_1} and r_{e_2} are equal to the first block multiplexing gains r_1 and r_2, respectively, and hence the proof is completed (See full proof in [7]). ∎

[2]A NACK at round $i-1$ implies a NACK at every round $l < i-1$.

IV. The Cooperative ARQ Protocol

Under cooperation, TX2 assists in relaying the message of TX1 in the event of a NACK reception at TX1. Here, we first consider a static decoding scheme where the decoding scheme at RX1, whether using the CMO or the TIAN decoding, is fixed and determined a priori according to the interference level β and the multiplexing gains r_1 and r_2. Next, we will consider a dynamic decoding scheme where RX1 dynamically decides at the beginning of *each* new transmission to use either the CMO or the TIAN decoding according to the channel gains, the interference level, and the multiplexing gains.

For the two cooperative ARQ schemes, if TX1 receives a NACK, TX2 will start listening to TX1 to decode its message regardless of its own feedback. Notice that we assume here a link exists between TX1 and TX2. We denote the time TX2 takes to decode TX1 message by T'. The following relation holds,

$$T' = \min\left\{T, \left\lceil \frac{TR_1}{\log_2(1+|h|^2\rho)} \right\rceil\right\}, \tag{17}$$

where h is the channel gain between TX1 and TX2.

Once TX2 has decoded TX1's message, it starts relaying this message using a codebook $\tilde{C}_1(\rho)$. If TX2 decodes the TX1's message in T' symbols, then it assists TX1 by relaying its message in the remaining time, $T - T'$ [7].

A. Cooperative ARQ with Static Decoding

We characterize here the achievable DMT of the cooperative ARQ scheme with static decoding considering a maximum of two transmission rounds for analytical tractability. We restrict ourselves to the use of CMO and TIAN schemes for simplicity. We will use the superscript ^{c} to refer to the cooperative setup.

Theorem 2. *The achievable DMT of the cooperative ARQ with static decoding for $L = 2$ under the use of the CMO scheme is given by*

$$\begin{aligned}
&d^c_{1,\mathrm{CMO}}(2) = \min\left\{d^c_{11,\mathrm{CMO}}(2), d^c_{12,\mathrm{CMO}}(2)\right\}, \quad \textit{where,}\\
&d^c_{11,\mathrm{CMO}}(2) =\\
&\begin{cases} 1-\frac{r_1}{2}, & \textit{if } r_1 \geq 2\beta \\ \min\left\{1+\frac{(1-r_1)\beta - r_1}{1+r_1}, 2-\frac{3r_1}{2}\right\}, & \textit{if } \frac{\beta}{1+\beta} \leq r_1 < 2\beta \\ \min\left\{2-\frac{3r_1}{2}, 2-\frac{\beta r_1}{\beta - r_1}, 1+\beta-\frac{r_1}{1-r_1}\right\}, & \textit{if } r_1 < \frac{\beta}{1+\beta}\end{cases}\\
&d^c_{12,\mathrm{CMO}}(2) = \left[1-\frac{r_1+r_2}{2}\right]^+ + \left[\beta_- \frac{r_1+r_2}{2}\right]^+.
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\textit{And,} \quad d^c_{2,\mathrm{CMO}}(2) = \min\left\{d^c_{21,\mathrm{CMO}}(2), d^c_{22}(2)\right\}, \quad \textit{where,}\\
&d^c_{21,\mathrm{CMO}}(2) =\\
&\min\left\{[1-r_1]^+, [1-r_1-r_2]^+ + [\beta - r_1 - r_2]^+\right\} + [1-r_2]^+\\
&d^c_{22}(2) = \left[1-\frac{r_2}{2}\right]^+.
\end{aligned}$$

For the TIAN scheme, the achievable DMT is

$$\begin{aligned}
&d^c_{1,\mathrm{TIAN}}(2) = \begin{cases} \left[1-\frac{r_1+\beta}{2}\right]^+, & \textit{if } r_1 \geq \beta \\ 2\left[1-r_1\right]^+, & \textit{if } r_1 < \frac{\beta}{2},\ \beta \geq 1 \\ [1-r_1]^+ + [\beta - r_1]^+, & \textit{if } r_1 < \frac{\beta}{2},\ \beta < 1 \\ \frac{(1-r_1)\beta}{r_1}, & \textit{if } r_1 > \frac{1}{2},\ \frac{\beta}{2} \leq r_1 < \beta \\ [1-r_1]^+ + [\beta - r_1]^+, & \textit{if } r_1 \leq \frac{1}{2},\ \frac{\beta}{2} \leq r_1 < \beta \end{cases}\\
&d^c_{2,\mathrm{TIAN}}(2) = \min\left\{d^c_{21,\mathrm{TIAN}}(2), d^c_{22}(2)\right\}\\
&\textit{where,} \quad d^c_{21,\mathrm{TIAN}}(2) = [1-r_2]^+ + [1-r_1-\beta]^+.
\end{aligned} \tag{19}$$

The overall achievable DMT curve, either between RX1 diversity and first user multiplexing gain r_1 or between RX2 diversity and second user multiplexing gain r_2, of the cooperative ARQ with static decoding scheme for $L = 2$ is the maximum of the achievable DMT using the CMO and the TIAN approaches.

Proof: It can be shown that the error event at RX1 is dominated by the outage event at the end of the second transmission round $\overline{\mathcal{A}}_2$ [2], [3], [7]. Thus, the probability of this outage event is exponentially equal to the error probability at RX1. We now derive this outage probability for the CMO scheme. Let us state the corresponding outage event as

$$\begin{aligned}
&\overline{\mathcal{A}}_2 = \{\mathcal{F}_{T'}, \{\mathcal{O}_1 \cup \mathcal{O}_2\}\}, \qquad \text{where,}\\
&\mathcal{F}_{T'} = \left\{h : \frac{T'}{T}\log\left(1+|h|^2\rho\right) = R_1\right\}\\
&\mathcal{O}_1 = \left\{h_{11}, h_{21} : \frac{T+T'}{T}\log\left(1+|h_{11}|^2\rho\right)\right.\\
&\qquad \left. + \frac{T-T'}{T}\log\left(1+|h_{11}|^2\rho + |h_{21}|^2\rho^\beta\right) < R_1\right\}\\
&\mathcal{O}_2 = \left\{h_{11}, h_{21} : \frac{T}{T}\log\left(1+|h_{11}|^2\rho + |h_{21}|^2\rho^\beta\right) + \frac{T'}{T}\log\left(1+|h_{11}|^2\rho\right)\right.\\
&\qquad \left. + \frac{T-T'}{T}\log\left(1+|h_{11}|^2\rho + |h_{21}|^2\rho^\beta\right) < R_1 + R_2\right\}.
\end{aligned} \tag{20}$$

Defining $f = \frac{T'}{T}$, $|h_{ij}|^2 = \rho^{-\gamma_{ij}}$, and $|h|^2 = \rho^{-u}$, the high-ρ approximation of the previous events can be written as

$$\begin{aligned}
&\mathcal{F}_{T'} = \{u : f\,[1-u]^+ = r_1\}\\
&\mathcal{O}_1 = \left\{\gamma_{11}, \gamma_{21}, f : (1+f)\,[1-\gamma_{11}]^+\right.\\
&\qquad \left. + (1-f)\max\left\{[1-\gamma_{11}]^+, [\beta-\gamma_{21}]^+\right\} < r_1\right\}\\
&\mathcal{O}_2 = \left\{\gamma_{11}, \gamma_{21}, f : (2-f)\max\left\{[1-\gamma_{11}]^+, [\beta-\gamma_{21}]^+\right\}\right.\\
&\qquad \left. + f\,[1-\gamma_{11}]^+ < r_1 + r_2\right\}.
\end{aligned} \tag{21}$$

Since we have $\max_{f\in[r_1,1]} \Pr(\mathcal{O}_1) \doteq \rho^{-d^c_{11,\mathrm{CMO}}(2)}$, thus,

$$d^c_{11,\mathrm{CMO}}(2) = \min_{\gamma_{11},\gamma_{21},u\in\mathcal{O}_1}\{\gamma_{11}+\gamma_{21}+u\} \tag{22}$$

By checking the above constraints for the different values of r_1, we can show that [7]

$$d^c_{11,\mathrm{CMO}}(2) = \min_{f\in[r_1,1]} \begin{cases} 2-\frac{r_1}{2}-\frac{r_1}{f}, & \text{if } r_1 \geq 2\beta \\ 2+\frac{(1-f)\beta - r_1}{1+f}-\frac{r_1}{f}, & \text{if } (1-f)\beta \leq r_1 < 2\beta \\ 2+\beta-\frac{r_1}{1-f}-\frac{r_1}{f}, & \text{if } r_1 < (1-f)\beta \end{cases} \tag{23}$$

The function $2-\frac{r_1}{2}-\frac{r_1}{f}$ is monotonically increasing in f, thus, its minimum is at $f = r_1$. For $r_1 < 2\beta$, we have

$$d^c_{11,\mathrm{CMO}}(2) = \min_{f\in[r_1,1]} \begin{cases} 2+\frac{(1-f)\beta - r_1}{1+f}-\frac{r_1}{f}, & \text{if } f \geq 1-\frac{r_1}{\beta} \\ 2+\beta-\frac{r_1}{1-f}-\frac{r_1}{f}, & \text{if } f < 1-\frac{r_1}{\beta} \end{cases} \tag{24}$$

The function $2+\frac{(1-f)\beta - r_1}{1+f}-\frac{r_1}{f}$ is a concave function over $f \in [r_1, 1]$, hence, it attains its minimum at the edges. Thus, for $r_1 \geq 1-\frac{r_1}{\beta}$, the function $2+\frac{(1-f)\beta - r_1}{1+f}-\frac{r_1}{f}$ attains its minimum at $f = r_1$ or $f = 1$. On the other hand, when $r_1 < 1-\frac{r_1}{\beta}$, it attains its minimum at $f = 1-\frac{r_1}{\beta}$ or $f = 1$. The function $2+\beta-\frac{r_1}{1-f}-\frac{r_1}{f}$ is also monotonically increasing in f over $f \in [r_1, 1]$ and attains its minimum at $f = r_1$.

Notice that the condition $f < 1 - \frac{r_1}{\beta}$ implies that $r_1 < 1 - \frac{r_1}{\beta}$ since $r_1 \leq f \leq 1$. Based on these arguments, we can easily derive the equation for $d^c_{11,\text{CMO}}(2)$ given in (18). Using similar arguments, we can derive the result for $d^c_{12,\text{CMO}}(2)$, where $\max_{f \in [r_1,1]} \Pr(\mathcal{O}_2) \doteq \rho^{-d^c_{12,\text{CMO}}(2)}$.

For the TIAN scheme, the outage event at RX1 at the end of the second transmission round can be expressed as

$$\overline{\mathcal{A}}_2 = \{\mathcal{F}_{T'}, \mathcal{O}_3\}, \qquad \text{where,}$$

$$\mathcal{O}_3 = \Bigg\{h_{11}, h_{21} : \log\left(1 + \frac{|h_{11}|^2\rho}{1 + |h_{21}|^2\rho^{\beta}}\right) + \frac{T'}{T}\log\left(1 + |h_{11}|^2\rho\right) + \frac{T - T'}{T}\log\left(1 + |h_{11}|^2\rho + |h_{21}|^2\rho^{\beta}\right) < R_1\Bigg\}. \tag{25}$$

Using similar arguments, we derive the result for $d^c_{1,\text{TIAN}}(2)$.

For both the CMO and TIAN schemes, it can be shown that the error event at RX2 $\{\mathcal{E}_2\}$ is dominated by the events $\{\mathcal{E}_2, \mathcal{A}_1, \overline{\mathcal{B}}_2\}$ and $\{\mathcal{E}_2, \overline{\mathcal{A}}_1, \overline{\mathcal{B}}_1\}$. The former event represents the error event at RX2 when RX1 receives an ACK at the end of round 1, while the latter represents the error event at RX2 when RX1 receives a NACK at the end of round 1 [7]. Thus,

$$\Pr(\mathcal{E}_2) \doteq \Pr(\overline{\mathcal{A}}_1, \overline{\mathcal{B}}_1) + \Pr(\mathcal{A}_1, \overline{\mathcal{B}}_2) \doteq \Pr(\overline{\mathcal{A}}_1)\Pr(\overline{\mathcal{B}}_1) + \Pr(\overline{\mathcal{B}}_2), \tag{26}$$

as the events $\overline{\mathcal{A}}_1$ and $\overline{\mathcal{B}}_1$ are independent, as well as the events $\mathcal{A}_1$ and $\overline{\mathcal{B}}_2$. Also, $\Pr(\mathcal{A}_1) \doteq 1$.

Using the definitions for $\overline{\mathcal{A}}_1$ and $\overline{\mathcal{B}}_1$ as in (9), (11) and (16), the results for $d^c_{2,\text{CMO}}(2)$ and $d^c_{2,\text{TIAN}}(2)$ can be derived. ■

B. Cooperative ARQ with Dynamic Decoding

In this case, each time *both* TX1 and TX2 begin to transmit new messages, RX1 decides to use either the CMO or the TIAN decoding according to the channel conditions revealed to it, h_{11} and h_{21}. The decoding scheme is no longer known a priori but is dynamically decided each time users transmit new messages. It is worthwhile noticing that the second transmitter has no CSI to dynamically change its splitting parameters according to the channel conditions [7].

Theorem 3. *The achievable DMT of the cooperative ARQ with dynamic decoding for $L = 2$ is characterized by*

$$\begin{aligned}
&d^c_{1,\text{DD}}(2) = \min\left\{d^c_{11,\text{DD}}(2), d^c_{12,\text{DD}}(2)\right\}, \qquad \textit{where,}\\
&d^c_{11,\text{DD}}(2) = d^c_{11,\text{CMO}}(2)\\
&d^c_{12,\text{DD}}(2) = \begin{cases} d^c_{1,\text{TIAN}}(2), & \textit{if } r_2 \geq \beta\\ d^c_{12,\text{CMO}}(2), & \textit{if } r_2 < \beta,\ r_1 \geq r_2\\ \left[\beta - \frac{(2r_1 - 1)r_2}{r_1}\right]^+, & \textit{if } r_2 < \beta,\ \frac{1}{2} \leq r_1 < r_2\\ [1 - r_1]^+ + [\beta - r_1]^+, & \textit{if } r_2 < \beta,\ r_1 < \min\left\{\frac{1}{2}, r_2\right\}. \end{cases}\\
&\textit{And,} \quad d^c_{2,\text{DD}}(2) = \min\left\{d^c_{21,\text{DD}}(2), d^c_{22,\text{DD}}(2)\right\}, \qquad \textit{where,}\\
&d^c_{21,\text{DD}}(2) = [1 - r_2]^+ + \max\left\{d_{1,\text{CMO}}(1), d_{1,\text{TIAN}}(1)\right\}\\
&d^c_{22,\text{DD}}(2) = \left[1 - \frac{r_2}{2}\right]^+,
\end{aligned} \tag{27}$$

where, $d_{1,\text{CMO}}(1)$, $d_{1,\text{TIAN}}(1)$, $d^c_{11,\text{CMO}}(2)$, $d^c_{12,\text{CMO}}(2)$, and $d^c_{1,\text{TIAN}}(2)$ are as given in (7), (6), (18), and (19), respectively. Recall that we can get $d_{1,\text{TIAN}}(1)$ from (6) via substitution with $b = 0$ and $t_2 = 0$.

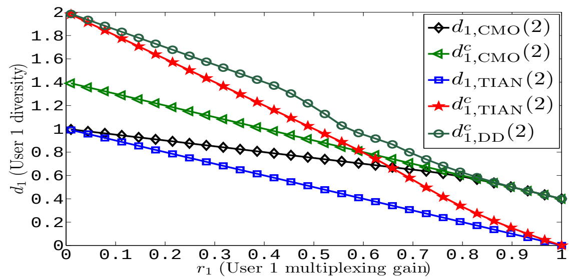

Fig. 2: The DMT of user 1 using the different non-cooperative and cooperative ARQ schemes for $\beta = 1.3$, $r_2 = 0.9$, and $L = 2$.

Proof: (Sketch) Based on the dynamic decoding scheme, outage at RX1 at the end of round 2 can be described as $\overline{\mathcal{A}}_2 = \{\mathcal{F}_{T'}, \{\mathcal{O}_1 \cup \mathcal{O}_2\}, \mathcal{O}_3\}$, where $\mathcal{F}_{T'}$, $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$ are as given in (20) and (25). The rest of the proof follows from analyzing the outage region above by analogy to the analysis of the static decoding scheme in the previous subsection and is omitted due to space limitations (See full proof in [7]). ■

To summarize our work, we show the DMT of the first user under the use of all the previously mentioned ARQ schemes for $L = 2$ in Fig. 2. It is obvious that the performance of the cooperative ARQ with dynamic decoding is better than the achievable performance of its static counterpart for some values of the first user multiplexing gain r_1.

V. Conclusion

We characterized the achievable diversity, multiplexing, and delay tradeoff for the outage limited two user single antenna Rayleigh fading ARQ ZIC under the use of non-cooperative and cooperative ARQ protocols. Under cooperation, we characterized the achievable tradeoff using static and dynamic decoders. We used the well-known HK approach as well as two special cases of it, where only a common or a private message is transmitted, to derive achievability results. Our characterization comes in closed-form expressions of the individual diversities as a function of the maximum number of transmission rounds (maximum delay), multiplexing gain pairs, interference level, rate and power splitting parameters.

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