

Asymptotic Behavior Analysis and Performance Optimization in Full Duplex Massive MIMO

Radwa Sultan[†], Karim G. Seddik^{*}, Zhu Han[†], and Behnaam Aazhang[‡]

[†] Electrical and Computer Engineering Department, University of Houston, TX, USA

^{*} Electronics and Communications Engineering Department, American University in Cairo, New Cairo, Egypt

[‡] Department of Electrical and Computer Engineering, Rice University, Houston, USA

Abstract—In this paper, we consider rate maximization of a single full-duplex (FD) massive MIMO base station (BS) with multiple downlink (DL) and uplink (UL) users. The BS applies transmit precoders on the DL transmission. These precoders are designed to reduce the multiuser interference among DL transmissions and to reduce the self-interference (SI) level from the DL transmissions at the BS UL receiving antennas. The self-interference is reduced by zero-nulling the DL transmissions at some of the UL receiving antennas. We derive lower bounds on the achievable DL and UL capacities. Based on the derived bounds, we optimize over the ratio of the transmit to receive antennas at the base station to maximize the achievable capacities. We also optimize over the portion of UL receive antennas that are zero-nulled by the DL transmit precoders to limit the effect of SI on the UL transmissions. Two different formulations of the rate region maximization problem are presented. The first formulation, which maximizes the UL rate for a given DL rate, is proved to be a convex optimization problem; for this formulation, an exact solution for the optimization problem is derived. The second formulation, which maximizes the DL rate for a given UL rate, is proved to be non-convex; therefore, the dual problem is formulated and solved. Numerical results validate the derived DL and UL bounds as well as the proposed rate maximization solutions.

I. INTRODUCTION

Full Duplex (FD) communication is envisioned as one of the candidates, that is capable of supplying the increase in communication links capacity required in the next generation cellular networks. Theoretically speaking, enabling FD communication can double the data rate achieved by half-duplex (HD) communication. In general, self-interference (SI), which is the interference from the FD node transmission on its reception, is considered the main obstacle for the FD feasibility until the recent evolution of SI cancellation techniques [1]–[3]. On the other hand, large-scale multiple-input-multiple-output (MIMO) or massive MIMO [4]–[7] has proven its ability to increase the spectral efficiency and communication reliability than the traditional MIMO as largely increasing the number of antennas helps in increasing the network’s densification. Additionally, with very large number of antennas, the simplest form of user detection, beamforming, as matched filter or zero-forcing, becomes optimal.

Applying FD communication in massive MIMO networks is expected to increase the communication network capacity. In [8], a theoretical framework for the study of FD massive MIMO cellular networks over Rician SI, Rayleigh and other interference fading channels is presented. A downlink (DL) linear zero-forcing with the SI nulling precoding scheme at the FD base stations (BSs) and an uplink (UL) SI-aware fractional power control mechanism at the FD user equipments are incorporated. In [9], a network with a single FD massive MIMO

BS is studied. The necessary conditions for FD operation to outperform HD operation are derived in terms of the number of transmitting antennas, co-channel interference, and self-interference cancellation. In [10], a multiuser-MIMO (MU-MIMO) system with a FD BS and a number of FD users is considered. It is shown that the detrimental impact of the SI can be eliminated by a very large number of antennas at the BS if the power scaling scheme is appropriately applied, as well as the effect of multi-users-interference (MUI) and inter-user interference.

It needs to be mentioned that, in the case of massive MIMO, the analog SI cancellation circuit will be very complicated. Accordingly, in order to prevent RF saturation of the receiver, SI cancellation via transmit beamforming is needed. Therefore, in the DL transmission, besides the necessity of decreasing the MUI, the SI power should be reduced by transmit beamforming. The main idea to suppress the SI is to design the precoder such that it simultaneously forms the beams of the DL signals to the users in DL mode and forms beams to the BS receive antennas to send all-zero signals, which will avoid SI. However, this extra requirement is expected to decrease the DL transmission gain at the DL users as the number of effective DL transmit antennas will decrease by increasing the number of nulls required. In [11], in order to solve the fundamental issue of SI cancellation in FD cellular communication systems, two schemes that exploit the excess of antennas present at the BS are proposed. Preliminary results show that large-scale MIMO is able to render full-duplex communication more resilient against inter-user interference and helps to mitigate the effects of residual TX-RF impairments. In [12], in order to improve the UL spectral efficiency, the utilization schemes of the spatial resources which can be used to enhance the UL performance are investigated. Results show that there are some tradeoffs between achieving receive diversity gain for the UL desired signal and suppressing the SI. In [13], an optimization problem in terms of the number of transmit and receive antennas to maximize the sum of DL and UL sum-rates is formulated. It is shown that the optimal antenna ratio converges to the ratio between the number of DL users and UL users.

In this paper, a single cell network with a single FD massive MIMO BS is considered. We study the trade-off between the DL transmission gain and SI cancellation via transmit beamforming. Lower bounds of both DL and UL capacities in massive MIMO systems are derived. In order to maximize the feasible rate region, two maximization problems are formulated to optimize both the ratio between transmit and receive antennas as well as the ratio of receive antennas at which SI is cancelled.

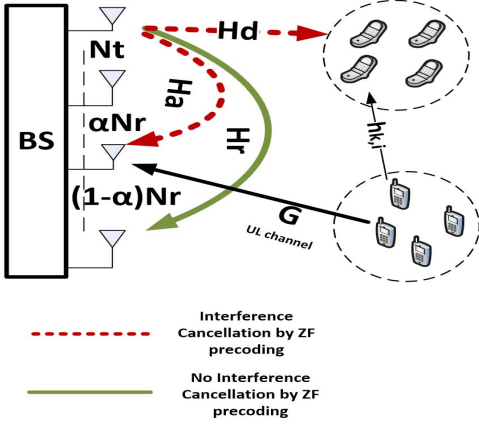


Fig. 1. System Model.

The rest of the paper is organized as follows. The system model is presented in Section II. The asymptotic analysis and the trade-off between DL capacity and SI cancellation are presented in Section III. The problem formulation for rate region maximization is presented in Section IV. Numerical analysis is presented in Section V. Finally, the paper is concluded in Section VI.

II. SYSTEM MODEL

We consider a wireless network with K users and a single N -antennas FD massive MIMO BS, in which $N \rightarrow \infty$. Under the assumption of FD transmission, the BS is equipped with a special FD radio to help suppress the SI. Additionally, the antennas are divided to N_t transmit antennas and N_r receive antennas to simultaneously serve K_d DL users and K_u uplink UL users, respectively.

In the DL transmission, it is assumed that transmit precoding is used to null the multiuser interference among the DL transmissions. However, in case of the FD transmission, we might need to null the SI on a proportion of the BS receive antennas. This can be achieved by adding this proportion of the BS receive antennas to the DL users and designing the transmit precoder to null the multiuser interference among the DL transmission and to null the DL transmissions at these receiving antennas by sending them zero signals. Since considering the SI cancellation in the precoder design is expected to decrease the DL transmission gain, we propose a variable $0 \leq \alpha \leq 1$ that controls the SI cancellation by transmit beamforming. When $\alpha = 0$, no BS receive antennas are considered to suppress SI. On the hand, when $\alpha = 1$, the precoder sends all-zero signals to all the N_r receiving antennas. Finally, when $0 < \alpha < 1$, the precoder sends all zero-signals to only αN_r receiving antennas. It should be mentioned that choosing specific αN_r antennas at which SI is cancelled is not required because, in very large antenna systems, the channel gains effect between the transmitting antennas and each receiving antenna are the same as a result of channel hardening, i.e., the signal to interference noise ration (SINR) at any receiving antenna will be the same [14]. Additionally, it should be guaranteed that $N_t > (K_d + \alpha N_r)$. The system model is shown in Fig. 1.

Based on the above assumptions, the received signal at the k^{th} DL user is given by

$$y_d = \underbrace{\sqrt{P_d} \mathbf{h}_k \mathbf{w}_k x_k}_{\text{Desired Signal}} + \sum_{\substack{j=1 \\ j \neq k \\ j \in \Phi_{dl}}} \underbrace{\sqrt{P_d} \mathbf{h}_k \mathbf{w}_j x_j}_{\text{MUI}} + \sum_{i \in \Phi_{ul}} \underbrace{\sqrt{P_u} h_{k,i} s_i}_{\text{CCI}} + n_k^d, \quad (1)$$

where, P_d is the DL transmission power, $\mathbf{h}_k \in \mathbb{C}^{1 \times (N_r)}$ is the channel vector between the k^{th} DL user and the BS transmit antennas. All channel coefficients are assumed to be independent and identically distributed (i.i.d.) zero mean complex Gaussian random variables with unit variance, i.e., the Rayleigh flat fading channel model. $\mathbf{w}_k \in \mathbb{C}^{(N_r) \times 1}$ is the precoding vector for the k^{th} DL user's data, we consider the zero-forcing (ZF) precoder in the DL, in which the precoding matrix $\mathbf{F}_d \in \mathbb{C}^{(N_r) \times (\alpha N_r + K_d)}$ and the K^{th} DL user precoding vector \mathbf{w}_k are given, respectively, by

$$\mathbf{F}_d = \mathbf{H}_t^H (\mathbf{H}_t \mathbf{H}_t^H)^{-1}, \quad (2)$$

$$\mathbf{w}_k = \frac{\mathbf{f}_k}{\|\mathbf{F}_d\|_F},$$

where, $\mathbf{H}_t = [\mathbf{H}_d \quad \mathbf{H}_a]^T \in \mathbb{C}^{(\alpha N_r + K_d) \times (N_r)}$ is the aggregated DL channel matrix which is composed of the DL channel matrix $\mathbf{H}_d = [\mathbf{h}_1 \quad \mathbf{h}_2 \cdots \mathbf{h}_{K_d}]^T \in \mathbb{C}^{K_d \times (N_r)}$ and $\mathbf{H}_a \in \mathbb{C}^{\alpha N_r \times (N_r)}$ is the proportion of the channel between the BS transmit and receive antennas at which the SI will be nulled, \mathbf{f}_k is the k^{th} DL user precoding vector before normalization, $\|\mathbf{F}_d\|_F$ is the Frobenius norm of the precoding matrix \mathbf{F}_d and determines the matrix normalization coefficient for the ZF precoder [15]. Furthermore, in (1), x_k is the k^{th} DL transmit signal where $\mathbb{E}\{\|x_k\|^2\} = 1$. The second term in (1) is the MUI from the other DL transmissions, Φ_{dl} is the set of users scheduled for DL transmission. The third term in (1) is the co-channel interference (CCI) from the scheduled UL transmissions, where $h_{k,i}$ is the channel coefficient between from the i^{th} UL user to the k^{th} DL user, s_i is the i^{th} UL user transmit signal with $\mathbb{E}\{\|s_i\|^2\} = 1$ and Φ_{ul} is the set of users scheduled for UL transmission. Finally, n_k^d is the additive white Gaussian noise (AWGN) term with variance σ^2 . Accordingly, the received SINR at the k^{th} DL user is given by

$$\Gamma_{dl} = \frac{P_d \|\mathbf{h}_k \mathbf{w}_k\|^2}{\sigma^2 + \sum_{\substack{j=1 \\ j \neq k \\ j \in \Phi_{dl}}} P_d \|\mathbf{h}_k \mathbf{w}_j\|^2 + \sum_{i \in \Phi_{ul}} P_u \|h_{k,i}\|^2}, \quad (3)$$

On the other hand, the received signal at the BS from the k^{th} UL user is given by

$$y_u = \underbrace{\sqrt{P_u} \mathbf{u}_k \mathbf{g}_k s_k}_{\text{Desired Signal}} + \sum_{\substack{i=1 \\ i \neq k \\ i \in \Phi_{ul}}} \underbrace{\sqrt{P_u} \mathbf{u}_k \mathbf{g}_i s_i}_{\text{MUI}} + \underbrace{\sqrt{C} \sqrt{P_d} \mathbf{u}_k \mathbf{H}_{si} \mathbf{W}_d \mathbf{x}_d}_{\text{SI}} + \mathbf{u}_k n_u, \quad (4)$$

where, P_u is the UL transmission power, $\mathbf{u}_k \in \mathbb{C}^{1 \times N_r}$ is the receive vector for the k^{th} UL user's data, $\mathbf{g}_k \in \mathbb{C}^{N_r \times 1}$ is the UL channel vector between the k^{th} UL user and the BS receive antennas. In this paper, we assume a ZF receiver, the UL receiving matrix $\mathbf{U}_{ul} \in \mathbb{C}^{K_u \times N_r}$ is given by

$$\mathbf{U}_{ul} = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H, \quad (5)$$

where $\mathbf{G} \in \mathbb{C}^{N_r \times K_u} = [\mathbf{u}_1 \ \mathbf{u}_2 \ \dots \ \mathbf{u}_{K_u}]^T$ is the UL channel matrix between the UL users and the BS receive antennas. The second term of the expression, in (4), is the MUI on the k^{th} UL user from other UL transmissions, the third term, in (4), is the residual self-interference (RSI) power, $0 \leq C \leq 1$ is the SI cancellation coefficient introduced by the FD radio [12], [13], $\mathbf{H}_{si} = [\mathbf{H}_a \ \mathbf{H}_r]^T \in \mathbb{C}^{N_r \times N_t}$ is the channel between the BS transmit and receive antennas with $\mathbf{H}_r \in \mathbb{C}^{(1-\alpha)N_r \times (N_r)}$ which is the remaining proportion of the channel between the BS transmit and receive antennas at which the SI will only be cancelled by the FD radio but not the transmit precoder, $\mathbf{W}_d = \mathbf{F}_d / \|\mathbf{F}_d\|_F$ is the DL normalized precoding matrix and $\mathbf{x}_d = [\mathbf{x} \ \mathbf{0}_{\alpha N_r}]^T \in \mathbb{C}^{K_d + \alpha N_r \times 1}$ is the transmitted DL signal which is composed of the DL signal to the users $\mathbf{x} \in \mathbb{C}^{K_d \times 1}$ and the zero signals sent to the αN_r BS receive antennas for SI cancellation. Accordingly the received SINR for the k^{th} UL user, at the BS receive antennas, is given by

$$\Gamma_{ul} = \frac{P_u \|\mathbf{u}_k \mathbf{g}_k\|^2}{\sigma^2 \|\mathbf{u}_k\|^2 + \sum_{\substack{i=1 \\ i \neq k \\ i \in \Phi_{ul}}} P_u \|\mathbf{u}_k \mathbf{g}_i\|^2 + CP_d \|\mathbf{u}_k \mathbf{H}_{si} \mathbf{W}_d \mathbf{x}_d\|^2}, \quad (6)$$

III. ASYMPTOTIC SUM RATE BEHAVIOR AND THE TRADE-OFF BETWEEN DL CAPACITY AND SI CANCELLATION

In the section, we study the asymptotic rate behavior when $N = N_t + N_r \rightarrow \infty$. First, we start by studying the behavior of the DL transmission.

Proposition 1: A lower bound for the total FD DL rate C_d is given by

$$C_d = K_d \log_2 \left(1 + \frac{N_t - (K_d + \alpha N_r)}{K_d + \alpha N_r} \left(\frac{P_d}{\sigma^2 + P_u K_u} \right) \right). \quad (7)$$

Proof: From (3) and by applying the Jensen's inequality for the convex function, we can get that

$$\begin{aligned} C_d &= K_d \mathbb{E} \{ \log_2(1 + \Gamma_{dl}) \}, \\ &\stackrel{(a)}{\geq} K_d \log_2 \left(1 + \left(\mathbb{E} \left\{ \frac{1}{\Gamma_{dl}} \right\} \right)^{-1} \right) \\ &\stackrel{(b)}{=} K_d \log_2 \left(1 + \left(\mathbb{E} \left\{ \frac{\sigma^2 + \sum_{i \in \Phi_{ul}} P_u \|h_{k,i}\|^2}{P_d / \|\mathbf{F}_d\|_F^2} \right\} \right)^{-1} \right) \\ &\stackrel{(c)}{=} K_d \log_2 \left(1 + P_d \frac{N_t - (K_d + \alpha N_r)}{(K_d + \alpha N_r)(\sigma^2 + \mathbb{E} \{ \sum_{i \in \Phi_{ul}} P_u \|h_{k,i}\|^2 \})} \right) \\ &\stackrel{(d)}{=} K_d \log_2 \left(1 + \frac{N_t - (K_d + \alpha N_r)}{K_d + \alpha N_r} \left(\frac{P_d}{\sigma^2 + P_u K_u} \right) \right) \end{aligned} \quad (8)$$

where (a) results from applying the Jensen inequality for the convex function $\log_2(1 + 1/x)$ and assuming that, in massive MIMO systems, each DL user from the K_d users will experience the receive the same SINR. (b) is obtained from the ZF array gain properties, in which $\mathbf{F}_{dl} \mathbf{H}_t = \mathbf{I}_{K_d + \alpha N_r} / \|\mathbf{F}_{dl}\|_F$. Therefore, $\mathbf{h}_k \mathbf{w}_i = \delta_{ki} / \|\mathbf{F}_{dl}\|_F$, where $\delta_{ki} = 1$ when $k = i$ and 0 otherwise. Additionally, (c) is obtained by applying the properties of the

central Wishart matrix [16, lemma 2.10] to obtain the value of $\|\mathbf{F}_d\|_F$ as follows

$$\begin{aligned} \|\mathbf{F}_d\|_F^2 &= \text{tr} \{ \mathbf{F}_d^H \mathbf{F}_d \}, \\ &= \text{tr} \{ (\mathbf{H}_t \mathbf{H}_t^H)^{-1} \mathbf{H}_r \mathbf{H}_r^H (\mathbf{H}_t \mathbf{H}_t^H)^{-1} \}, \\ &= \text{tr} \{ (\mathbf{H}_t \mathbf{H}_t^H)^{-1} \} \\ &= \frac{K_d + \alpha N_r}{N_t - (K_d + \alpha N_r)}. \end{aligned} \quad (9)$$

Finally, (d) is obtained by knowing that $\sum_{i \in \Phi_{ul}} \|h_{k,i}\|^2 \sim E(K_u, 1)$ which is the Erlang distribution with K_u shape parameter and unity rate parameter.

The next step is to analyze the UL capacity when $N \rightarrow \infty$.

Proposition 2: A lower bound for the FD UL rate C_u is given by

$$C_u = K_u \log_2 \left(1 + \frac{P_u(N_r - K_u)}{\sigma^2 + CP_d(1 - \alpha)} \right). \quad (10)$$

Proof: From (6) and by applying the Jensen's inequality for the convex function, we can get that

$$\begin{aligned} C_u &= K_u \mathbb{E} \{ \log_2(1 + \Gamma_{ul}) \}, \\ &\stackrel{(a)}{\geq} K_u \log_2 \left(1 + \left(\mathbb{E} \left\{ \frac{1}{\Gamma_{ul}} \right\} \right)^{-1} \right) \\ &\stackrel{(b)}{=} K_u \log_2 \left(1 + \left(\frac{\mathbb{E} \{ \sigma^2 \|\mathbf{u}_k\|^2 + CP_d \|\dot{\mathbf{u}}_k \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x}\|^2 \}}{P_u} \right)^{-1} \right) \\ &\stackrel{(c)}{=} K_u \log_2 \left(1 + \frac{P_u}{\sigma^2 / (N_r - K_u) + CP_d \mathbb{E} \{ \|\dot{\mathbf{u}}_k \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x}\|^2 \}} \right) \\ &\stackrel{(d)}{=} K_u \log_2 \left(1 + \frac{P_u(N_r - K_u)}{\sigma^2 + CP_d(1 - \alpha)} \right) \end{aligned} \quad (11)$$

where (a) results from applying the Jensen inequality for the convex function $\log_2(1 + 1/x)$. (b) is obtained from the ZF array gain properties, in which $\mathbf{G}_{U_{ul}} = \mathbf{I}_{K_u}$. Therefore, $\mathbf{u}_k \mathbf{g}_i = \delta_{ki}$. Furthermore, after the transmit beamforming, the SI is completely cancelled at αN_r and the only remaining term in the RSI term is $\mathbb{E} \{ \|\dot{\mathbf{u}}_k \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x}\|^2 \}$, where $\dot{\mathbf{u}}_k \in \mathbb{C}^{(1-\alpha)N_r \times 1}$ is the proportion of the receive vector corresponding to the $(1 - \alpha)N_r$ receive antennas at which the SI was only cancelled by the FD radio, $\mathbf{W}_{dl} \in \mathbb{C}^{K_d \times N_r}$ is the transmit precoding matrix applied for the K_d DL user's data. It is assumed that $\mathbb{E} \{ \|\mathbf{W}_{dl} \mathbf{x}\|^2 \} = 1$. The derivation of the remaining RSI term from the initial term calculated in (6) is presented in Appendix A. Additionally, equality (c) in (11) is obtained from the work presented in [17] proving that $\mathbb{E} \{ \|\mathbf{u}_k\|^2 \} = 1 / (N_r - K_u)$. Finally, equality (d) is obtained by calculating $\mathbb{E} \{ \|\dot{\mathbf{u}}_k \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x}\|^2 \}$ as follows

$$\begin{aligned} &\mathbb{E} \{ \|\dot{\mathbf{u}}_k \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x}\|^2 \} \\ &= \mathbb{E} \{ \mathbf{x}^H \mathbf{W}_{dl}^H \mathbf{H}_r^H \dot{\mathbf{u}}_k \dot{\mathbf{u}}_k^H \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x} \} \\ &= \frac{1}{N_r(N_r - K_u)} \mathbb{E} \{ \mathbf{x}^H \mathbf{W}_{dl}^H \mathbf{H}_r^H \mathbf{H}_r \mathbf{W}_{dl} \mathbf{x} \} \\ &= \frac{1 - \alpha}{N_r - K_u} \mathbb{E} \{ \mathbf{x}^H \mathbf{W}_{dl}^H \mathbf{W}_{dl} \mathbf{x} \} \\ &= \frac{1 - \alpha}{N_r - K_u}. \end{aligned} \quad (12)$$

From expressions (7) and (10), we can see the effect of varying α on the lower bounds of the DL and UL capacities, respectively. In (7), any increase in α , which corresponds to including more receive antennas in SI cancellation by transmit beamforming, will cause a degradation in the DL capacity; however, this increase in α will lead to lower RSI, and hence, better UL capacity as shown in (10). In order to maximize the sum rate, the value of α as well as the value of N_t and N_r should be optimized.

IV. ANTENNA RATIO AND SELF-INTERFERENCE CANCELLATION OPTIMIZATION FOR MAXIMIZING THE FEASIBLE RATE REGION

In this section, we find the optimal $\beta = N_t/N_r$ and α that maximize the feasible rate region for the proposed system model. First, we start by rewriting the DL and UL capacities defined in (7) and (10), respectively, in terms of β and α . The DL and UL capacities are given, respectively, by

$$\begin{aligned} C_d(\beta, \alpha) &= K_d \log_2 \left(1 + \left(\frac{P_d}{\sigma^2 + P_u K_u} \right) \left(\frac{\beta}{\frac{K_d(\beta+1)}{N} + \alpha} - 1 \right) \right), \\ C_u(\beta, \alpha) &= K_u \log_2 \left(1 + \frac{P_u \left(\frac{N}{\beta+1} - K_u \right)}{\sigma^2 + C P_d (1 - \alpha)} \right). \end{aligned} \quad (13)$$

Since $N_t > (K_d + \alpha N_r)$, then β should be larger than $(\alpha + K_d/N)/(1 - K_d/N)$. The feasible rate region is obtained by setting the value of C_d to a certain constant value, and then by sweeping C_d over its all possible range while maximizing C_u value that can be achieved for each value of C_d . Therefore, for a constant C_d , the relation between β and α is given by

$$\beta \left(1 - \Psi \frac{K_d}{N} \right) = \Psi \left(\frac{K_d}{N} + \alpha \right), \quad (14)$$

where $\Psi = 1 + \frac{(\sigma^2 + P_u K_u)(2^{C_d/K_d} - 1)}{P_d}$. Since $\Psi > 1$, then the relation in (14) satisfies that $\beta > (\alpha + K_d/N)/(1 - K_d/N)$. Therefore, the feasible rate region maximization problem is given by

$$\begin{aligned} \max_{\beta, \alpha} \quad & C_u(\beta, \alpha) \\ \text{subject to} \quad & 0 \leq \alpha \leq 1, \\ & \beta > \frac{\alpha + \frac{K_d}{N}}{1 - \frac{K_d}{N}}, \\ & \beta \left(1 - \Psi \frac{K_d}{N} \right) = \Psi \left(\frac{K_d}{N} + \alpha \right). \end{aligned} \quad (15)$$

To find the solution of rate region maximization problem in (15), we will first study the convexity of the objective function.

Proposition 3: The total UL capacity $C_u(\beta, \alpha)$, defined in (13), is a convex function in both β and α given that $N_r \geq \frac{4}{3} K_u$.

The proof is presented in Appendix B

Therefore, we can reformulate the rate maximization in terms of β or α without affecting the convexity of the problem. After substituting the value of β in terms of α in (15), the rate maximization problem is now given by

$$\begin{aligned} \max_{\alpha} \quad & C_u(\alpha) \\ \text{subject to} \quad & 0 \leq \alpha \leq 1, \end{aligned} \quad (\mathbf{P1})$$

Since the objective function is a convex function, the maximization problem's solution lies on the boundaries of the feasible set defined by the constraints in **(P1)**. Therefore, after some simple mathematical steps, it can be shown that the values of α and β maximizing C_u for given C_d are given, respectively, by

$$\begin{aligned} \alpha &= 1, \\ \beta &= \frac{\Psi \left(1 + \frac{K_d}{N} \right)}{1 - \Psi \left(\frac{K_d}{N} \right)}. \end{aligned} \quad (16)$$

On the other hand, the rate maximization problem can also be derived by setting the value of C_u to a certain constant value, and then by sweeping C_u over its all possible range while maximizing C_d value that can be achieved for each value of C_u . Therefore, for a constant C_u , the relation between β and α is given by

$$\beta = \frac{N P_u}{P_u K_u + (2^{C_u/K_u} - 1)(\sigma^2 + C P_d (1 - \alpha))} - 1. \quad (17)$$

It should be noticed that in order to guarantee that $\beta > (\alpha + K_d/N)/(1 - K_d/N)$, then the FD radio should satisfy the following condition

$$C_{max} = \frac{1}{P_d} \left(\frac{P_u(N - K_u)}{2^{C_u/K_u} - 1} - \sigma^2 \right). \quad (18)$$

Accordingly, the rate maximization problem is given by

$$\begin{aligned} \max_{\alpha} \quad & C_d(\alpha) \\ \text{subject to} \quad & 0 \leq \alpha \leq 1. \end{aligned} \quad (\mathbf{P2})$$

In that case, the optimization problem defined in **(P2)** is not a convex optimization problem. Therefore, we will derive the lagrangian function and apply the KKT conditions to solve the dual problem. Accordingly, the dual problem of the rate maximization problem in **(P2)** is given by

$$\begin{aligned} \max_{\alpha, \lambda_1, \lambda_2} \quad & \mathcal{L}(\alpha, \lambda_1, \lambda_2) = C_d - \lambda_1(\alpha) + \lambda_2(1 - \alpha) \\ \text{subject to} \quad & 0 \leq \alpha \leq 1. \end{aligned} \quad (19)$$

Proposition 4: The solution of the dual problem using the KKT conditions is given by

$$\begin{aligned} \alpha &= 0, \\ \lambda_1 &= \lambda_2 = 0, \end{aligned} \quad (20)$$

The solution is obtained by differentiating the Lagrangian function and solving the system of equations obtained from applying the KKT conditions.

It is clear that obtaining the feasible rate region by solving the problem in **(P1)** is more straightforward than solving the problem in **(P2)**, as a result of the convexity of the objective function in **(P1)** with respect to α

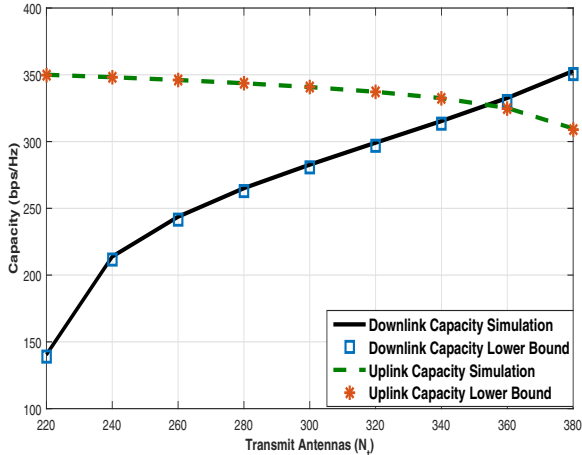


Fig. 2. Validating Derived Downlink and Uplink Bounds for Different Transmit Antennas. Parameters used to generate this figure: $N = 400$, $K_d = 30$, $K_u = 10$, $\alpha = 1$, $\sigma^2 = -110\text{dB}$, $C = 80\text{dB}$, $P_d = 10\text{W}$ and $P_u = 2\text{mW}$

V. NUMERICAL ANALYSIS

In this section, we start by validating the bounds derived in (7) and (10). The system is simulated at $N = 400$ antennas, $K_d = 30$ users, $K_u = 10$ users, and $\alpha = 1$ to show the variation of the DL and UL capacities with different numbers of transmit antennas N_t . The results in Fig. 2 verify that the derived DL and UL lower bounds are tight bounds, as for different numbers of N_t , both the derived bounds and the capacities obtained from simulations match. Also, the results show how C_d increases and C_u decreases with increasing N_t .

In Fig. 3, we verify the values of β and α derived in (16). The system is simulated at $N = 200$ antennas, $K_d = 20$ users, $K_u = 15$ users, and $C_d = 200\text{bps/Hz}$. The results are showing the variation of C_u with α and the corresponding β values calculated from the relation in (14), for different values of cancellation parameter C . The following points can be noticed from the results in Fig. 3, First, the convexity of C_u with respect to α and β can be verified. Second, the uplink capacity C_u is always maximized at $\alpha = 1$ which correspond to a value of $\beta = 7.55$ which match the calculated values from the formulas in (16). Third, from (16), the value of the optimum β maximizing C_u is independent of C value which can be verified from the results in Fig. 3. Additionally, decreasing C increases the value of C_u as a result of the decreased RSI. However, at $\alpha = 1$, the SI is completely cancelled, and therefore, the value of C will not affect the UL performance.

In Fig. 4, we validate the solution of the dual problem derived in (20). The system is simulated at $N = 200$ antennas, $K_d = 10$ users, $K_u = 15$ users for different values of C_u , showing the variation of $C_d(\alpha)$ with α . In the beginning, the results validate that $C_d(\alpha)$ is neither convex nor concave function in α . Furthermore, it can be validated from the results in Fig. 4 that, for different values of C_u , $C_d(\alpha)$ is always maximized at $\alpha = 0$ which validates the solution derived in (20)

Finally, in Fig. 5, the system is simulated at $N = 200$ antennas, $K_d = 20$ users, and $K_u = 10$ showing the feasible rate regions for different FD and HD systems. The results

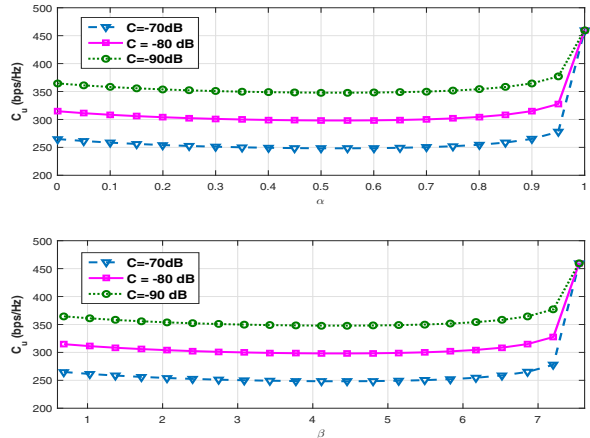


Fig. 3. Validating Rate Maximization Problem Solution when Fixing C_d and Maximizing C_u . Parameters used to generate this figure: $N = 200$, $K_d = 20$, $K_u = 15$, $\sigma^2 = -110\text{dB}$, $C_d = 200\text{bps/Hz}$, $P_d = 10\text{W}$ and $P_u = 2\text{mW}$

compare the derived bounds with the capacities obtained from numerical simulations. Additionally, it compares the rate region obtained from solving the optimization problems in (P1) and (P2) and compares it with the rate region obtained by setting $\beta = K_d/K_u$ and $\alpha = 0$ as suggested in [13], when maximizing the rate region by only optimizing the antennas ratio and without considering the SI cancellation in the DL precoder design, and with the HD rate region. It can be noticed that solving the dual problem of the problem (P2) results in an upper bound with a very small duality gap. Additionally, it can be seen the derived solutions achieve better rate region than that achieved by the HD system and that achieved when setting $\beta = K_d/K_u$ and $\alpha = 0$ which validates the necessity of optimizing both the antennas ratio and the SI cancellation in the rate region maximization problem.

VI. CONCLUSION

In this paper, the asymptotic behavior of full-duplex massive MIMO systems is presented. We derive lower bounds for the downlink and uplink capacities. Afterwards, we show the trade-off between the downlink capacity and decreasing the self-interference using transmit beamforming. Accordingly, a rate region maximization problem optimizing the transmit and receive antennas ratio and the proportion of self-interference cancellation is formulated. From the numerical results, the derived bounds and the rate maximization solution are verified to be very tight. Additionally, it is shown that the proposed scheme achieves better rate region than the scheme with antennas ratio set to the ratio between downlink and uplink users.

APPENDIX A

CALCULATING RSI AFTER TRANSMIT BEAMFORMING SI CANCELLATION

The RSI term in (6) can be rewritten as

$$\begin{aligned}
 RSI &= \mathbf{u}_k \mathbf{H}_{si} \mathbf{W}_d \mathbf{x}_d \\
 &= \begin{bmatrix} \mathbf{u}_k'' & \mathbf{u}_k' \\ 1 \times \alpha N_r & 1 \times (1-\alpha) N_r \end{bmatrix} \begin{bmatrix} \mathbf{H}_d \\ \mathbf{H}_r \\ (1-\alpha) N_r \times N_r \end{bmatrix} \begin{bmatrix} \mathbf{W}_{d1} & \mathbf{W}_{d2} \\ N_r \times K_d & N_r \times \alpha N_r \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{0} \\ \alpha N_r \times 1 \end{bmatrix}. \quad (21)
 \end{aligned}$$

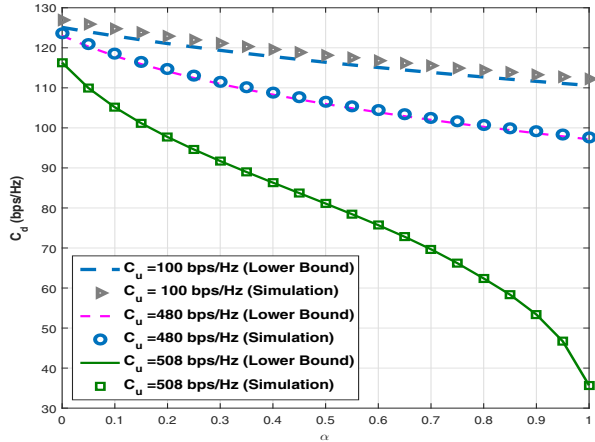


Fig. 4. Validating Rate Maximization Problem Solution when Fixing C_u and Maximizing C_d . Parameters used to generate this figure: $N = 200$, $K_d = 10$, $K_u = 15$, $\sigma^2 = -110\text{dB}$, $P_d = 10\text{W}$ and $P_u = 2\text{mW}$, for $C_u = 100, 480, 508\text{bps/Hz}$

By applying the ZF matrix properties, the RSI is reduced to

$$\begin{aligned}
 &= \begin{bmatrix} \mathbf{u}_k'' & \mathbf{u}_k' \\ 1 \times \alpha N_r & 1 \times (1-\alpha)N_r \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{H}_r \mathbf{W}_{d_{di}} & \mathbf{H}_r \mathbf{W}_{d_{si}} \\ (1-\alpha)N_r \times K_d & (1-\alpha)N_r \times \alpha N_r \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ K_d \times 1 \\ \mathbf{0} \\ \alpha N_r \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{u}_k'' & \mathbf{u}_k' \\ 1 \times \alpha N_r & 1 \times (1-\alpha)N_r \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \alpha N_r \\ \mathbf{H}_r \mathbf{W}_{d_{di}} \mathbf{x} \end{bmatrix} \quad (22) \\
 &= \mathbf{u}_k' \mathbf{H}_r \mathbf{W}_{d_{di}} \mathbf{x} \\
 & \quad 1 \times (1-\alpha)N_r
 \end{aligned}$$

This completes the proof of the expression in (11) in Proposition 2.

APPENDIX B PROOF OF PREPOSITION 3

To prove the convexity of $C_u(\beta, \alpha)$ in β and α , we need to prove that the Hessian matrix of $C_u(\beta, \alpha)$ with respect to β and α is a positive semidefinite, i.e., $\nabla_{\beta, \alpha}^2 C_u(\beta, \alpha) \geq 0$. Due to space limitations, we will state the results obtained from calculating $\nabla_{\beta, \alpha}^2 C_u(\beta, \alpha)$:

- 1) $\frac{\delta^2 C_u}{\delta \alpha^2} > 0$,
- 2) $\frac{\delta^2 C_u}{\delta \beta^2} \geq 0$, given that $N_r \geq \frac{4}{3} K_u$,
- 3) $\det \nabla_{\beta, \alpha}^2 C_u(\beta, \alpha) > 0$, given that $N_r \geq \frac{4}{3} K_u$.

Therefore, from the results obtained above, by setting $N_r \geq \frac{4}{3} K_u$, the UL capacity $C_u(\beta, \alpha)$ will be convex in both β and α .

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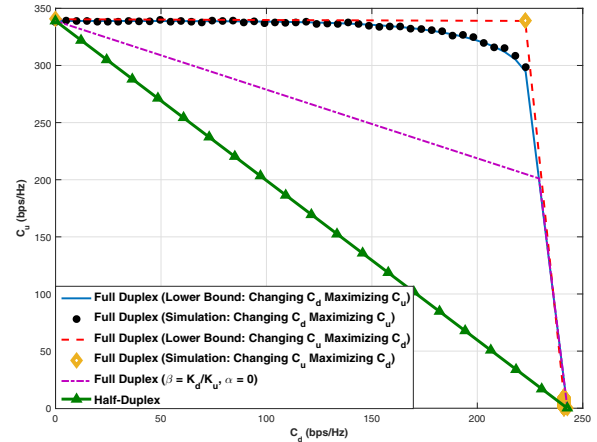


Fig. 5. Feasible Rate Region for Different Full Duplex and Half Duplex Systems. Parameters used to generate this figure: $N = 200$, $K_d = 20$, $K_u = 10$, $\sigma^2 = -110\text{dB}$, $P_d = 10\text{W}$ and $P_u = 2\text{mW}$

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