

# Hybrid Feedback-Based Access Scheme for Cognitive Radio Systems

Sara A. Attalla<sup>‡</sup>, Karim G. Seddik<sup>‡</sup>, Amr A. El-Sherif<sup>++†</sup>, and Sherif I. Rabia<sup>\*⊥</sup>

<sup>‡</sup>Electronics and Communications Engineering Department, American University in Cairo, AUC Avenue, New Cairo 11835, Egypt.

<sup>+</sup>Wireless Intelligent Networks Center (WINC), Nile University, Giza 12588, Egypt.

<sup>†</sup>Department of Electrical Engineering, Alexandria University, Alexandria 21544, Egypt.

<sup>\*</sup>Department of Engineering Mathematics and Physics, Alexandria University, Alexandria 21544, Egypt.

<sup>⊥</sup> Department of Computer Science and Engineering, Egypt-Japan University of Science and Technology (E-JUST), New Borg El-Arab city, Alexandria 21934, Egypt.

Email: saraattalla@aucegypt.edu, kseddik@aucegypt.edu, aelsherif@nu.edu.eg, sherif.rabia@alexu.edu.eg

**Abstract**—In this paper, a cognitive radio system is studied in which the secondary user (SU) leverages the primary user (PU) channel quality indicator feedback (CQI) and the PU automatic repeat request (ARQ). The SU randomly accesses the PU channel with access probabilities based on its spectrum sensing outcome and the PU feedbacks. The SU's access probabilities are selected though an optimization problem with the objective to maximize the SU's throughput while ensuring the stability of the PU's packet queue. This system is modeled using a multi-dimensional Markov chain. This model enabled us to derive a closed-form expression for the SU's throughput, which is used in the throughput maximization problem. The proposed scheme is shown to improve the SU service rate compared to the system where no PU feedback is exploited by the SU, the system where the SU utilizes only the PU CQI feedback, and the system where the SU utilizes only the PU ARQ feedback.

## I. INTRODUCTION

Cognitive radio (CR) is a technology used in wireless communication that enables the secondary user (SU) to share the spectrum with the primary user (PU) without harmful effect on the PU network. Therefore, CR solves the spectrum scarcity problem and the inefficient use of the radio spectrum [1].

There are different approaches to enable cognitive radio networks. In the interweave approach [2] the SU senses the PU existence and accesses the PU channel depending on the sensing information. The problem with this model is that the SU has no information about its interference level on the PU network. One solution to this problem is to allow the SU to overhear the feedback sent from the primary receiver to its transmitter, and accesses the PU channel based on this feedback. PU feedback provides extra information for the SU to know if the PU is transmitting in the current time slot or not. The authors in [3] have proposed a system where the SU observes the automatic repeat request (ARQ) sent from the primary receiver and uses this information to stay within its interference level. In [4], the authors designed a scheme where the SU accesses the PU channel based on random access probabilities depending on the PU ACK/NACK feedback, which enhances the system performance in terms of SU's throughput. In [5], the authors presented a system in which

the SU accesses the PU network with access probabilities that are function of soft spectrum sensing information and the ACK/NACK feedback from the PU receiver. In [6], based on the ACK/NACK received, the authors devised optimal transmission strategies for the cognitive radio so as to maximize a weighted sum of PU and SU throughput, which is determined by the degree of protection for the primary link.

There is a different type of feedback information, namely, the channel quality indicator (CQI) feedback that can be used in a CR system. It informs the PU transmitter of the state of its receiver channel. The PU adjusts its transmission parameters to achieve the maximum transmission rate or the minimum packet loss rate using this feedback information. In [7], the authors developed a spectrum sharing scheme for the SU based on primary CQI feedback. They also derived the optimal transmit power and transmission rate for the SU when no or perfect primary CQI is available at the secondary transmitter by maximizing its average throughput while satisfying the rate loss constraint of the primary system. In a previous work [8], closed form expression for the SU throughput as well as the PU delay in cognitive radio system based on a primary CQI feedback are derived. This system outperforms other systems, where no feedback is exploited, in terms of the SU throughput under a PU QoS guarantee.

In this work, we study the effect of combining two types of PU feedbacks, namely, the CQI feedback and the ARQ feedback, to design a SU access scheme in a cognitive radio system. The ARQ feedback has two messages: ACK and NACK. The SU refrains from accessing the PU channel upon hearing NACK, assuming that the PU has a good channel, as the PU has to retransmit its undelivered packet to the PU receiver. Moreover, the SU tries to access the PU channel if ACK is overheard as the PU may not have a new packet to send. The CQI feedback is assumed to have only two states, informing the PU transmitter whether the channel is good and a successful transmission is expected or bad and any transmission is most likely to fail. In the proposed scheme, based on CQI feedback and the ACK/NACK feedback, the SU can exploit the time slots when the PU channel is bad to access the channel with a high access probability knowing

that the PU is idle for sure. Therefore, the SU accesses the PU's channel with no interference to the PU. The proposed scheme is compared with three systems. Namely, the system where the SU does not exploit any feedback information, the system where the SU exploits the PU ARQ feedback [5], and the system where the SU exploits the PU CQI feedback, [8]. It is assumed in all systems that the SU accesses the channel based on spectrum sensing decisions [9]. In all systems, the SU access probability is determined by solving an optimization problem that maximizes the SU's throughput subject to a constraint on the PU's queue stability. We study and compare the performance of the three systems by finding closed form expressions of the secondary throughput for each system. Our results show significant performance gains of our proposed scheme as compared to the other three systems.

## II. SYSTEM MODEL

We consider a cognitive system consisting of one PU and one SU. The channel between the PU transmitter and receiver is modeled as a two-state Markov chain as shown in Fig. 1. The two states are the good state and the bad state. When the channel is in the good state, it is most likely that the PU packet is transmitted successfully. However, the PU packet is not transmitted successfully in the bad state. The probabilities of the channel staying in the good state and in the bad state are  $p_g$  and  $p_B$ , respectively. The probability of the channel moving from a good state to a bad state is  $1 - p_g$  and moving from a bad state to a good state is  $1 - p_B$ . The steady state probabilities of the channel being in the good state and in the bad state are  $\zeta_g$  and  $\zeta_B$ , respectively and can be calculated using the following equations:

$$\zeta_g = \frac{1 - p_B}{2 - p_B - p_g}, \text{ and } \zeta_B = \frac{1 - p_g}{2 - p_B - p_g}. \quad (1)$$

The system is time-slotted, and it is assumed that the duration of one time slot equals the time of one packet transmission. It is assumed that the packets arrive at the start of the time slot, which means that a packet can be served in the same time slot it arrives at. The PU accesses the channel at the start of each time slot whenever it has a packet to transmit and the channel is in the good state. It is assumed that the channel state does not change during one time slot. Furthermore, collision channel model is assumed, i.e., if both the PU and SU transmit in the same time slot, then a collision occurs and both packets are lost. The PU and SU have an infinite buffer for storing fixed length packets. The arrival process at the PU queue is a Bernoulli process with mean  $0 < \lambda_p < 1$ . The SU is assumed to always have packets to transmit in its queue.

In our model, the SU employs hard-decision energy detection for sensing the PU's presence. The SU accesses the channel with access probability  $a_s$  when the detected energy is less than the detection threshold. These access probabilities are selected such that the SU throughput is maximized and the stability of the PU queue is guaranteed. Stability can be loosely defined as having a certain quantity of interest kept bounded, in

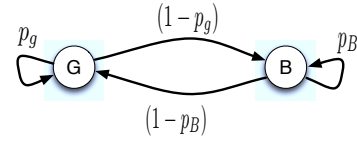


Fig. 1: The channel model

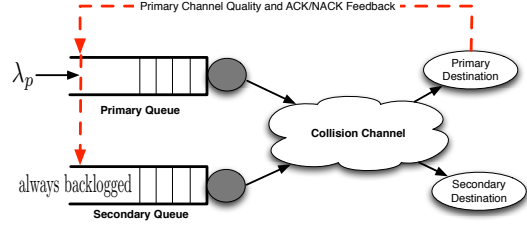


Fig. 2: The system model

our case, the queue size. For more information about stability, see [10] and [11]. If the arrival and service processes of a queuing system are strictly stationary, one can apply Loynes' theorem to check for stability [12]. This theorem states that if the average arrival rate is less than the average service rate of a queuing system, whose arrival and service processes are strictly stationary, then the queue is stable, otherwise it is unstable.

In the following subsections, different SU access schemes are presented.

### A. The No Feedback System

In this system, the PU has ACK/NACK ARQ feedback but has no CQI feedback information. Therefore, the PU transmits its packets regardless of the channel state. In this system it is assumed that the SU does not access the PU feedback. Therefore, it accesses the channel with an access probability  $a_s$  in every time slot based on its spectrum sensing.

### B. The ARQ Feedback-based System

In this system, the SU has access to the PU's ARQ feedback. Observing a NACK, the SU backs-off since it knows that the PU will retransmit its undelivered packet from the previous time slot, thus avoiding sure collisions with the PU. However, upon hearing an ACK, the SU accesses the channel with an access probability  $a_s$  based on its spectrum sensing.

### C. The CQI Feedback-based System

In this system, the PU has ARQ feedback, and CQI feedback of the channel state in the next time slot, which is an indicator of how good/bad the channel between the PU transmitter and receiver is. If a good CQI feedback is observed, the PU transmits whenever it has packets in its queue. Observing a bad PU CQI feedback, the PU backs-off since it knows that the packet will not be received correctly. The SU monitors only the PU CQI feedback. Hearing a bad PU CQI feedback, the SU accesses the channel with probability 1. If the SU observes a good PU CQI feedback, it accesses the channel with an access probability  $a_s$  based on its spectrum sensing.

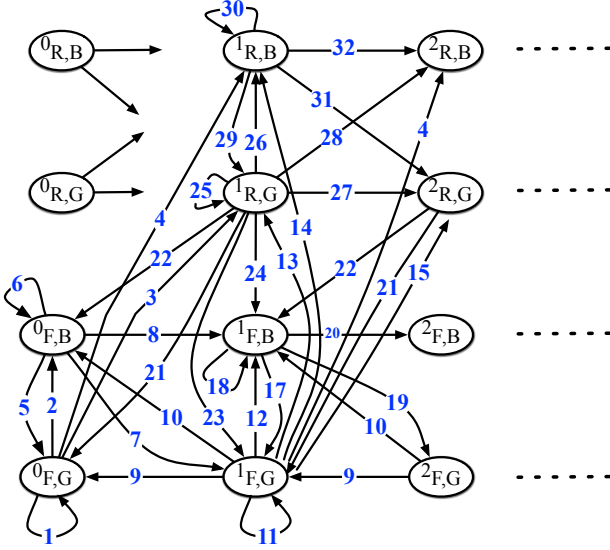


Fig. 3: The PU queue Markov chain model

#### D. Proposed Hybrid Feedback-based Access System

The proposed system model is shown in Fig. 2, in which the PU has both CQI and ARQ feedbacks; the SU listens to these two types of feedback. The SU accesses the channel depending on the hard-decision sensing scheme and the PU feedbacks. If a good CQI feedback and ACK message is observed, and the SU does not detect the PU's existence, the SU accesses the channel with access probability  $a_s$ . If a bad CQI feedback is observed, the SU exploits the knowledge that the PU will be in "back-off" state during the next time slot to transmit with probability 1 irrespective of the ARQ feedback. In the case of PU NACK with a good CQI, the SU backs-off to allow for collision-free transmission for the PU. In the case of PU NACK with bad CQI, the SU accesses the channel with probability 1, as the SU knows for sure that the PU will be in the "back-off" state, although it has packets to transmit.

### III. PERFORMANCE ANALYSIS

In this section, we present the analysis of the PU's queue for the different access schemes described above.

#### A. The Proposed System

The PU's queue in the proposed system is modeled by the three-dimensional Markov chain  $\{X(n), n = 0, 1, 2, \dots\}$  shown in Fig. 3, whose state space is given by  $S = \{(K, D, T) : K = 0, 1, 2, \dots, D \in \{F, R\}, T \in \{G, B\}\}$ . Where  $K$  is the number of PU packets in the queue,  $D$  is the ARQ feedback, where  $F$  means that the packet at the head of the PU queue is being transmitted for the first time, while  $R$  means that the packet at the head of the PU is being retransmitted due to failure in the previous time slot. Finally,  $T$  is the CQI feedback, where  $G$  means that the PU channel is in the good state and  $B$  means that it is in the bad state.

1) *The Transition Probabilities:* The transitions between states are as follows:

— From  $(K, F, G)$  to  $(K - 1, F, G)$ ,  $K > 0$ : the transition in this case occurs according to the following equation:

$\Pr(X(n+1) = (K - 1, F, G) | X(n) = (K, F, G)) = \Pr(\text{no new packet arrives at the PU queue}) \cap (\text{SU does not detect the PU presence and decides not to access the channel}) \cap (\text{the channel in the next time slot remains in the good state}) \cup \Pr(\text{no new packet arrives at the PU queue}) \cap (\text{SU detects the PU presence}) = (1 - \lambda_p)((1 - a_s)(1 - p_d) + p_d)p_g$ .

— From  $(K, F, G)$  to  $(K - 1, F, B)$ ,  $K > 0$ : it is the same as the previous transition but  $p_g$  is replaced by  $1 - p_g$ . Therefore the transition probability equals to  $(1 - \lambda_p)((1 - a_s)(1 - p_d) + p_d)(1 - p_g)$ .

— From  $(K, R, G)$  to  $(K, F, G)$ ,  $K > 0$ : as the MC is in  $(K, R, G)$ , the packet at the head of the queue will be transmitted successfully with probability 1. The transition in this case occurs according to this equation:

$\Pr(X(n+1) = (K, F, G) | X(n) = (K, R, G)) = \Pr(\text{new packet arrives at the PU queue}) \cap (\text{the channel in the next time slot remains in the good state}) = \lambda_p p_g$

— The rest of the transition probabilities are shown in Fig. 3 and can be deduced easily.

The complete transition probabilities of the Markov chain are shown in Table I.

2) *The Steady State Distribution Calculation:* To get an expression for the SU throughput of the proposed system, we start by calculating the steady state distribution of the Markov chain shown in Fig. 3.

The steady state distribution vector is given by

$$\mathbf{v} = [\pi_0^{FG}, \pi_0^{FB}, 0, 0, \pi_1^{FG}, \pi_1^{FB}, \pi_1^{RG}, \pi_1^{RB}, \dots].$$

It is clear that the PU can not be in the retransmission state if it has no packets. Therefore,  $\pi_0^{RG} = 0$  and  $\pi_0^{RB} = 0$ . Define

the vector  $\mathbf{v}_k = \begin{pmatrix} \pi_k^{FG} \\ \pi_k^{FB} \\ \pi_k^{RG} \\ \pi_k^{RB} \\ \pi_k \end{pmatrix}$ , note that  $\mathbf{v}_0 = \begin{pmatrix} \pi_0^{FG} \\ \pi_0^{FB} \\ 0 \\ 0 \end{pmatrix}$ . The state

transition matrix of the Markov chain shown in Fig. 3 can be written as

$$\Phi = \begin{pmatrix} B & A_0 & 0 & 0 & \dots \\ A_2 & A_1 & A_0 & 0 & \dots \\ 0 & A_2 & A_1 & A_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (2)$$

where  $B, A_0, A_1, A_2$  are shown in equation (3) at the top of the next page. The state transition matrix  $\Phi$  is a block-tridiagonal matrix; therefore the Markov chain shown in 3 is a homogeneous quasi birth-and-death (QBD) Markov chain. To make the state transition matrix a block-tridiagonal matrix, a transition from  $\pi_0^{RB}$  to  $\pi_1^{RG}$  and a transition from  $\pi_0^{RB}$  to  $\pi_1^{RB}$  are added. Adding these transitions do not affect the markov chain as the probabilities of being in  $\pi_0^{RG}$  and  $\pi_0^{RB}$  are equal to zero. The steady state distribution of the Markov chain shown in Fig. 3 satisfies the following equation [13]:

$$\mathbf{v}_k = \mathbf{R}^k \mathbf{v}_0, \quad k > 0, \quad (4)$$

TABLE I: Transition Probabilities of the MC for the Proposed System.

Number	Transition Probability	Number	Transition Probability
1	$[(1-\lambda_p) + \lambda_p((1-a_s)(1-p_d) + p_d)]p_g$	10	$(1-\lambda_p)[(1-a_s)(1-p_d) + p_d](1-p_g)$
2	$[(1-\lambda_p) + \lambda_p((1-a_s)(1-p_d) + p_d)](1-p_g)$	11	$\lambda_p[(1-a_s)(1-p_d) + p_d]p_g$
3	$\lambda_p a_s(1-p_d)p_g$	12	$\lambda_p[(1-a_s)(1-p_d) + p_d](1-p_g)$
4	$\lambda_p a_s(1-p_d)(1-p_g)$	13	$(1-\lambda_p)a_s(1-p_d)p_g$
5	$(1-\lambda_p)(1-p_B)$	14	$(1-\lambda_p)a_s(1-p_d)(1-p_g)$
6	$(1-\lambda_p)p_B$	15	$(1-\lambda_p)p_g$
7	$\lambda_p(1-p_B)$	16	$(1-\lambda_p)(1-p_g)$
8	$\lambda_p p_B$	17	$\lambda_p p_g$
9	$(1-\lambda_p)[(1-a_s)(1-p_d) + p_d]p_g$	18	$\lambda_p(1-p_g)$

$$\begin{aligned}
 \mathbf{B} &= \begin{pmatrix} (1-\lambda_p) + \lambda_p((1-a_s)(1-p_d) + p_d)p_g & (1-\lambda_p)(1-p_B) & 0 & 0 \\ (1-\lambda_p) + \lambda_p((1-a_s)(1-p_d) + p_d)(1-p_g) & (1-\lambda_p)p_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \\
 \mathbf{A}_0 &= \begin{pmatrix} (1-\lambda_p)((1-a_s)(1-p_d) + p_d)p_g & 0 & (1-\lambda_p)p_g & 0 \\ (1-\lambda_p)((1-a_s)(1-p_d) + p_d)(1-p_g) & 0 & (1-\lambda_p)(1-p_g) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \\
 \mathbf{A}_1 &= \begin{pmatrix} \lambda_p((1-a_s)(1-p_d) + p_d)p_g & (1-\lambda_p)(1-p_B) & \lambda_p p_g & 0 \\ \lambda_p((1-a_s)(1-p_d) + p_d)(1-p_g) & (1-\lambda_p)p_B & \lambda_p(1-p_g) & 0 \\ (1-\lambda_p)a_s(1-p_d)p_g & 0 & 0 & (1-\lambda_p)(1-p_B) \\ (1-\lambda_p)a_s(1-p_d)(1-p_g) & 0 & 0 & (1-\lambda_p)p_B \end{pmatrix}. \\
 \mathbf{A}_2 &= \begin{pmatrix} 0 & \lambda_p(1-p_B) & 0 & 0 \\ 0 & \lambda_p p_B & 0 & 0 \\ \lambda_p a_s(1-p_d)p_g & 0 & 0 & \lambda_p(1-p_B) \\ \lambda_p a_s(1-p_d)(1-p_g) & 0 & 0 & \lambda_p p_B \end{pmatrix}. \tag{3}
 \end{aligned}$$

where the rate matrix  $\mathbf{R}$ :

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix},$$

is the solution of

$$A_2 + (A_1 - I_4)R + A_0R^2 = 0_{4 \times 4}, \tag{5}$$

which can be obtained by substituting equation (4) in the next equation

$$\mathbf{v}_k = A_2\mathbf{v}_{k-1} + A_1\mathbf{v}_k + A_0\mathbf{v}_{k+1}, \quad k \geq 1. \tag{6}$$

Equation (6) can be easily derived using the states balance equations.

By solving equation (5), the matrix  $\mathbf{R}$  can be obtained. The details of this calculation is omitted due to the lack of the space. To get the steady state distribution of the Markov chain, the following normalization requirement is applied:

$$\sum_{k=0}^{\infty} (\pi_k^{FG} + \pi_k^{FB} + \pi_k^{RG} + \pi_k^{RB}) = 1,$$

and using equation (4), we have

$$\bar{\mathbf{1}} \left( \sum_{k=0}^{\infty} \mathbf{R}^k \right) \mathbf{v}_0 = 1, \text{ where } \bar{\mathbf{1}} = [1 \quad 1 \quad 1 \quad 1].$$

$$\text{So, } \bar{\mathbf{1}} \left( \sum_{k=0}^{\infty} \mathbf{R}^k \right) \mathbf{v}_0 = \bar{\mathbf{1}}(\mathbf{I}_4 - \mathbf{R})^{-1} \begin{pmatrix} \pi_0^{FG} \\ \pi_0^{FB} \\ 0 \\ 0 \end{pmatrix} = 1,$$

where  $\mathbf{I}_4$  is the  $4 \times 4$  identity matrix. The last equation is a function of both  $\pi_0^{FG}$  and  $\pi_0^{FB}$ . Therefore, the relationship between  $\pi_0^{FG}$  and  $\pi_0^{FB}$  has to be obtained so that this equation becomes a function of one variable only. To get the relationship between  $\pi_0^{FG}$  and  $\pi_0^{FB}$ , the balance equations around  $(0, F, G)$  and  $(0, F, B)$  are solved.

The balance equation around state  $(0, F, G)$  is given by:

$$\begin{aligned}
 [a_s \lambda_p p_g - p_g - a_s \lambda_p p_d p_g + 1] \pi_0^{FG} &= (1-\lambda_p)(1-p_B) \pi_0^{FB} \\
 + [p_g(p_d + (1-a_s)(1-p_d))(1-\lambda_p)] \pi_1^{FG} \\
 + (1-\lambda_p) p_g \pi_1^{RG}, \tag{7}
 \end{aligned}$$

The balance equation around state  $(0, F, B)$  is given by:

$$\begin{aligned}
 [\lambda_p p_B - p_B + 1] \pi_0^{FB} &= \\
 [(1-\lambda_p)(1-p_g) + \lambda_p((1-a_s)(1-p_d) + p_d)(1-p_g)] \pi_0^{FG} \\
 + [(1-\lambda_p)((1-a_s)(1-p_d) + p_d)(1-p_g)] \pi_1^{FG} \\
 + (1-\lambda_p)(1-p_g) \pi_1^{RG}. \tag{8}
 \end{aligned}$$

Solving equation (7) and equation (8), we get

$$\pi_0^{FB} = \frac{(1-p_g)\pi_0^{FG}}{\lambda_p p_B - p_B - \lambda_p + \lambda_p p_g + 1}, \quad (9)$$

and  $\pi_0^{FG}$  is obtained as shown in equation (10) shown at the top of next page.

### Secondary Throughput Analysis:

The closed-form expressions for the SU throughput of the proposed system are derived as follows. In this system, the SU has access to the PU CQI and ARQ feedback messages; therefore, the SU accesses the channel with probability 1 under bad PU CQI feedback irrespective of the PU ARQ messages, since the PU is backing off due to the bad channel. However, under good PU CQI feedback and PU first transmission state the SU accesses the channel with probability  $a_s$  if the SU decides that the PU is absent through sensing. Therefore, the SU transmits its packets collision-free in the bad states  $(K, F, B)$  and  $(K, R, B)$  with probability 1 and in the empty first transmission good state,  $(0, F, G)$ , with probability  $a_s(1-p_f)$ , where  $p_f$  is the false alarm probability of the spectrum sensor. Hence, the SU throughput in this system,  $\mu_{s3}$ , is given by,

$$\begin{aligned} \mu_{s3} &= a_s(1-\lambda_p)(1-p_f)\pi_0^{FG} + \sum_{k=0}^{\infty} [\pi_k^{FB} + \pi_k^{RB}] \\ &= a_s(1-\lambda_p)(1-p_f)\pi_0^{FG} + [0 \quad 1 \quad 0 \quad 1](\mathbf{I}_2 - \mathbf{R})^{-1} \begin{pmatrix} \pi_0^{FG} \\ \pi_0^{FB} \\ \pi_0^{RG} \\ \pi_0^{RB} \end{pmatrix} \\ &= \frac{B_1}{p_B + p_g - 2}, \end{aligned}$$

where  $B_1$  is shown in equation (11) at the top of next page.

3) *Access Probabilities Calculation:* The access probability  $a_s$  has to be selected to maximize the secondary throughput,  $\mu_{si}$ ,  $i = 1, 2, 3$ , while keeping the PU queue stable. Stability of the PU queue is determined by the value of  $\pi_0^{FG}$  and  $\pi_0^{FB}$ . If these probabilities are greater than zero, it means that the probability of the PU queue being empty is also greater than zero. Therefore, the problem can be formulated as follows:

$$\max_{a_s} \mu_{si}$$

subject to

$$\pi_0^{FG} > 0 \text{ and } \pi_0^{FB} > 0.$$

By differentiating the expression of  $\mu_{si}$  with respect to  $a_s$  and equating the derivative to zero, the optimal access probability  $a_s^*$  can be obtained. For all systems, the differentiation of  $\mu_{si}$  with respect to  $a_s$  results in a second degree polynomial in  $a_s$ . Therefore, there are two solutions of this maximization problem. The solution in the range from 0 to 1 is selected as the value of  $a_s$  that results in the maximum secondary user throughput will always guarantee the stability of the PU queue; since if this  $a_s$  causes the PU queue to be unstable, this will reduce the SU throughput since the SU will never transmit

any packets in the good channel states, as the PU queue will always be backlogged. The maximum secondary throughput is obtained by substitution of  $a_s^*$  in the equation of  $\mu_{si}$  to get the maximum secondary throughput,  $\mu_s^{max}$ .

The closed-form expressions of the access probabilities to maximize the secondary throughput of the proposed system are as follows,

$$a_s^* = \frac{\lambda_p p_B - p_B - 2\lambda_p + \lambda_p p_g + 1}{4\lambda_p - 4\lambda_p p_d - 2\lambda_p p_B - 2\lambda_p p_g + 2\lambda_p p_d p_B + 2\lambda_p p_d p_g}. \quad (12)$$

### B. The ARQ System

In [5], the authors have done the analysis of ARQ system utilizing the soft sensing scheme. In this paper, the soft sensing scheme is converted to the hard sensing scheme so the proposed scheme can be compared with the ARQ system. After modifying the results in [5] to match our model the SU throughput is given by,

$$\mu_s = \left(1 - \frac{\lambda_p}{1 - (1-p_d)(1-\zeta_B)a_s}\right)(1-p_f)a_s, \quad (13)$$

where  $p_d$  is the detection probability of the spectrum sensor and  $p_f$  is the probability of false alarm.

The access probability that maximizes the SU service rate  $a_s^*$  is obtained by differentiating the SU service rate in equation (13) with respect to  $a_s$  and equating the result to zero, so

$$a_s^* = \frac{(\sqrt{\lambda_p} - 1)(p_B + p_g - 2)}{(1-p_d)(1-p_B)}. \quad (14)$$

### C. The NO FB system, and the CQI system

In a previous work [8], this paper's authors studied the system where the SU exploits the PU CQI feedback to access the PU channel. A closed form expression of the SU throughput for the system with no feedback and the CQI system were obtained, which was omitted due to the lack of the space.

### D. The SU Perfect Sensing with PU CQI Feedback Based-Access System

In this system, which is an upper bound system, the PU has a CQI feedback. The PU accesses the channel if there is a new arrival based on the CQI feedback. The SU accesses the PU channel in the bad channel states with probability 1. When the PU channel is in the good state and the PU's queue is empty, the SU accesses the channel with probability 1 as well (because of perfect sensing). The analysis of the PU's queue for the SU perfect sensing with the PU CQI feedback based access system was provided in [8]. The same expression for the SU service rate, shown in equation (15), is used in this work as the ACK does not add information to the SU in the perfect sensing system.

$$\mu_{sp} = \begin{cases} 1 - \lambda_p, & \text{if } \lambda_p < \zeta_B - 1. \\ \zeta_B, & \text{otherwise.} \end{cases} \quad (15)$$

$$\pi_0^{FG} = \frac{\lambda_p p_B - p_B - 2a_s \lambda_p - 2\lambda_p + \lambda_p p_g + 2a_s \lambda_p p_d + a_s \lambda_p p_B + a_s \lambda_p p_g - a_s \lambda_p p_d p_B - a_s \lambda_p p_d p_g + 1}{(\lambda_p - 1)(p_B + p_g - 2)} \quad (10)$$

$$B_1 = p_g - a_s + 2a_s \lambda_p + a_s p_f + a_s p_B + 2a_s^2 \lambda_p - 2a_s \lambda_p p_f - a_s \lambda_p p_B - a_s \lambda_p p_g - a_s p_f p_B - 2a_s^2 \lambda_p p_d - 2a_s^2 \lambda_p p_f - a_s^2 \lambda_p p_B - a_s^2 \lambda_p p_g + a_s \lambda_p p_f p_B + a_s \lambda_p p_f p_g + 2a_s^2 \lambda_p p_d p_f + a_s^2 \lambda_p p_d p_B + a_s^2 \lambda_p p_f p_B + a_s^2 \lambda_p p_d p_g + a_s^2 \lambda_p p_f p_g - a_s^2 \lambda_p p_d p_f p_B - a_s^2 \lambda_p p_d p_f p_g - 1. \quad (11)$$

#### IV. RESULTS AND DISCUSSION

In this section, we compare the performance of the proposed hybrid feedback SU access scheme with the no-FB scheme, the ARQ FB-based scheme and the CQI FB-based scheme. We also compare it with the performance of the perfect-sensing system where at each time slot the SU is able to sense the channel occupancy without error. Therefore, the perfect sensing system provides an upper bound on the performance of any access scheme.

Fig. 4 and Fig. 5 depict the SU throughput as a function of the PU arrival rate for the different access schemes. The figures differ in the steady state probability of the channel being in the bad or good states, the probability of detection and the probability of false alarm. At zero PU arrival rate, the perfect sensing scheme achieves a SU throughput of 1 since it can access all the time slots. The hybrid and CQI FB-based schemes benefit from the CQI feedback to access the channel without sensing when the PU channel is in the bad state. Therefore, achieving a throughput of  $\zeta_B + \zeta_G(1 - p_F)$ , which is equal to 0.81 in Fig. 5. The ARQ FB-based and No-FB schemes are limited by the false alarm rate and will start at the value of 0.7, which is  $(1 - p_F)$ . At high PU arrival rates, the hybrid, ARQ FB-based schemes and the perfect sensing scheme achieve a minimum SU throughput equal to the steady state probability of the PU channel being in the bad state. This minimum value is guaranteed as the PU is backing off under bad channel conditions allowing the SU to access the channel with probability 1. By combining the information of the CQI and ARQ feedbacks, the proposed hybrid scheme outperforms the other schemes for all values of the PU arrival rate. In Fig. 4, the SU throughput for the ARQ FB-based scheme is slightly better than that of the CQI FB-based one for relatively low PU arrival rates. This is because in Fig. 4 the probability of the PU channel being in the bad state is 0.125. Thus, the gain achieved by preventing the collisions with the PU in the ARQ FB-based scheme outweigh the gain of accessing the channel when the PU refrains from accessing it when it is in the bad state in the CQI FB-based scheme. At high PU arrival rates, the only opportunity for the SU to access the channel is when the PU refrains from using it in the bad state, hence, the CQI FB-based schemes performing better than the ARQ FB-based one. In Fig. 5 when the the probability of the PU channel being in the bad state is 0.3636, the gain of exploiting the channel when it is in the bad state outweigh that of preventing the collisions with the PU. Therefore, the CQI FB-based scheme outperforms the ARQ FB-based scheme for all values of the

PU arrival rates.

In Fig. 6 and Fig. 7, the SU access probabilities are plotted against the PU arrival rate for different access schemes. The steady state probability of the channel being in the bad state in Fig. 6 is 0.125 and in Fig. 7 is 0.3636. The results in Fig. 6 and Fig. 7 show very interesting insight. Comparing the No-FB and the CQI FB-based system, we can see that the optimum SU access probabilities are always the same. Note that the difference between these two systems lies in the fact that the SU accesses the channel with probability 1 under bad PU channel state in the CQI FB-based system and with probability  $a_s$  in the no-FB system. Under good PU channel state, both access the channel with some  $a_s$ . So both systems will have the same effect on the PU (since under PU bad state, whatever action that will be taken by the SU will not affect the PU, since it will be either in the back-off state or it will transmit a packet with failure). Therefore, it is expected that these two systems will have the same access probability. Comparing the ARQ FB-based scheme with the CQI FB-based, we can see that the access probability of the ARQ FB-based system is higher. This can be attributed to the fact that in the ARQ FB-based system the SU can be more aggressive in accessing the channel, and under collision, it will go to a back-off state to allow for collision-free transmission from the PU user; this is not the case for the CQI FB-based system, since under collision, there is no guarantee for the PU in the next time-slot. Therefore, the SU should limit its collisions with the PU in the CQI FB-based system as much as it can. Also, in the CQI FB-based system, the SU is always guaranteed to receive an access to the channel whenever the PU channel becomes in the bad state, therefore, it can limit its access probability in the good PU states. As clear from these two figures, the gap between the ARQ FB-based and the CQI FB-based access probabilities is bigger when the probability of the PU channel being in the bad state becomes higher. Again this is expected, since as the probability of the PU channel being in the bad state becomes higher, the SU will get a higher service in the bad state in the CQI FB-based system, so it will be less aggressive in accessing the PU channel under good PU channel state. Finally, comparing the hybrid FB-based system and the ARQ FB-based system access probabilities, we can see that none of them will be always higher than the other. For the case of high probability of the PU being in the bad channel, the access probability of the ARQ FB-based system will always be higher since in the hybrid FB system the SU benefits from the PU bad channel states, so it can be less aggressive in accessing the channel when the PU channel becomes good (cf.



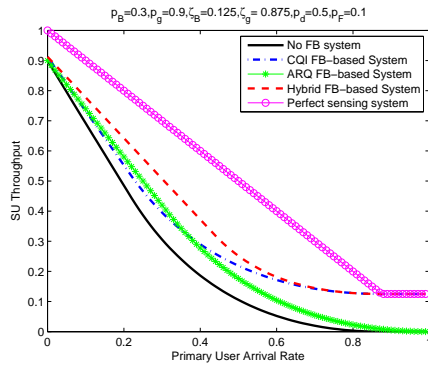


Fig. 4: The SU throughput for different access schemes

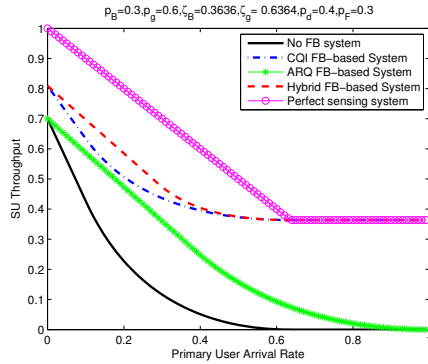


Fig. 5: The SU throughput for different access schemes

Fig. 7). For the other case of having lower probability of the PU channel being in the bad state, knowing more about the PU activity through the PU feedback might mean more aggressive access under small PU arrival rates and less aggressive access under higher PU arrival rates (cf. Fig. 6).

## V. CONCLUSIONS

In this paper, a hybrid feedback based hard decision access scheme for cognitive radio systems has been developed. The SU optimizes its access strategy based on information collected from the PU CQI and ARQ feedbacks. The ARQ feedback enables the SU to prevent collisions with the PU when the PU is retransmitting failed packets. The CQI feedback enables the SU to transmit when the PU is in a bad state, knowing that the PU is refraining from transmission is this case. The throughput of the SU is obtained by modeling the system as a three-dimensional Markov chain. Results reveal that the proposed scheme outperforms the access schemes in which the SU does not exploit the PU feedback information, or only exploits one type of PU feedback.

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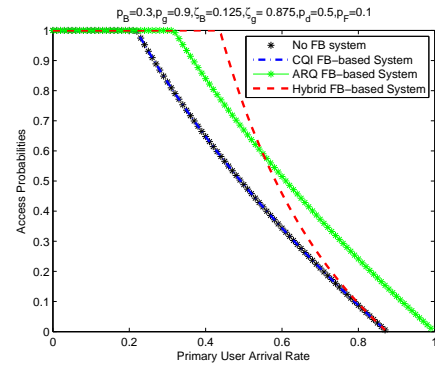


Fig. 6: The SU optimal access probabilities for different access schemes

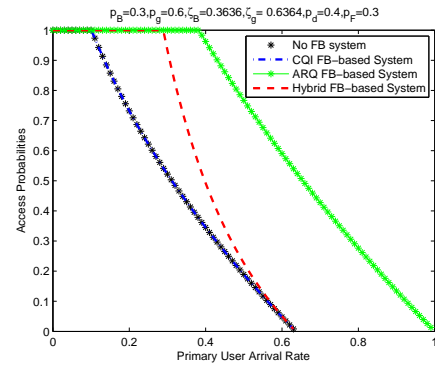


Fig. 7: The SU optimal access probabilities for different access schemes

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