A Feedback-Based Access Scheme for Cognitive Radio Systems

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Abstract— **In this paper, we consider the design of access schemes for secondary users in cognitive radio systems based on the primary user feedback information. We consider a secondary user employing a random access scheme with an access probability that depends on the primary user feedback state. We show that the proposed scheme can enhance the system performance in terms of the secondary throughput and/or primary user delay while guaranteeing a certain quality of service (QoS) for the primary user; this is due to the fact that the proposed scheme avoids sure collisions between the primary and secondary users. The proposed scheme can be deployed with any other random access based scheme and it always results in a performance gain using the extra piece of information coming from the primary user feedback.**

I. INTRODUCTION

Cognitive radio technology is a transmission paradigm making use of the underutilized radio spectrum in order to solve the problem of spectrum scarcity and enhance the overall wireless transmission efficiency and throughput. In cognitive radio networks, secondary users coexist with licensed, primary users in such a way that a minimum quality of service represented by certain performance metrics is guaranteed.

In a typical cognitive radio setting, the cognitive transmitter senses primary activity and decides on accessing the channel on the basis of the sensing outcome. This approach is problematic because sensing does not inform the cognitive terminal about its impact on primary receiver. This problem has induced interest in leveraging the feedback from the primary receiver to the primary transmitter to optimize the secondary transmission strategies taking into account the effect it has on the primary receiver. For instance, in [1], the secondary user observes the automatic repeat request (ARQ) feedback from the primary receiver. The ARQs reflect the primary user achieved packet rate. The cognitive radio's objective is to maximize secondary throughput under the constraint of guaranteeing a certain packet rate for primary user. In [2], the authors use a partially observable Markov decision process (POMDP) to optimize the secondary action on the basis of sensing and primary ARQ feedback. Secondary power control on the basis of primary link control feedback is investigated in [3]. In [4], the optimal transmission policy for the secondary user when the primary user adopts a retransmission based error control scheme is investigated. The policy of the secondary user determines how often it transmits according to the retransmission state of the packet being served by the primary user. The resulting optimal strategy of the secondary user is proven to have a unique structure. In particular, the optimal throughput is achieved by the secondary user by concentrating its interference to the primary user in the first transmission of a packet.

In this paper, we consider a secondary network that employs a random access scheme where the access probability is adjusted based on the primary user feedback state while guaranteeing a certain QoS for the primary network. In this paper, we make the following contributions. We analyze the primary user's queue in the presence of a secondary terminal which changes its transmission strategy based on the primary feedback. In contrast with [4], the primary user can have new packet arrivals during the retransmission phase of the packets delivered unsuccessfully. Based on the queueing analysis, we provide an expression for the secondary throughput and find the access probability that maximizes it.

The rest of the paper is organized as follows. The system model is presented in Section II. The performance of the proposed scheme is investigated in Section III. We provide numerical results in Section IV and conclude the paper in Section V.

II. SYSTEM MODEL

We consider a system that has one primary user (PU) and one secondary user (SU). The PU has an infinite buffer for storing fixed length packets. The channel is slotted in time and a slot duration equals the packet transmission time. The arrival process at the PU queue is a Bernoulli process with mean λ_p. Under our system model assumptions, the average arrival rate of the PU is λ_p packets per time slot, and is bounded as $0 < \lambda_p < 1$.

In our system, the PU will access the channel whenever it has a packet to send while the secondary user employs a random access based approach. In the presented schemes, the SU does not employ any spectrum sensing technique[1]; it randomly selects whether to transmit or not whenever it has a packet to send. The access probability will be selected in order to keep a certain quality of the service (QoS) for the PU. In our system the quality of service will be measured in terms

[1]The presented scheme can be combined with any sensing-based random access scheme for performance improvement utilizing the available PU feedback information

of the stability of the PU queue. The SU will select its access probability to maximize the SU throughput while guaranteeing the stability of the PU queue. A packet will be lost only when a collision occurs between the PU and SU transmissions, i.e., when the PU has a packet to send and the SU has a packet and decided to access the channel. If only one user is transmitting we assume that the packet will be delivered correctly; this assumption is used for simplicity of analysis and presentation. The analysis can be readily extended to allow for considering channel packet errors under no collisions.

Stability can be loosely defined as having a certain quantity of interest kept bounded. In our case, we are interested in the queue size being bounded. For an irreducible and aperiodic Markov chain with countable number of states, the chain is stable if and only if it is positive recurrent, which implies the existence of its stationary distribution. For a rigorous definition of stability under more general scenarios see [5] and [6]).

If the arrival and service processes of a queueing system are strictly stationary, then one can apply Loynes's theorem to check for stability conditions [7]. This theorem states that if the arrival process and the service process of a queueing system are strictly stationary, and the average arrival rate is less than the average service rate, then the queue is stable, otherwise it is unstable.

Next, we will present two access schemes for the SU user, namely, the conventional access scheme and the feedback-based access scheme.

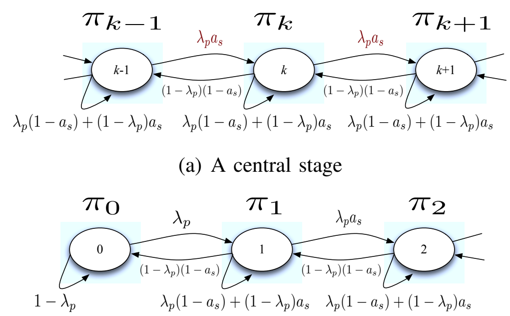
A. Conventional Access Scheme

In the conventional scheme, the SU accesses the channel with probability a_s ($0 \leq a_{\rm s} \leq 1$) independent of the PU activity. The access probability $a_{\rm s}$ is selected to maximize the SU throughput with the primary user guaranteed that its queue remains stable.

B. Feedback-Based Access Scheme

In the feedback-based access scheme, the SU utilizes the available primary feedback information for accessing the channel. The cognitive part of the SU comes from having the SU always monitoring the PU feedback channel to learn about the PU activity.

In the proposed scheme, the SU monitors the PU feedback channel. If an 'ACK' or 'Nothing' is observed on the feedback channel at the SU, the SU randomly accesses the channel with access probability $a_{\rm s}$. If a 'NACK' is observed, then the SU backs-off allowing for the retransmission of the lost primary packet. This way the SU avoids a clear collision with the primary transmission since a NACK means that the primary will be transmitting with probability 1. So if we assume that in the new access scheme we use the same access probability as the conventional scheme, then the service rate for the PU will be clearly higher in the proposed new scheme by virtue of avoiding the collisions with the PU after the reception of a NACK.



(a) A central stage

(b) Around the empty state

Fig. 1. The conventional PU queue evolution Markov chain

III. Performance Analysis

In this section, the performance of the schemes presented above is analyzed. The objective is to maximize the secondary throughput under a QoS constraint for the PU. The QoS is provided to the PU by ensuring the stability of the PU queue. We assume that the SU always has packets to send. A transmission error occurs whenever the primary and secondary terminals are active simultaneously. Moreover, if one terminal transmits while the other is silent, the probability of packet delivery failure is negligible.

A. Performance Analysis for the Conventional Access Scheme

1) Secondary Throughput: In the conventional system, the SU tries to randomly access the channel in each slot with probability $a_{\rm s}$. The PU successfully transmits a packet if there is no collision with the SU. The primary throughput, defined as the probability of successful transmission when the terminal has a packet to send, is then given by

$$\mu_{\rm p} = 1 - a_{\rm s}. \tag{1}$$

The Markov chain modelling the evolution of the PU queue length under the conventional scheme is shown in Fig. 1. The transition probabilities are based on the assumption that packet arrivals occur near the end o fthe time slot, therefore, if the queue is empty an arriving packet cannot be transmitted during the same time slot. The probability of the queue length increasing by one is $\lambda_{\rm p} a_{\rm s}$. If the queue is nonempty, the probability of the queue length decreasing by one is $(1-\lambda_{\rm p})(1-a_{\rm s})$. For a queue length $k > 0$, if π_k is the probability of being in state k, we have the following balance equation

$$\begin{aligned}\pi_k \Big(\lambda_{\rm p} a_{\rm s} + (1-\lambda_{\rm p})(1-a_{\rm s}) \Big) \\ = \pi_{k-1} \lambda_{\rm p} a_{\rm s} + \pi_{k+1} (1-\lambda_{\rm p})(1-a_{\rm s}).\end{aligned} \tag{2}$$

The balance equation between state 0 and state 1 is

$$\pi_0 \lambda_p = \pi_1 (1-\lambda_{\rm p})(1-a_{\rm s}). \tag{3}$$

In this case, for $k \geq 1$ we have

$$\pi_k = \pi_0 \frac{1}{a_s} \left(\frac{\lambda_{\rm p} a_{\rm s}}{(1-\lambda_{\rm p})(1-a_s)} \right)^k \tag{4}$$

The normalization condition $\sum_{k=0}^{\infty} \pi_k = 1$ means that

$$\pi_0 = \frac{1 - \frac{\lambda_{\rm p} a_{\rm s}}{(1-\lambda_{\rm p})(1-a_{\rm s})}}{1 - \frac{\lambda_{\rm p} a_{\rm s}}{(1-\lambda_{\rm p})(1-a_{\rm s})} + \frac{\lambda_{\rm p}}{(1-\lambda_{\rm p})(1-a_{\rm s})}} = \frac{1-\lambda_{\rm p} - a_{\rm s}}{1-a_{\rm s}} \tag{5}$$

That is,

$$\pi_0 = 1 - \frac{\lambda_{\rm p}}{1-a_{\rm s}}. \tag{6}$$

Note that for the sum $\sum_{k=0}^{\infty} \pi_k$ to exist, the term $\frac{\lambda_{\rm p} a_{\rm s}}{(1-\lambda_{\rm p})(1-a_{\rm s})}$ must be less than unity. This means that for a stationary distribution to exist for the Markov chain, the condition $\lambda_{\rm p} + a_{\rm s} < 1$ must hold; this is the condition for the primary queue stability. Given (1), this condition is equivalent to $\lambda_{\rm p} < \mu_{\rm p}$.

The SU successfully delivers a packet to its destination if it decides to access the channel when the PU is not transmitting. Therefore, the secondary throughput $\mu_{\rm s}$ is given by

$$\mu_{\rm s} = \pi_0 a_{\rm s} = \left(1 - \frac{\lambda_{\rm p}}{1-a_{\rm s}}\right) a_{\rm s} \tag{7}$$

Our objective is to select $a_{\rm s}$ to maximize $\mu_{\rm s}$ while keeping the PU queue stable. The problem can be formulated as follows

$$\max_{a_{\rm s}} \mu_{\rm s} \quad \text{subject to } \mu_{\rm p} > \lambda_{\rm p}. \tag{8}$$

Differentiating the expression for $\mu_{\rm s}$ with respect to $a_{\rm s}$ and equating the differential to 0 we can get the optimal access probability $a_{\rm s}^*$ as

$$\left.\frac{\partial \mu_{\rm s}}{\partial a_{\rm s}}\right|_{a_{\rm s}=a_{\rm s}^*} = 0 \rightarrow a_{\rm s}^* = 1 - \sqrt{\lambda_{\rm p}}. \tag{9}$$

The function $\mu_{\rm s}$ can be easily proved to be concave in $a_{\rm s}$ for $0 \leq a_s \leq 1$, which implies that $a_{\rm s}^*$ is the SU access probability that maximizes $\mu_{\rm s}$. The maximum secondary throughput $\mu_{\rm s}^{\max}$ is given by $\mu_{\rm s}^{\max} = \mu_{\rm s}(a_{\rm s}^*) = \left(1 - \sqrt{\lambda_{\rm p}}\right)^2$.

The primary throughput with SU access probability $a_{\rm s}^*$ is $\mu_{\rm p} = \sqrt{\lambda_{\rm p}} > \lambda_{\rm p}$, which always achieves the stability of the PU queue since $0 < \lambda_{\rm p} < 1$.

2) Primary User Packet Delay: By Little's law the average number of packets in the queue is $\lambda_{\rm p} D$, where D is the average delay. This means that $D = \frac{1}{\lambda_{\rm p}} \sum_{k=0}^{\infty} k\pi_k$, which can be calculated as

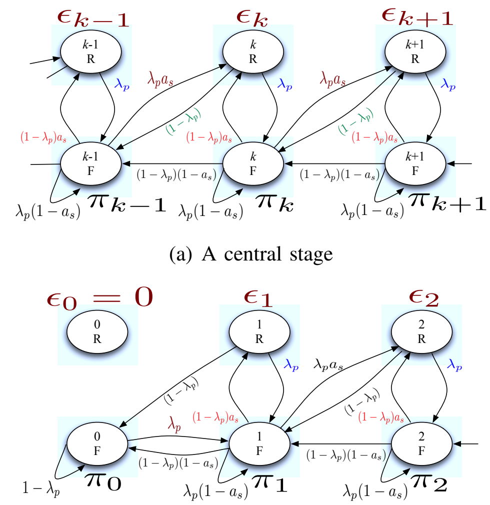
$$D = \frac{1-\lambda_{\rm p}}{1-a_{\rm s}-\lambda_{\rm p}}, \tag{10}$$

B. Performance Analysis for the Feedback-Based Access Scheme

1) Secondary Throughput: The PU queue evolution Markov chain is shown in Fig. 2. The probability of the queue having k packets and transmitting for the first time is π_k, where F in Fig. 2 denotes first transmission. The probability of the queue having k packets and retransmitting is ϵ_k, where R in Fig. 2 denotes retransmission. The balance equations are

$$\pi_k\left(\bar{\lambda}_{\rm p}\bar{a}_{\rm s} + \bar{\lambda}_{\rm p} a_{\rm s} + \lambda_{\rm p} a_{\rm s}\right) = \pi_{k+1}\bar{\lambda}_{\rm p}\bar{a}_{\rm s} + \epsilon_{k+1}\bar{\lambda}_{\rm p} + \epsilon_k \lambda_{\rm p} \tag{11}$$

$$\epsilon_k\left(\bar{\lambda}_{\rm p} + \lambda_{\rm p}\right) = \pi_{k-1}\lambda_{\rm p} a_{\rm s} + \pi_k \bar{\lambda}_{\rm p} a_{\rm s}, \tag{12}$$



(a) A central stage

(b) Around the empty state

Fig. 2. The PU queue evolution Markov chain

where we have used $\bar{\lambda}_{\rm p} = 1 - \lambda_{\rm p}$ and $\bar{a}_{\rm s} = 1 - a_{\rm s}$. Then ϵ_k can be written as

$$\epsilon_k = \pi_{k-1}\lambda_{\rm p} a_{\rm s} + \pi_k \bar{\lambda}_{\rm p} a_{\rm s} \tag{13}$$

We use this in the formula for π_k as

$$\begin{aligned}\pi_k\left(\bar{\lambda}_{\rm p} + \lambda_{\rm p} a_{\rm s}\right) =& \pi_{k+1}\bar{\lambda}_{\rm p}\bar{a}_{\rm s} + \bar{\lambda}_{\rm p}\left(\pi_k \lambda_{\rm p} a_{\rm s} + \pi_{k+1}\bar{\lambda}_{\rm p} a_{\rm s}\right) \\ &+ \lambda_{\rm p}\left(\pi_{k-1}\lambda_{\rm p} a_{\rm s} + \pi_k \bar{\lambda}_{\rm p} a_{\rm s}\right)\end{aligned} \tag{14}$$

$$\pi_k\left(\bar{\lambda}_{\rm p} + \lambda_{\rm p} a_{\rm s} - 2\lambda_{\rm p}\bar{\lambda}_{\rm p} a_{\rm s}\right) = \pi_{k-1}\lambda_{\rm p}^2 a_{\rm s} + \pi_{k+1}\bar{\lambda}_{\rm p}\left(\bar{a}_{\rm s} + \bar{\lambda}_{\rm p} a_{\rm s}\right). \tag{15}$$

Then we have

$$\bar{a}_{\rm s} + \bar{\lambda}_{\rm p} a_{\rm s} = 1 - a_{\rm s}\lambda_{\rm p}$$

and

$$\bar{\lambda}_{\rm p} + \lambda_{\rm p} a_{\rm s} - 2\lambda_{\rm p}\bar{\lambda}_{\rm p} a_{\rm s} = \bar{\lambda}_{\rm p}\left(1 - \lambda_{\rm p} a_{\rm s}\right) + \lambda_{\rm p}^2 a_{\rm s}.$$

Substituting in (15), we get

$$\pi_k\left(a_{\rm s}\lambda_{\rm p}^2 + \bar{\lambda}_{\rm p}\left(1 - a_{\rm s}\lambda_{\rm p}\right)\right) = \pi_{k-1} a_{\rm s}\lambda_{\rm p}^2 + \pi_{k+1}\bar{\lambda}_{\rm p}\left(1 - a_{\rm s}\lambda_{\rm p}\right) \tag{16}$$

We write now the balance equation for the empty state, which according to the assumption of packet arrivals near the end of the slot , corresponds always to the first-transmission case, i.e., $\epsilon_0 = 0$.

$$\pi_0 \lambda_{\rm p} = \pi_1 \bar{\lambda}_{\rm p}\bar{a}_{\rm s} + \epsilon_1 \bar{\lambda}_{\rm p} \tag{17}$$

$$\epsilon_1\left(\lambda_{\rm p} + \bar{\lambda}_{\rm p}\right) = \epsilon_1 = \pi_1 \bar{\lambda}_{\rm p} a_{\rm s} \tag{18}$$

$$\pi_0 \lambda_{\rm p} = \pi_1 \bar{\lambda}_{\rm p}\bar{a}_{\rm s} + \pi_1 \bar{\lambda}_{\rm p}^2 a_{\rm s} \tag{19}$$

$$\pi_1 = \pi_0 \frac{\lambda_{\rm p}}{\bar{\lambda}_{\rm p}\left(\bar{a}_{\rm s} + \bar{\lambda}_{\rm p} a_{\rm s}\right)}$$

$$\pi_1 = \pi_0 \frac{\lambda_{\rm p}}{\bar{\lambda}_{\rm p}\left(1 - a_{\rm s}\lambda_{\rm p}\right)} \tag{20}$$

Now to the balance equation of state new-transmission 1,

$$\begin{aligned}\pi_1 & \left(\bar{\lambda}_\mathrm{p}+\lambda_\mathrm{p}a_\mathrm{s}\right)\\ &=\pi_0\lambda_\mathrm{p}+\epsilon_1\lambda_\mathrm{p}+\epsilon_2\bar{\lambda}_\mathrm{p}+\pi_2\bar{\lambda}_\mathrm{p}\bar{a}_\mathrm{s}\\ &=\pi_0\lambda_\mathrm{p}+\pi_1\bar{\lambda}_\mathrm{p}a_\mathrm{s}\lambda_\mathrm{p}+\left(\pi_1 a_\mathrm{s}\lambda_\mathrm{p}+\pi_2 a_\mathrm{s}\bar{\lambda}_\mathrm{p}\right)\bar{\lambda}_\mathrm{p}+\pi_2\bar{\lambda}_\mathrm{p}\bar{a}_\mathrm{s}\end{aligned} \tag{21}$$

$$\begin{aligned}\pi_2\bar{\lambda}_\mathrm{p}\left(\bar{a}_\mathrm{s}+a_\mathrm{s}\bar{\lambda}_\mathrm{p}\right) &= \pi_2\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)\\ &=\pi_1\left(\bar{\lambda}_\mathrm{p}+\lambda_\mathrm{p}a_\mathrm{s}-2a_\mathrm{s}\lambda_\mathrm{p}\bar{\lambda}_\mathrm{p}\right)-\pi_0\lambda_\mathrm{p}\end{aligned} \tag{22}$$

As shown above, we have

$$\bar{\lambda}_\mathrm{p}+\lambda_\mathrm{p}a_\mathrm{s}-2\lambda_\mathrm{p}\bar{\lambda}_\mathrm{p}a_\mathrm{s}=\bar{\lambda}_\mathrm{p}\left(1-\lambda_\mathrm{p}a_\mathrm{s}\right)+\lambda_\mathrm{p}^2a_\mathrm{s}$$

Therefore,

$$\pi_2\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)=\pi_1\left(\bar{\lambda}_\mathrm{p}\left(1-\lambda_\mathrm{p}a_\mathrm{s}\right)+\lambda_\mathrm{p}^2a_\mathrm{s}\right)-\pi_0\lambda_\mathrm{p}. \tag{23}$$

Substituting from (20) we get

$$\pi_2=\pi_0\frac{1}{a_\mathrm{s}\lambda_\mathrm{p}}\frac{a_\mathrm{s}^2\lambda_\mathrm{p}^4}{\left[\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)\right]^2} \tag{24}$$

In general, we have

$$\pi_k=\pi_0\frac{1}{a_\mathrm{s}\lambda_\mathrm{p}}\frac{a_\mathrm{s}^k\lambda_\mathrm{p}^{2k}}{\left[\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)\right]^k}, \tag{25}$$

which can be shown to verify the difference equation (16). Defining

$$\rho=\frac{a_\mathrm{s}\lambda_\mathrm{p}^2}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)}$$

we have for $k\geq 1$

$$\pi_k=\pi_0\frac{1}{a_\mathrm{s}\lambda_\mathrm{p}}\rho^k \tag{26}$$

Using equation (13), for $k\geq 2$,

$$\begin{aligned}\epsilon_k &=\pi_0\rho^{k-1}+\pi_0\frac{\bar{\lambda}_\mathrm{p}}{\lambda_\mathrm{p}}\rho^k\\ &=\pi_0\rho^{k-1}\left(1+\frac{a_\mathrm{s}\lambda_\mathrm{p}}{1-a_\mathrm{s}\lambda_\mathrm{p}}\right)\end{aligned}$$

$$\epsilon_k=\pi_0\frac{1}{1-a_\mathrm{s}\lambda_\mathrm{p}}\rho^{k-1}\text{ for }k\geq 2,\quad \epsilon_0=0,\quad \epsilon_1=\pi_0\frac{\bar{\lambda}_\mathrm{p}}{\lambda}\rho \tag{27}$$

For $k\geq 2$ we have

$$\begin{aligned}\pi_k+\epsilon_k &=\pi_0\rho^{k-1}\left(\frac{1}{a_\mathrm{s}\lambda_\mathrm{p}}\rho+\frac{1}{1-a_\mathrm{s}\lambda_\mathrm{p}}\right)\\ &=\pi_0\frac{1}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{s}\right)}\rho^{k-1}.\end{aligned} \tag{28}$$

After some mathematical manipulation we can show that

$$\begin{aligned}\sum_{k=2}^{\infty}\Big[\pi_k+\epsilon_k\Big] &=\pi_0\frac{1}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{s}\right)}\sum_{k=2}^{\infty}\rho^{k-1}\\ &=\pi_0\frac{1}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{s}\right)}\ \frac{a_\mathrm{s}\lambda_\mathrm{p}^2}{1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p}}\end{aligned} \tag{29}$$

From the probability normalization condition

$$\pi_0+\pi_1+\epsilon_1+\sum_{k=2}^{\infty}\Big[\pi_k+\epsilon_k\Big]=1.$$

Provided that $\rho<1$, we have

$$\pi_0+\pi_0\frac{1}{a_\mathrm{s}\lambda_\mathrm{p}}\rho+\pi_0\frac{\bar{\lambda}_\mathrm{p}}{\lambda_\mathrm{p}}\rho+\pi_0\frac{1}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{s}\right)}\ \frac{a_\mathrm{s}\lambda_\mathrm{p}^2}{1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p}}=1$$

After some involved mathematical manipulation, we can show that

$$\pi_0=1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p} \tag{30}$$

Note that the condition $\rho<1$ for the stationary distribution to exist means that $\lambda_\mathrm{p}\left(1+a_\mathrm{s}\right)<1$. The secondary throughput is given by

$$\mu_\mathrm{s}=\pi_0 a_\mathrm{s}=\left(1-\lambda_\mathrm{p}(1+a_\mathrm{s})\right)a_\mathrm{s} \tag{31}$$

Employing primary feedback increases π_0 and, consequently, μ_s. Using (7) and (31), the probability of the primary queue being empty is increased by the factor

$$\lambda_\mathrm{p}\Big[\frac{1}{1-a_\mathrm{s}}-\left(1+a_\mathrm{s}\right)\Big]=\lambda_\mathrm{p}\frac{a_\mathrm{s}^2}{1-a_\mathrm{s}} \tag{32}$$

Note that the factor $\frac{a_\mathrm{s}^2}{1-a_\mathrm{s}}$ is monotonically increasing in a_s.

The optimal access probability of the new scheme can be easily proved to be given by

$$a_\mathrm{s}^*=\min\left(\frac{1-\lambda_\mathrm{p}}{2\lambda_\mathrm{p}},1\right). \tag{33}$$

Therefore, for $\lambda_\mathrm{p}\leq 1/3$, the optimal access probability is always $a_\mathrm{s}=1$, which is another interesting property of the proposed scheme; for $\lambda_\mathrm{p}\leq 1/3$, we do not need to know the exact value of the PU arrival rate. This is different from the conventional system where to get the optimal access probability we need to know the exact value of λ_p. Note that the optimal access probability a_s^* guarantees the stability of the PU queue which requires the access probability to satisfy $\lambda_\mathrm{p}<\frac{1}{1+a_\mathrm{s}}$.

2) Primary User Packet Delay: Applying Little's law, the average number of packets in queue is given by

$$D\lambda_\mathrm{p}=\pi_1+\epsilon_1+\sum_{k=2}^{\infty}k\Big[\pi_k+\epsilon_k\Big]. \tag{34}$$

and the delay can be proved to be given by

$$D=\frac{a_\mathrm{s}\lambda_\mathrm{p}\Big(\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)+1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p}\Big)+\Big(1+a_\mathrm{s}\bar{\lambda}_\mathrm{p}\Big)\Big(1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p}\Big)^2}{\bar{\lambda}_\mathrm{p}\left(1-a_\mathrm{s}\lambda_\mathrm{p}\right)\left(1-\lambda_\mathrm{p}-a_\mathrm{s}\lambda_\mathrm{p}\right)}$$

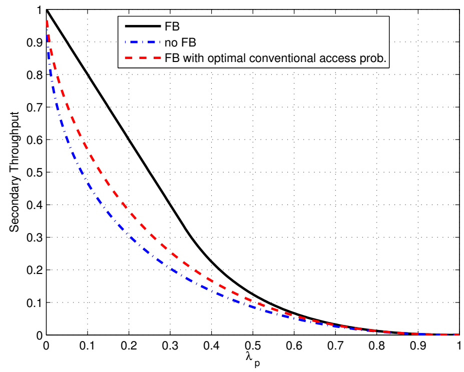


Fig. 3. The secondary throughput for the different access schemes.

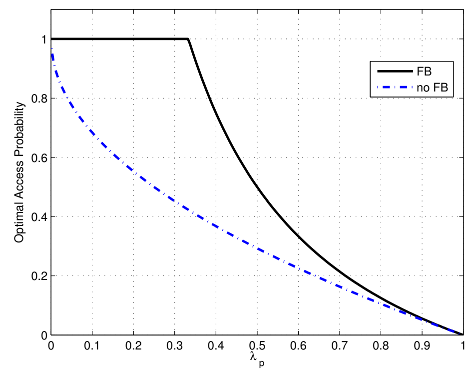


Fig. 4. The SU optimal access probabilities for the different access schemes.

IV. Simulation Results

In this section, we will present some simulation results. Fig. 3 shows the secondary throughput for the conventional and feedback-based schemes as a function of the PU arrival rate. We can see that the use of the feedback information can highly increase the secondary throughput. For example, at a PU arrival rate of $\lambda_{\mathrm{p}} = 0.3$, the feedback-based scheme can achieve secondary throughput which is double that of the conventional scheme. Note that if we use the feedback-based scheme with the the conventional system's optimal access probability we will also gain in terms of the secondary throughput because in the feedback-based scheme we avoid clear collisions with the PU. Fig. 4 shows the optimal access probability for the two schemes presented in the paper. As mentioned earlier for the feedback-based scheme, the optimal access probability is 1 for $\lambda_{\mathrm{p}} \leq 1/3$.

Finally, Fig. 5 shows the delay for both schemes. Note that the feedback-based scheme with the same access probability as the conventional scheme will always have a lower delay compared to the conventional scheme since the queue length will be always lower for the feedback-based scheme. However, this is not the case for the feedback-based scheme with the optimal access probability derived above for maximizing the secondary throughput. Maximizing the secondary throughput might result in increasing the SU's access probability at the expense of slightly increasing the delay incurred by the PU and without affecting its stability.

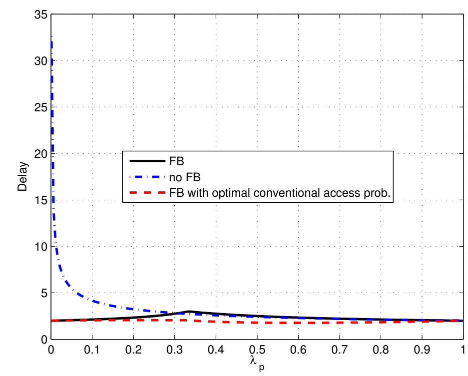


Fig. 5. The PU delay for the different access schemes.

V. Conclusions

We have introduced a new access scheme for cognitive radio systems utilizing the available primary user feedback information. The secondary user employs a random access scheme where the access probability of the secondary user is adjusted according to the primary user feedback state. The proposed scheme can be used with any sensing-based access scheme and it will always result in a performance improvement by avoiding sure collisions with the Primary user. When a NACK is sent from the primary receiver, the secondary user backs-off allowing for collision free transmission from the primary user and this can highly improve the system performance in terms of the achievable secondary throughput and/or the primary user packet delay.

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