Design Criteria and Performance Analysis for Distributed Space-Time Coding

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Abstract—In this paper, the design of distributed space-time (ST) codes for wireless relay networks is considered. Distributed ST coding (DSTC) can be achieved through node cooperation to emulate a multiple-antenna transmitter. First, the decode-and-forward (DAF) protocol, in which each relay node decodes the symbols received from the source node before retransmission, is considered. An ST code designed to achieve full diversity and maximum coding gain over multiple-input–multiple-output (MIMO) channels is proven to achieve full diversity but not necessarily maximize the coding gain if used with the DAF protocol. Next, the amplify-and-forward (AAF) protocol is considered; each relay node can only perform simple operations, such as linear transformation of the received signal and amplification of the signal before retransmission. An ST code designed to achieve full diversity and maximum coding gain over MIMO channels is proven to achieve full diversity and maximum coding gain if used with the AAF protocol. Next, the design of DSTC that can mitigate the relay node synchronization errors is considered. Most of the previous works on cooperative transmission assume perfect synchronization between the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays are identical. Perfect synchronization is difficult to achieve among randomly located relay nodes. To simplify the synchronization in the network, a diagonal structure is imposed on the ST code used. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay node to transmit at any time slot. Hence, it is not necessary to synchronize simultaneous "in-phase" transmissions of randomly located relay nodes, which greatly simplifies the synchronization among the relay nodes. The code design criterion for distributed ST codes based on the diagonal structure is derived. This paper shows that the code design criterion maximizes the minimum product distance.

Index Terms—Coding gain, distributed ST coding (DSTC), space-time (ST) coding, spatial diversity, wireless relay networks.

I. INTRODUCTION

RECENTLY, there has been much interest in modulation techniques to achieve transmit diversity motivated by the increased capacity of multiple-input–multiple-output (MIMO) channels [1]. To achieve transmit diversity, the transmitter needs to be equipped with more than one antenna. The antennas should be well separated to have uncorrelated fading

among the different antennas; hence, higher diversity orders and higher coding gains are achievable. It is affordable to equip base stations with more than one antenna, but it is difficult to equip the small mobile units with more than one antenna with uncorrelated fading. In such a case, transmit diversity can only be achieved through user cooperation, leading to what is known as cooperative diversity [2], [3].

Designing protocols that allow several single-antenna terminals to cooperate via forwarding each others' data can increase system reliability by achieving spatial diversity. Another benefit results from boosting the system throughput by employing cooperation between nodes in the network. In [2] and [3], various node cooperation protocols were proposed, and outage probability analyses for these protocols were provided. The concepts of the decode-and-forward (DAF) and amplify-and-forward (AAF) protocols have been introduced in these works. Symbol error rate (SER) performance analyses for the single-node and multinode DAF cooperation protocols were provided in [4] and [5]. Performance analyses for the single-node and multinode AAF cooperation protocols can be found in [6] and [7]. In [8], Pabst emphasized the importance of studying distributed multistage relaying, in which each stage acts as a virtual antenna, and he envisioned this to be a promising direction to achieve the very high data rate requirements of future wireless systems.

The main problem with the multinode DAF protocol [5] and the multinode AAF protocol [6], [9] is the loss in data rate as the number of relay nodes increases. The use of orthogonal subchannels for the relay node transmissions, either through time-division multiple access (TDMA) or frequency-division multiple access, results in a high loss of the system spectral efficiency. This leads to the use of what is known as distributed space-time coding (DSTC), where relay nodes are allowed to simultaneously transmit over the same channel by emulating a space-time (ST) code. The term *distributed* comes from the fact that the virtual multiantenna transmitter is distributed between randomly located relay nodes. The ST code design criteria for virtual antenna arrays was considered in [10], where the design of ST trellis codes was considered. It was proposed in [2] to use relay nodes to form a virtual multiantenna transmitter to achieve diversity. In addition, an outage analysis was presented for the system.

Several works have considered the application of the existing ST codes in a distributed fashion for the wireless relay network [11]–[14]. All of these works have considered a two-hop relay network where a direct link between the source and destination nodes does not exist, as shown in Fig. 1. In [11], ST block
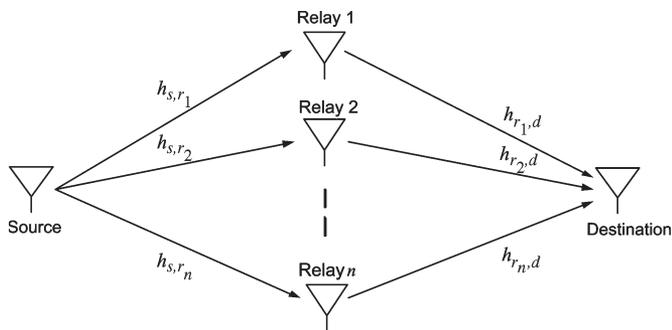
Fig. 1. Simplified system model for the two-hop distributed ST codes.

Fig. 2. Time frame structure for (a) conventional distributed ST codes and (b) DDSTCs.

codes were used in a completely distributed fashion. Each relay node transmits a randomly selected column from the ST code matrix. This system achieves a diversity of order one, as the signal-to-noise ratio (SNR) tends to infinity and is limited by the probability of having all of the relay nodes selecting transmission of the same column of the ST code matrix. In [12], DSTC based on the Alamouti scheme and AAF cooperation protocol was analyzed. An expression for the average SER was derived. In [13], a performance analysis of the gain of using cooperation among nodes was considered, assuming that the number of relays that are available for cooperation is a Poisson random variable. The authors compared the performance of different distributed ST codes designed for the MIMO channels under this assumption. In [14], the performance of the linear dispersion (LD) ST codes of [15] was analyzed when used for DSTC in wireless relay networks. These works did not account for the code design criteria for the ST codes when employed in a distributed fashion. This paper is an answer to the question of whether an ST code, which achieves full diversity and maximum coding gain over MIMO channels, can also achieve full diversity and maximum coding gain if used in a distributed fashion.

In this paper, the two-hop relay network model depicted in Fig. 1, where the system lacks a direct link from the source node to the destination node, is considered. The results apply to that system model, and other relaying network models might have different results. Every node is assumed to be equipped with only one antenna. In the analysis, the wireless channel between any two nodes is assumed to be a Rayleigh flat-fading channel. First, DSTC in conjunction with the DAF cooperation protocol is considered. In this scheme, the relay node forwards the source symbols if it has correctly decoded. Hence, not all of the relays assigned to help the source will forward the source information. An ST code designed to achieve full diversity and maximum coding gain over MIMO channels will achieve full diversity if used with the DAF cooperation protocol. However, it will not necessarily maximize the coding gain.

The design of distributed ST codes used in conjunction with the AAF cooperation protocol is considered. In this scheme, the relay nodes do not decode the received signals from the source node, but they can perform simple operations to the received signal such as linear transformation. Each relay node amplifies the received signal after processing and retransmits it to the destination. In the AAF cooperation protocol, all of the assigned relay nodes for helping the source will always forward the source information. An ST code designed to achieve full diversity and maximum coding gain over MIMO channels is proven to achieve full diversity and maximum coding gain when used with the AAF protocol [16].
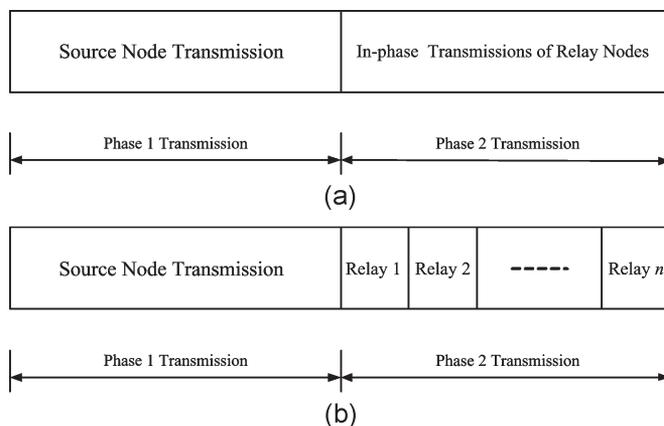
Most of the previous works on cooperative transmission assume perfect synchronization among the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays are identical. Perfect synchronization is difficult to achieve among randomly located relay nodes. Synchronization mismatches can result in intersymbol interference, which can highly degrade system performance. However, if the receiver is able to estimate the synchronization mismatches, it can apply a maximum likelihood (ML) detector, and the system will incur a lower performance degradation. This comes at the expense of increased receiver complexity to estimate the synchronization mismatches and increased overhead in the system in terms of the required training symbols.

To simplify the synchronization in the network, a diagonal structure is imposed on the distributed ST code. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay to transmit at any time slot (assuming TDMA). Hence, it is not necessary to synchronize simultaneous in-phase transmissions of randomly located relay nodes, which greatly simplifies the synchronization among the relay nodes. Fig. 2 shows the time frame structure for the conventional distributed ST codes and the diagonal distributed ST codes (DDSTCs). From Fig. 2, it is clear that perfect propagation delay synchronization and carrier frequency synchronization are not needed since only one relay is transmitting at any given time slot. The code design criterion for the distributed ST code based on the diagonal structure is derived. It turns out that the code design criterion maximizes the minimum product distance of the code, which was previously used to design the diagonal algebraic ST (DAST) codes [17] and to design full-rate full-diversity space frequency (SF) codes [18], [19].

The rest of this paper is organized as follows: In Section II, the DSTC with DAF protocol system model is described, and the performance analysis of the system is provided. In Section III, the DSTC with AAF protocol system model is described, and the performance analysis of the system is provided. In Section IV, the code design criterion for distributed

ST code based on the diagonal structure is derived. In Section V, simulation results are presented. Finally, Section VI concludes this paper.

II. DSTC WITH THE DAF PROTOCOL

In this section, the system model for DSTC with DAF cooperation protocol is presented, and a system performance analysis is provided. The notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{C})$ is used to denote that random vector $\mathbf{x}$ is a circularly symmetric complex Gaussian random vector with mean $\mathbf{m}$ and covariance matrix $\mathbf{C}$.

A. DSTC With the DAF Protocol System Model

The source node is assumed to have n relay nodes assigned for cooperation. The system has two phases given as follows: In phase 1, the source transmits data to the relay nodes with power P_1. The received signal at the kth relay is modeled as

$$\mathbf{y}_{s,r_k} = \sqrt{P_1} h_{s,r_k} \mathbf{s} + \mathbf{v}_{s,r_k}, \qquad k = 1, 2, \ldots, n \quad (1)$$

where $\mathbf{s}$ is an $L \times 1$ transmitted data vector with a power constraint $\|\mathbf{s}\|_F^2 \leq L$, where, in turn, $\|\cdot\|_F^2$ denotes the Frobenius norm,[1] and $h_{s,r_k} \sim \mathcal{CN}(0, \delta_{s,r_k}^2)$ denotes the channel gain between the source node and the kth relay node. The channel gains from the source node to the relay nodes are assumed to be independent. All channel gains are fixed during the transmission of one data packet and can vary from one packet to another, i.e., a block flat-fading channel model is assumed. In (1), $\mathbf{v}_{s,r_i} \sim \mathcal{CN}(\mathbf{0}, N_o \mathbf{I}_n)$ denotes additive white Gaussian noise (AWGN), where $\mathbf{I}_n$ denotes the $n \times n$ identity matrix.

The n relay nodes try to decode the received signals from the source node. Each relay node is assumed to be capable of deciding whether it has correctly decoded. If a relay node correctly decodes, it will forward the source data in the second phase of the cooperation protocol; otherwise, it remains idle. This can be achieved through the use of cyclic redundancy check codes [20]. Alternatively, this performance can be approached by setting an SNR threshold at the relay nodes, and the relay will only forward the source data if the received SNR is larger than that threshold [5]. For the analysis in this section, the relay nodes are assumed to be synchronized by either a centralized or a distributed algorithm.

In phase 2, the relay nodes that have correctly decoded reencode data vector $\mathbf{s}$ with a preassigned code structure. In the subsequent development, no specific code design will be assumed; instead, a generic ST code structure is considered. The ST code is distributed among the relays such that each relay will emulate a single antenna in a multiple-antenna transmitter. Hence, each relay will generate a column in the corresponding ST code matrix. Let $\mathbf{X}_r$ denote the $K \times n$ ST code matrix, with $K \geq n$. Column k of $\mathbf{X}_r$ represents the code transmitted

from the kth relay node. The signal received at the destination is given by

$$\mathbf{y_d} = \sqrt{P_2} \mathbf{X}_r \mathbf{D_I} \mathbf{h}_d + \mathbf{v}_d \quad (2)$$

where

$$\mathbf{h}_d = [h_{r_1,d}, h_{r_2,d}, \ldots, h_{r_n,d}]^T$$

is an $n \times 1$ channel gain vector from the n relays to the destination, $h_{r_k,d} \sim \mathcal{CN}(0, \delta_{r_k,d}^2)$, and P_2 is the relay node power where equal power allocation among the relay nodes is assumed. The channel gains from the relay nodes to the destination node are assumed to be statistically independent, as the relays are spatially separated. The $K \times 1$ vector $\mathbf{v}_d \sim \mathcal{CN}(\mathbf{0}, N_o \mathbf{I}_K)$ denotes AWGN at the destination node. Matrix $\mathbf{D_I}$ is the state matrix, which will be defined later.

The state of the kth relay, i.e., whether it has correctly decoded or not, is denoted by random variable I_k ($1 \leq k \leq n$), which takes values 1 or 0 if the relay correctly or erroneously decodes, respectively. Let

$$\mathbf{I} = [I_1, I_2, \ldots, I_n]^T$$

denote the state vector of the relay nodes and $n_{\mathbf{I}}$ denote the number of relay nodes that have correctly decoded corresponding to a certain realization $\mathbf{I}$. The random variables I_k are statistically independent, as the state of each relay depends only on its channel conditions to the source node, which are independent from other relays. Matrix

$$\mathbf{D_I} = \operatorname{diag}(I_1, I_2, \ldots, I_n)$$

in (2) is defined as the state matrix of the relay nodes. An energy constraint is imposed on the generated ST code such that $\|\mathbf{X}_r\|_F^2 \leq L$, and this guarantees that the transmitted power per source symbol is less than or equal to $P_1 + P_2$.

B. DSTC With the DAF Protocol Performance Analysis

In this section, the pairwise error probability (PEP) performance analysis for the cooperation scheme described in Section II-A is provided. The diversity and coding gain achieved by the protocol are then analyzed.

The random variable I_k can be easily seen to be a Bernoulli random variable. Therefore, the probability distribution of I_k is given by

$$I_k = \begin{cases} 0, & \text{with probability} = 1 - (1 - SER_k)^L \\ 1, & \text{with probability} = (1 - SER_k)^L \end{cases} \quad (3)$$

where SER_k is the uncoded SER at the kth relay node and is modulation dependent. For M-ary quadrature amplitude modulation (M-QAM, $M = 2^p$ with p even), the exact expression can be shown to be upper bounded by [21]

$$SER_k \leq \frac{2N_{og}}{bP_1 \delta_{s,r_k}^2} \quad (4)$$

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[1] The Frobenius norm of the $m \times n$ matrix $\mathbf{A}$ is defined as $\|\mathbf{A}\|_F^2 = \sum_{i=1}^m \sum_{j=1}^n |\mathbf{A}(i,j)|^2 = \operatorname{tr}(\mathbf{A}\mathbf{A}^{\mathcal{H}}) = \operatorname{tr}(\mathbf{A}^{\mathcal{H}}\mathbf{A})$, where $\operatorname{tr}(\cdot)$ is the trace of a matrix, and $\mathbf{A}^{\mathcal{H}}$ is the Hermitian transpose of $\mathbf{A}$.

where $b = 3/(M-1)$, and $g = (4R/\pi) \int_0^{\pi/2} \sin^2 \theta d\theta - (4R^2/\pi) \int_0^{\pi/4} \sin^2 \theta d\theta$, in which $R = 1 - (1/\sqrt{M})$.

The destination is assumed to have perfect channel state information (CSI) as well as the relay node state vector. The destination applies an ML receiver, which will be a minimum distance rule. The conditional PEP is given by

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I}, \mathbf{h_d}\right) = \Pr\left(\|\mathbf{y_d} - \sqrt{P_2}\mathbf{X}_1\mathbf{D_I}\mathbf{h_d}\|_F^2 > \|\mathbf{y_d}\right.$$

$$\left. - \sqrt{P_2}\mathbf{X}_2\mathbf{D_I}\mathbf{h_d}\|_F^2 | \mathbf{I}, \mathbf{h_d}, \mathbf{X}_1 \text{was transmitted}\right) \quad (5)$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are two possible transmitted codewords. The conditional PEP can be expressed as a quadratic form of a complex Gaussian random vector as

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I}, \mathbf{h_d}\right) = \Pr\left(q < 0 | \mathbf{I}, \mathbf{h_d}\right) \quad (6)$$

where

$$q = \begin{bmatrix} \mathbf{w}_1^{\mathcal{H}} & \mathbf{w}_2^{\mathcal{H}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_n \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix}$$

$\mathbf{w}_1 = \sqrt{P_2}(\mathbf{X}_1 - \mathbf{X}_2)\mathbf{D_I}\mathbf{h_d} + \mathbf{v}_d$, and $\mathbf{w}_2 = \mathbf{v}_d$. The random vectors $\mathbf{h_d}$ and $\mathbf{I}$ are mutually independent, as they arise from independent processes. First, the conditional PEP was averaged over channel realizations $\mathbf{h_d}$. By defining the signal matrix

$$\mathbf{C_I} = (\mathbf{X}_1 - \mathbf{X}_2)\mathbf{D_I}\text{diag}(\delta_{r_1,d}^2, \delta_{r_2,d}^2, \ldots, \delta_{r_n,d}^2)$$
$$\times \mathbf{D_I}(\mathbf{X}_1 - \mathbf{X}_2)^{\mathcal{H}} \quad (7)$$

the conditional PEP in (6) can be tightly upper bounded by [22]

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I}\right) \leq \frac{\binom{2\Delta(\mathbf{I})-1}{\Delta(\mathbf{I})-1} N_0^{\Delta(\mathbf{I})}}{P_2^{\Delta(\mathbf{I})} \prod_{i=1}^{\Delta(\mathbf{I})} \lambda_i^{\mathbf{I}}} \quad (8)$$

where $\Delta(\mathbf{I})$ is the number of nonzero eigenvalues of the signal matrix, and the $\lambda_i^{\mathbf{I}}$'s are the nonzero eigenvalues of the signal matrix corresponding to state vector $\mathbf{I}$. The nonzero eigenvalues of the signal matrix are the same as the nonzero eigenvalues of the matrix [23], i.e.,

$$\Gamma(\mathbf{X}_1, \mathbf{X}_2) = \text{diag}(\delta_{r_1,d}, \delta_{r_2,d}, \ldots, \delta_{r_n,d})\mathbf{D_I}\Phi(\mathbf{X}_1, \mathbf{X}_2)$$
$$\times \mathbf{D_I}\text{diag}(\delta_{r_1,d}, \delta_{r_2,d}, \ldots, \delta_{r_n,d})$$

where

$$\Phi(\mathbf{X}_1, \mathbf{X}_2) = (\mathbf{X}_1 - \mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1 - \mathbf{X}_2).$$

The employed ST code is assumed to achieve full diversity and maximum coding gain over MIMO channels, which means that matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ is full rank of order n for any pair of distinct codewords $\mathbf{X}_1$ and $\mathbf{X}_2$. Achieving maximum coding gain means that the minimum of the products $\prod_{i=1}^n \lambda_i$, where the λ_i's are the eigenvalues of matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$, is maximized over all the pairs of distinct codewords [24].

Clearly, if matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ has a rank of order n, then matrix $\Gamma(\mathbf{X}_1, \mathbf{X}_2)$ will have a rank of order $n_{\mathbf{I}}$, which is the number of relays that have correctly decoded. Equation (8) can now be rewritten as

$$\Pr\left(\mathbf{X}_1 \to \mathbf{X}_2 | \mathbf{I}\right) \leq \frac{\binom{2n_{\mathbf{I}}-1}{n_{\mathbf{I}}-1} N_0^{n_{\mathbf{I}}}}{P_2^{n_{\mathbf{I}}} \prod_{i=1}^{n_{\mathbf{I}}} \lambda_i^{\mathbf{I}}}. \quad (9)$$

Second, the conditional PEP was averaged over the relays' state vector $\mathbf{I}$. The dependence of the expression in (9) on $\mathbf{I}$ appears through the set of nonzero eigenvalues $\{\lambda_i^{\mathbf{I}}\}_{i=1}^{n_{\mathbf{I}}}$, which depends on the number of relays that have correctly decoded and their realizations. The state vector $\mathbf{I}$ of the relay nodes determines which columns from the ST code matrix are replaced with zeros and, thus, affect the resulting eigenvalues.

The probability of having a certain realization of $\mathbf{I}$ is given by

$$\Pr(\mathbf{I}) = \left(\prod_{k \in \text{CR}(\mathbf{I})} (1 - SER_k)^L\right)$$
$$\times \left(\prod_{k \in \text{ER}(\mathbf{I})} \left(1 - (1 - SER_k)^L\right)\right) \quad (10)$$

where $\text{CR}(\mathbf{I})$ is the set of relays that have correctly decoded, and $\text{ER}(\mathbf{I})$ is the set of relays that have erroneously decoded, corresponding to the $\mathbf{I}$ realization. For simplicity of presentation, symmetry is assumed between all relays, i.e., $\delta_{s,r_k}^2 = \delta_{s,r}^2$ and $\delta_{r_k,d}^2 = \delta_{r,d}^2$ for all k. Averaging over all realizations of the states of the relays gives the PEP at high SNR as

$$\text{PEP} = \Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq \sum_{k=0}^n \left((1 - SER)^L\right)^k$$
$$\times \left(1 - (1 - SER)^L\right)^{n-k} \sum_{\mathbf{I}:n_{\mathbf{I}}=k} \frac{\binom{2k-1}{k-1} N_0^k}{P_2^k \prod_{i=1}^k \lambda_i^{\mathbf{I}}} \quad (11)$$

where SER is now the SER at any relay node due to the symmetry assumption.

The diversity order of a system determines the average rate with which the error probability decays at high-enough SNR. To compute the diversity order of the system, the PEP in (11) is rewritten in terms of the SNR, which is defined as $\text{SNR} = P/N_o$, where $P = P_1 + P_2$ is the transmitted power per source symbol. Let $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where $\alpha \in (0,1)$. Substituting these definitions, along with the SER expressions at the relay nodes from (4), into (11) and considering high SNR, the PEP can be upper bounded as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq \text{SNR}^{-n} \sum_{k=0}^n \left(\frac{2Lg}{b\alpha\delta_{s,r}^2}\right)^{n-k}$$
$$\times \sum_{\mathbf{I}:n_{\mathbf{I}}=k} \frac{\binom{2k-1}{k-1}}{(1-\alpha)^k \prod_{i=1}^k \lambda_i^{\mathbf{I}}} \quad (12)$$

where, at high SNR, $1 - (1 - SER)^L \approx L \cdot SER$ and upper bounds $1 - L \cdot SER$ by 1. The diversity gain is defined as $d = \lim_{\text{SNR} \to \infty} -(\log(\text{PEP})/\log(\text{SNR}))$. Applying this definition to the PEP in (12), when the number of cooperating nodes is n, gives

$$d_{\text{DF}} = \lim_{\text{SNR} \to \infty} -\frac{\log(\text{PEP})}{\log(\text{SNR})} = n. \tag{13}$$

Hence, any code that is designed to achieve full diversity over MIMO channels will achieve full diversity in the distributed relay network if it is used in conjunction with the DAF protocol. Some of these codes can be found in [15], [17], and [24]–[26].

If full diversity is achieved, the coding gain is

$$C_{\text{DF}} = \left(\sum_{k=0}^{n} \left(\frac{2ng}{b\alpha\delta_{s,r}^2} \right)^{n-k} \sum_{\mathbf{I}:n_{\mathbf{I}}=k} \frac{\binom{2k-1}{k-1}}{(1-\alpha)^k \prod_{i=1}^{k} \lambda_i^{\mathbf{I}}} \right)^{-\frac{1}{n}} \tag{14}$$

which is a term that does not depend on the SNR. To minimize the PEP bound, the coding gain of the distributed ST code needs to be maximized. This is different from the determinant criterion in the case of MIMO channels [24]. Hence, an ST code designed to achieve full diversity and maximum coding gain over MIMO channels will achieve full diversity but not necessarily maximize the coding gain if used in a distributed fashion with the DAF protocol. Intuitively, the difference is due to the fact that, in the case of distributed ST codes with DAF protocol, not all of the relays will always transmit their corresponding code matrix columns. The design criterion used in the case of distributed ST codes makes sure that the coding gain is significant over all sets of possible relays that have correctly decoded. Although it is difficult to design codes to maximize the coding gain, as given by (14), this expression gives insight on how to design good codes. The code design should take into consideration the fact that not all of the relays will always transmit in the second phase.

III. DSTC WITH THE AAF PROTOCOL

In this section, the DSTC based on the AAF protocol is introduced. In this case, the relay nodes do not perform any hard-decision operation on the received data vectors. The system model is presented, and a performance analysis is provided.

A. DSTC With the AAF Protocol System Model

The system has two phases, which are given as follows: In phase 1, if n relays are assigned for cooperation, the source transmits data to the relays with power P_1, and the signal received at the kth relay is as modeled in (1), with $L = n$. For simplicity of presentation, symmetry of the relay nodes is assumed, i.e., $h_{s,r_k} \sim \mathcal{CN}(0, \delta_{s,r}^2)$ $\forall k$, and $h_{r_k,d} \sim \mathcal{CN}(0, \delta_{r,d}^2)$ $\forall k$. In the AAF protocol, relay nodes do not decode the received signals. Instead, the relays can only amplify the received signal and perform simple operations such as permutations of the received symbols or other forms of *unitary* linear transformations. Let $\mathbf{A}_k$ denote the $n \times n$ unitary transforma-

tion matrix at the kth relay node. Each relay will normalize the received signal by the factor $\sqrt{(P_2/n)/(P_1\delta_{s,r}^2 + N_0)}$ to satisfy a long-term power constraint. It can be easily shown that this normalization will give a transmitted power per symbol of $P = P_1 + P_2$.

The $n \times 1$ received data vector from the relay nodes at the destination node can be modeled as

$$\mathbf{y_d} = \sqrt{\frac{P_2/n}{P_1\delta_{s,r}^2 + N_0}} \tilde{\mathbf{X}}_r \mathbf{h}_d + \mathbf{v}_d \tag{15}$$

where $\mathbf{h}_d = [h_{r_1,d}, h_{r_2,d}, \ldots, h_{r_n,d}]^T$ is an $n \times 1$ vector channel gain from the n relays to the destination, where $h_{r_i,d} \sim \mathcal{CN}(0, \delta_{r,d}^2)$, $\tilde{\mathbf{X}}_r$ is the $n \times n$ code matrix given by

$$\tilde{\mathbf{X}}_r = [h_{s,r_1}\mathbf{A}_1\mathbf{s}, h_{s,r_2}\mathbf{A}_2\mathbf{s}, \ldots, h_{s,r_n}\mathbf{A}_n\mathbf{s}]$$

and $\mathbf{v}_d$ denotes the AWGN. Each element of $\mathbf{v}_d$, given the channel coefficients, has the distribution of $\mathcal{CN}(0, N_0(1 + ((P_2/n)/(P_1\delta_{s,r}^2 + N_o)) \sum_{i=1}^{n} |h_{r_i,d}|^2))$, and $\mathbf{v}_d$ accounts for both the noise propagated from the relay nodes and the noise generated at the destination. It can be easily shown that restricting the linear transformations at the relay nodes to be unitary causes the elements of vector $\mathbf{v}_d$, given the channel coefficients, to be mutually independent.

Now, the received vector in (15) can be rewritten as

$$\mathbf{y_d} = \sqrt{\frac{P_2P_1/n}{P_1\delta_{s,r}^2 + N_0}} \mathbf{X}_r \mathbf{h} + \mathbf{v}_d \tag{16}$$

where

$$\mathbf{h} = [h_{s,r_1}h_{r_1,d}, h_{s,r_2}h_{r_2,d}, \ldots, h_{s,r_n}h_{r_n,d}]^T$$
$$\mathbf{X}_r = [\mathbf{A}_1\mathbf{s}, \mathbf{A}_2\mathbf{s}, \ldots, \mathbf{A}_n\mathbf{s}]$$

plays the role of the ST codeword.

B. DSTC With the AAF Protocol Performance Analysis

In this section, a PEP analysis is made to derive the code design criteria. With the ML decoder, the PEP of mistaking $\mathbf{X}_1$ by $\mathbf{X}_2$ can be upper bounded by the following Chernoff bound:

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2)$$
$$\leq E \left\{ \exp \left(-\frac{P_1P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|2\right)} \right. \right.$$
$$\left. \left. \times \mathbf{h}^{\mathcal{H}}(\mathbf{X}_1 - \mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1 - \mathbf{X}_2)\mathbf{h} \right) \right\} \tag{17}$$

where the expectation is over the channel coefficients. Taking the expectation in (17) over the source-to-relay channel

coefficients, which are complex Gaussian random variables, gives

$$
\Pr(\mathbf{X}_1 \rightarrow \mathbf{X}_2)
$$

$$
\leq E\left\{ \det^{-1}\left[\mathbf{I}_n + \frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)} \right. \right.
$$

$$
\times (\mathbf{X}_1 - \mathbf{X}_2)^{\mathcal{H}} \times (\mathbf{X}_1 - \mathbf{X}_2)
$$

$$
\left.\left. \times \operatorname{diag}\left(|h_{r_1,d}|^2, |h_{r_2,d}|^2, \ldots, |h_{r_n,d}|^2\right) \right] \right\}
$$
(18)

where $\mathbf{I}_n$ is the $n \times n$ identity matrix.

To evaluate the expectation in (18), define the matrix

$$
\mathbf{M} = \frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}
$$

$$
\times \Phi(\mathbf{X}_1, \mathbf{X}_2)\operatorname{diag}\left(|h_{r_1,d}|^2, |h_{r_2,d}|^2, \ldots, |h_{r_n,d}|^2\right)
$$

where

$$
\Phi(\mathbf{X}_1, \mathbf{X}_2) = (\mathbf{X}_1 - \mathbf{X}_2)^{\mathcal{H}}(\mathbf{X}_1 - \mathbf{X}_2).
$$

The bound in (18) can be written in terms of the eigenvalues of $\mathbf{M}$ as

$$
\Pr(\mathbf{X}_1 \rightarrow \mathbf{X}_2) \leq E\left\{ \frac{1}{\prod_{i=1}^{n}(1 + \lambda_{M_i})} \right\} \tag{19}
$$

where λ_{M_i} is the ith eigenvalue of matrix $\mathbf{M}$. If $P_1 = \alpha P$ and $P_2 = (1 - \alpha)P$, where P is the power per symbol for some $\alpha \in (0, 1)$ and $\mathrm{SNR} = P/N_0$, the eigenvalues of $\mathbf{M}$ increase with the increase in the SNR. Now, assuming that matrix $\mathbf{M}$ is

full rank of order n, (20), shown at the bottom of the page, holds at high SNR, where the λ_i's are the eigenvalues of matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$. The determinant of a matrix is equal to the product of the matrix eigenvalues, and the determinant of the multiplication of two matrices is equal to the product of the individual matrices' determinants.

The PEP in (19) can now be approximated at high SNR as (21), shown at the bottom of the page. Consider now the term $h = \sum_{i=1}^{n}|h_{r_i,d}|^2$ in (21), which can be reasonably approximated as $\sum_{i=1}^{n}|h_{r_i,d}|^2 \approx n\delta_{r,d}^2$, particularly for large n [14] (by the strong law of large numbers). Averaging the expression in (21) over the exponential distribution of $|h_{r_i,d}|^2$ gives (22), shown at the bottom of the next page, where $\mathbf{Ei}(.)$ is the exponential integral function defined as [27]

$$
\mathbf{Ei}(\mu) = \int_{-\infty}^{\mu} \frac{\exp(t)}{t}dt, \qquad \mu < 0. \tag{23}
$$

The exponential integral function that can be approximated as μ tends to 0 as $-\mathbf{Ei}(\mu) \approx \ln(-1/\mu)$, $\mu < 0$ [27]. At high SNR (high P), $\exp(-(4N_0(P_1\delta_{s,r}^2 + N_o + P_2\delta_{r,d}^2))/((\delta_{s,r}^2\delta_{r,d}^2 P_1 P_2/n)\lambda_i)) \approx 1$, and using the approximation for the $\mathbf{Ei}(.)$ function provides the bound in (22) as

$$
\Pr(\mathbf{X}_1 \rightarrow \mathbf{X}_2) \leq \prod_{i=1}^{n}\left(\frac{\left(\delta_{s,r}^2\delta_{r,d}^2 P_1 P_2/n\right)\lambda_i}{4N_0\left(P_1\delta_{s,r}^2 + P_2\delta_{r,d}^2\right)} \right)^{-1}
$$

$$
\times \prod_{i=1}^{n}\ln\left(\frac{(\delta_{s,r}^2\delta_{r,d}^2 P_1 P_2/n)\lambda_i}{4N_0\left(P_1\delta_{s,r}^2 + P_2\delta_{r,d}^2\right)} \right). \tag{24}
$$

Let $P_1 = \alpha P$ and $P_2 = (1 - \alpha)P$, where P is the power per symbol, for some $\alpha \in (0, 1)$. With the definition of the SNR as

$$
\prod_{i=1}^{n}(1 + \lambda_{M_i}) \simeq 1 + \prod_{i=1}^{n}\lambda_{M_i}
$$

$$
= 1 + \left(\frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)} \right)^n \prod_{i=1}^{n}\lambda_i \prod_{i=1}^{n}|h_{r_i,d}|^2
$$

$$
\simeq \prod_{i=1}^{n}\left(1 + \frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\lambda_i|h_{r_i,d}|^2 \right) \tag{20}
$$

$$
\Pr(\mathbf{X}_1 \rightarrow \mathbf{X}_2) \leq E\left\{ \frac{1}{\prod_{i=1}^{n}\left(1 + \frac{\delta_{s,r}^2 P_1 P_2/n}{4N_0\left(P_1\delta_{s,r}^2 + N_o + \frac{P_2}{n}\sum_{i=1}^{n}|h_{r_i,d}|^2\right)}\lambda_i|h_{r_i,d}|^2\right)} \right\} \tag{21}
$$

$\mathrm{SNR} = P/N_0$, the bound in (24) can be given as

$$
\begin{aligned}
\Pr(\mathbf{X}_1 \to \mathbf{X}_2) &\leq a_{\mathrm{AF}} \frac{1}{\prod_{i=1}^{n} \lambda_i} \mathrm{SNR}^{-n} \prod_{i=1}^{n} (\ln(\mathrm{SNR}) \\
&\quad + \ln(C_i)) \\
&\simeq a_{\mathrm{AF}} \frac{1}{\prod_{i=1}^{n} \lambda_i} \mathrm{SNR}^{-n} (\ln(\mathrm{SNR}))^n \quad (25)
\end{aligned}
$$

where

$$
C_i = \frac{\left(\delta_{s,r}^2 \delta_{r,d}^2 \alpha(1-\alpha)/n \right) \lambda_i}{4 \left(\alpha \delta_{s,r}^2 + (1-\alpha)\delta_{r,d}^2 \right)}, \qquad i = 1, \dots, n
$$

are constant terms that do not depend on the SNR, and a_{AF} is a constant that depends on power allocation parameter α and the variances of the channels. The $\ln(C_i)$ terms are neglected at high SNRs, resulting in the last bound in (25). The diversity order of the system can be calculated as $d_{\mathrm{AF}} = \lim_{\mathrm{SNR} \to \infty} -(\log(\mathrm{PEP})/\log(\mathrm{SNR})) = n$. The system will achieve a full diversity of order n if matrix $\mathbf{M}$ is full rank, i.e., code matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ must be full rank of order n over all distinct pairs of codewords $\mathbf{X}_1$ and $\mathbf{X}_2$. It can be easily shown, following the same approach, that, if the code matrix $\Phi(\mathbf{X}_1, \mathbf{X}_2)$ is rank deficient, then the system will not achieve full diversity. Thus, any code that is designed to achieve full diversity over MIMO channels will achieve full diversity in the case of the AAF DSTC scheme.

If full diversity is achieved, the coding gain is given as

$$
C_{\mathrm{AF}} = \left(a_{\mathrm{AF}} \frac{1}{\prod_{i=1}^{n} \lambda_i} \right)^{-\frac{1}{n}}.
$$

To maximize the coding gain of the AAF distributed ST codes, the product $\prod_{i=1}^{n} \lambda_i$ needs to be maximized, which is the same as the determinant criterion used over MIMO channels [24]. Thus, if an ST code is designed to maximize the coding gain over MIMO channels, it will also maximize the coding gain if it can be used in a distributed fashion with the AAF protocol.

IV. SYNCHRONIZATION-AWARE DISTRIBUTED ST CODES

In this section, the design of distributed ST codes that relax the stringent synchronization requirement is considered. Most of the previous work on cooperative transmission assumed perfect synchronization between the relay nodes, which means that the relays' timings, carrier frequencies, and propagation delays
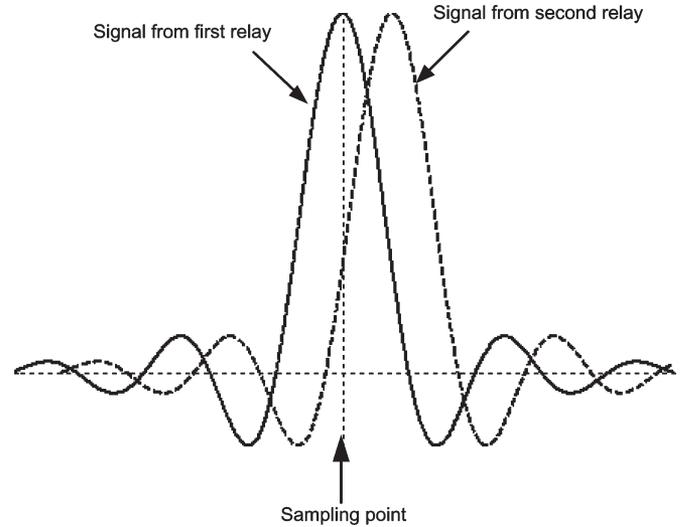
Fig. 3.   Baseband signals (each is a raised-cosine pulse-shaped) from two relays at the receiver.

are identical. To simplify the synchronization in the network, a diagonal structure is imposed on the ST code used. The diagonal structure of the code bypasses the perfect synchronization problem by allowing only one relay to transmit at any time slot. Hence, synchronizing simultaneous in-phase transmissions of randomly distributed relay nodes is not necessary.

This greatly simplifies the synchronization since nodes can maintain slot synchronization, which means that coarse slot synchronization is available [28].[2] However, fine synchronization is more difficult to achieve. Guard intervals are introduced to ensure that the transmissions from different relays are not overlapped. One relay is allowed to consecutively transmit its part of the ST code from different data packets. This allows the overhead introduced by the guard intervals to be neglected. Fig. 3 shows the effect of propagation delay on the received signal from two relays. The sampling time in Fig. 3 is the optimum sampling time for the first relay signal, but clearly, it is not optimal for the second relay signal. Some work has been done on selecting the optimal sampling time [29], but this only works for the case of two relays. The code design criterion for the DDSTC is derived. An AAF system model is considered, which simplifies the relay node design and prevents the propagation of relay errors.

---

[2]For example, any synchronization scheme that is used for TDMA systems can be employed to achieve synchronization in the network.

$$
\begin{aligned}
\Pr(\mathbf{X}_1 \to \mathbf{X}_2) &\leq \prod_{i=1}^{n} \left(\frac{(\delta_{s,r}^2 \delta_{r,d}^2 P_1 P_2 / n) \lambda_i}{4N_0 \left(P_1 \delta_{s,r}^2 + N_o + P_2 \delta_{r,d}^2 \right)} \right)^{-1} \\
&\quad \times \prod_{i=1}^{n} \left[-\exp \left(-\frac{4N_0 \left(P_1 \delta_{s,r}^2 + N_o + P_2 \delta_{r,d}^2 \right)}{\left(\delta_{s,r}^2 \delta_{r,d}^2 P_1 P_2 / n \right) \lambda_i} \right) \mathbf{Ei} \left(-\frac{4N_0 \left(P_1 \delta_{s,r}^2 + N_o + P_2 \delta_{r,d}^2 \right)}{\left(\delta_{s,r}^2 \delta_{r,d}^2 P_1 P_2 / n \right) \lambda_i} \right) \right] \quad (22)
\end{aligned}
$$

A. DDSTC System Model

In this section, the system model with n relay nodes, which helps the source by emulating a diagonal space-time code (STC), is introduced. The system has two phases with the time frame structure shown in Fig. 2(b). In phase 1, the received signals at the relay nodes are modeled as in (1), with $L = n$.

In phase 2, the kth relay applies a linear transformation $\mathbf{t}_k$ to the received data vector, where $\mathbf{t}_k$ is an $1 \times n$ row vector, as

$$
\begin{aligned}
y_{r_k} &= \mathbf{t}_k \mathbf{y}_{s,r_k} \\
&= \sqrt{P_1} h_{s,r_k} \mathbf{t}_k \mathbf{s} + \mathbf{t}_k \mathbf{v}_{s,r_k} \\
&= \sqrt{P_1} h_{s,r_k} x_k + v_{r_k}
\end{aligned}
\tag{26}
$$

where $x_k = \mathbf{t}_k \mathbf{s}$, and $v_{r_k} = \mathbf{t}_k \mathbf{v}_{s,r_k}$. If the linear transformations are restricted to have unit norm, i.e., $\|\mathbf{t}_k\|^2 = 1$ for all k, then v_{r_k} is $\mathcal{CN}(0, N_o)$. The relay then multiplies y_{r_k} by the factor

$$
\beta_k \leq \sqrt{\frac{P_2}{P_1 |h_{s,r_k}|^2}}
\tag{27}
$$

to satisfy a power constraint of $P = P_1 + P_2$ transmitted power per source symbol [3]. The received signal at the destination due to the kth relay transmission is given by

$$
\begin{aligned}
y_k &= h_{r_k,d} \beta_k \sqrt{P_1} h_{s,r_k} x_k + h_{r_k,d} \beta_k v_{r_k} + \tilde{v}_k \\
&= h_{r_k,d} \beta_k \sqrt{P_1} h_{s,r_k} x_k + z_k, \qquad k = 1, \ldots, n
\end{aligned}
\tag{28}
$$

where $\tilde{v}_k$ is modeled as $\mathcal{CN}(0, N_0)$; hence, z_k, given the channel coefficients, is $\mathcal{CN}(0, (\beta_k^2 |h_{r_k,d}|^2 + 1) N_0)$, $k = 1, \ldots, n$.

B. DDSTC Performance Analysis

In this section, the code design criterion of the DDSTC based on the PEP analysis is derived. In the following, the power constraint in (27) is set to be satisfied with equality.

Now, we start deriving a PEP upper bound to derive the code design criterion. Let σ_k^2 denote the variance of z_k in (28), which is given by

$$
\sigma_k^2 = \left(\frac{P_2 |h_{r_k,d}|^2}{P_1 |h_{s,r_k}|^2} + 1 \right) N_0, \qquad k = 1, \ldots, n.
\tag{29}
$$

Then, define the codeword vector $\mathbf{x}$ from (26) as

$$
\mathbf{x} = \underbrace{\left[\mathbf{t}_1^T, \mathbf{t}_2^T, \ldots, \mathbf{t}_n^T \right]^T}_{\mathbf{T}} \mathbf{s} = \mathbf{T} \mathbf{s}
\tag{30}
$$

where $\mathbf{T}$ is an $n \times n$ linear transformation matrix. From $\mathbf{x}$, define the $n \times n$ code matrix $\mathbf{X} = \mathrm{diag}(\mathbf{x})$, which is a diagonal matrix with the elements of $\mathbf{x}$ on its diagonal. Let $\mathbf{y} = [y_1, y_2, \ldots, y_n]^T$ denote the received data vector at the destination node, as given in (28).

Using our system model assumptions, the probability density function (pdf) of $\mathbf{y}$, given the source data vector $\mathbf{s}$ and the CSI, is given by

$$
p(\mathbf{y}|\mathbf{s}, \mathrm{CSI}) = \left(\prod_{i=1}^{n} \frac{1}{\pi \sigma_i^2} \right)
$$

$$
\times \exp\left(-\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \left| y_i - \sqrt{\frac{P_1 P_2}{P_1 |h_{s,r_i}|^2}} h_{s,r_i} h_{r_i,d} x_i \right|^2 \right)
\tag{31}
$$

from which the ML decoder can be expressed as

$$
\arg\max_{\mathbf{s} \in \mathcal{S}} p(\mathbf{y}|\mathbf{s}, \mathrm{CSI})
$$

$$
= \arg\min_{\mathbf{s} \in \mathcal{S}} \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \left| y_i - \sqrt{\frac{P_1 P_2}{P_1 |h_{s,r_i}|^2}} h_{s,r_i} h_{r_i,d} x_i \right|^2
\tag{32}
$$

where $\mathcal{S}$ is the set of all possible transmitted source data vectors.

The PEP of mistaking $\mathbf{X}_1$ by $\mathbf{X}_2$ can be upper bounded as [30]

$$
\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E \left\{ \exp\left(\lambda \left[\ln p(\mathbf{y}|\mathbf{s}_2) - \ln p(\mathbf{y}|\mathbf{s}_1) \right] \right) \right\}
\tag{33}
$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are the code matrices corresponding to the source data vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. Equation (33) applies for any λ, which is a parameter that can be adjusted to get the tightest bound. Now, the PEP can be written as (34), shown at the bottom of the page, where the expectation is over the noise and channel coefficients statistics, and x_{ij} is the jth element of the ith code vector.

To average the expression in (34) over the noise statistics, define receiver noise vector $\mathbf{z} = [z_1, z_2, \ldots, z_n]^T$, where the

$$
\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E \Bigg\{ \exp\Bigg(-\lambda \Bigg[\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \Bigg(\sqrt{\frac{P_1 P_2}{P_1 |h_{s,r_i}|^2}} h_{s,r_i} h_{r_i,d} (x_{1i} - x_{2i}) z_i^*
$$

$$
+ \sqrt{\frac{P_1 P_2}{P_1 |h_{s,r_i}|^2}} h_{s,r_i}^* h_{r_i,d}^* (x_{1i} - x_{2i})^* z_i + \frac{P_1 P_2}{P_1 |h_{s,r_i}|^2} |h_{s,r_i}|^2 |h_{r_i,d}|^2 |x_{1i} - x_{2i}|^2 \Bigg) \Bigg] \Bigg) \Bigg\}
\tag{34}
$$

z_i's are as defined in (28). The pdf of $\mathbf{z}$, given the CSI, is given by

$$p(\mathbf{z}|\text{CSI}) = \left(\prod_{i=1}^{n} \frac{1}{\pi\sigma_i^2}\right) \exp\left(-\sum_{i=1}^{n} \frac{1}{\sigma_i^2} z_i z_i^*\right). \quad (35)$$

Taking the expectation in (34) over $\mathbf{z}$, given the channel coefficients, yields (36), shown at the bottom of the page. Choose $\lambda = 1/2$ that maximizes the term $\lambda(1-\lambda)$, i.e., minimizes the PEP upper bound. Substituting for the σ_i^2's from (29), the PEP can be upper bounded as (37), shown at the bottom of the page.

To get the expression in (37), let us define the variable

$$\gamma_i = \frac{P_1|h_{s,r_i}|^2 P_2|h_{r_i,d}|^2}{(P_1|h_{s,r_i}|^2 + P_2|h_{r_i,d}|^2)N_0}, \qquad i = 1, \ldots, n$$

which is the scaled harmonic mean[3] of the two exponential random variables $((P_1|h_{s,r_i}|^2)/N_0)$ and $((P_2|h_{r_i,d}|^2)/N_0)$. Averaging the expression in (37) over the channel coefficients, the upper bound on the PEP can be expressed as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq \prod_{i=1, x_{1i}\neq x_{2i}}^{n} M_{\gamma_i}\left(\frac{1}{4}|x_{1i} - x_{2i}|^2\right) \quad (38)$$

where $M_{\gamma_i}(.)$ is the moment-generating function (MGF) of random variable γ_i. The problem now is getting an expression for $M_{\gamma_i}(.)$. To get $M_{\gamma_i}(.)$, let y_1 and y_2 be two independent exponential random variables with parameters α_1 and α_2, respectively. Let $y = (y_1 y_2/(y_1 + y_2))$ be the scaled harmonic mean of y_1 and y_2. Then, the MGF of y is [4]

$$M_y(s) = \frac{(\alpha_1 - \alpha_2)^2 + (\alpha_1 + \alpha_2)s}{\Delta^2}$$
$$+ \frac{2\alpha_1\alpha_2 s}{\Delta^3} \ln \frac{(\alpha_1 + \alpha_2 + s + \Delta)^2}{4\alpha_1\alpha_2} \quad (39)$$

[3]The scaling factor is 1/2 since the harmonic mean of two numbers g_1 and g_2 is $(2g_1g_2/(g_1 + g_2))$.

where

$$\Delta = \sqrt{(\alpha_1 - \alpha_2)^2 + 2(\alpha_1 + \alpha_2)s + s^2}.$$

Using the expression in (39), the MGF for γ_i can be approximated at a high-enough SNR to be [4]

$$M_{\gamma_i}(s) \simeq \frac{\zeta_i}{s} \quad (40)$$

where

$$\zeta_i = \frac{N_0}{P_1\delta_{s,r}^2} + \frac{N_0}{P_2\delta_{r,d}^2}.$$

The PEP can now be upper bounded as

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq N_0^n \left(\prod_{i=1, x_{1i}\neq x_{2i}}^{n} \left(\frac{1}{P_1\delta_{s,r}^2} + \frac{1}{P_2\delta_{r,d}^2}\right)\right)$$
$$\times \left(\prod_{i=1, x_{1i}\neq x_{2i}}^{n} \frac{1}{4}|x_{1i} - x_{2i}|^2\right)^{-1}. \quad (41)$$

Let $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where P is the power per symbol, for some $\alpha \in (0,1)$, and define $\text{SNR} = P/N_0$. The diversity order d_{DDSTC} of the system is

$$d_{\text{DDSTC}} = \lim_{SNR\to\infty} -\frac{\log(\text{PEP})}{\log(\text{SNR})} = \min_{m\neq j} rank(\mathbf{X}_m - \mathbf{X}_j) \quad (42)$$

where $\mathbf{X}_m$ and $\mathbf{X}_j$ are two possible code matrices. To achieve a diversity order of n, matrix $\mathbf{X}_m - \mathbf{X}_j$ should be full rank for any $m \neq j$ (i.e., $x_{mi} \neq x_{ji} \; \forall m \neq j \; \forall i = 1, \ldots, n$). Intuitively, if two code matrices exist, for which the rank of matrix $\mathbf{X}_m - \mathbf{X}_j$ is not n, this means that they have at least one diagonal element that is the same in both matrices. Clearly, this element cannot be used to decide between these two possible transmitted code matrices; hence, the diversity order of the

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\exp\left(-\lambda(1-\lambda)\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \frac{P_1 P_2}{P_1|h_{s,r_i}|^2}\left(|h_{s,r_i}|^2|h_{r_i,d}|^2|x_{1i} - x_{2i}|^2\right)\right)\right.$$
$$\times \int_{\mathbf{z}} \left(\prod_{i=1}^{n} \frac{1}{\pi\sigma_i^2}\right) \exp\left(-\sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left|z_i + \lambda\sqrt{\frac{P_1 P_2}{P_1|h_{s,r_i}|^2}}h_{s,r_i}h_{r_i,d}(x_{1i} - x_{2i})\right|^2\right) d\mathbf{z}\right\}$$
$$= E\left\{\exp\left(-\lambda(1-\lambda)\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \frac{P_1 P_2}{P_1|h_{s,r_i}|^2}\left(|h_{s,r_i}|^2|h_{r_i,d}|^2|x_{1i} - x_{2i}|^2\right)\right)\right\} \quad (36)$$

$$\Pr(\mathbf{X}_1 \to \mathbf{X}_2) \leq E\left\{\exp\left(-\frac{1}{4}\sum_{i=1}^{n} \frac{P_1|h_{s,r_i}|^2 P_2|h_{r_i,d}|^2}{(P_1|h_{s,r_i}|^2 + P_2|h_{r_i,d}|^2)N_0}|x_{1i} - x_{2i}|^2\right)\right\} \quad (37)$$

system is reduced. This criterion implies that each element in the code matrix is unique to that matrix and that any other matrix will have a different element at that same location, which is really the source of diversity. Furthermore, to minimize the PEP bound in (41), we need to maximize

$$\min_{m \neq j} \left(\prod_{i=1}^{n} |x_{mi} - x_{ji}|^2 \right)^{1/n} \tag{43}$$

which is called the minimum product distance of the set of symbols $\mathbf{s} = [s_1, s_2, \ldots, s_n]^T$ [31], [32]. A linear mapping is used to form the transmitted codeword, i.e.,

$$\mathbf{x} = \mathbf{T}\mathbf{s}. \tag{44}$$

Several works have considered the design of the $n \times n$ transformation matrix $\mathbf{t}$ to maximize the minimum product distance. It was proposed in [33] and [34] to use both Hadamard transforms and Vandermonde matrices to design the $\mathbf{t}$ matrix. The transforms based on the Vandermonde matrices were shown to give larger minimum product distance than the Hadamard-based transforms. Some of the best known transforms based on the Vandermonde matrices [18] are summarized. Two classes of optimum transforms were proposed in [33].

1) If $n = 2^k$ $(k \geq 1)$, the optimum transform is given by $\mathbf{T}_{\text{opt}} = (1/\sqrt{n})\text{vander}(\theta_1, \theta_2, \ldots, \theta_n)$, where $\theta_1, \theta_2, \ldots, \theta_n$ are the roots of the polynomial $\theta^n - j$ over the field $\mathbf{Q}[j] \triangleq \{c + dj \ : \ \text{both } c \text{ and } d \text{ are rational numbers}\}$, and they are determined as $\theta_i = e^{j(4i-3/2n)\pi}, \ i = 1, 2, \ldots, n$.

2) If $n = 3 \cdot 2^k$ $(k \geq 0)$, the optimum transform is given by $\mathbf{T}_{\text{opt}} = \frac{1}{\sqrt{n}} \text{vander}(\theta_1, \theta_2, \ldots, \theta_n)$, where $\theta_1, \theta_2, \ldots, \theta_n$ are the roots of the polynomial $\theta^n + w$ over the field $\mathbf{Q}[w] \triangleq \{c + dw \ : \ \text{both } c \text{ and } d \text{ are rational numbers}\}$, and they are determined as $\theta_i = e^{j\frac{6i-1}{3n}\pi}, \ i = 1, 2, \ldots, n$.

The signal constellation from $\mathbf{Z}[j]$, such as QAM, M-ary phase-shift keying, and pulse amplitude modulation constellations, are of practical interest. Moreover, in [34], some nonoptimal transforms were proposed for some n's not satisfying any of the aforementioned two cases.

V. SIMULATION RESULTS

In this section, simulation results for the DSTC schemes from the previous sections are presented. In the simulations, the variance of any source–relay or relay–destination channel is taken as 1. The performance of the different schemes with two relays helping the source is compared. Fig. 4 shows the simulations for two DAF systems using the Alamouti scheme (DAF Alamouti) and the diagonal STC (DAF DAST); distributed ST codes based on the LD ST codes (LD-DSTC) [14], which are based on the AAF scheme; the orthogonal distributed ST codes (O-DSTC) proposed in [35] and [36]; and DDSTC. The O-DSTCs are based on a generalized AAF scheme, where relay nodes apply linear transformation to the received data as well as their complex conjugate. All of these systems have a data rate of 1/2. Quaternary phase-shift keying (QPSK) modulation is used, which means that a rate of one
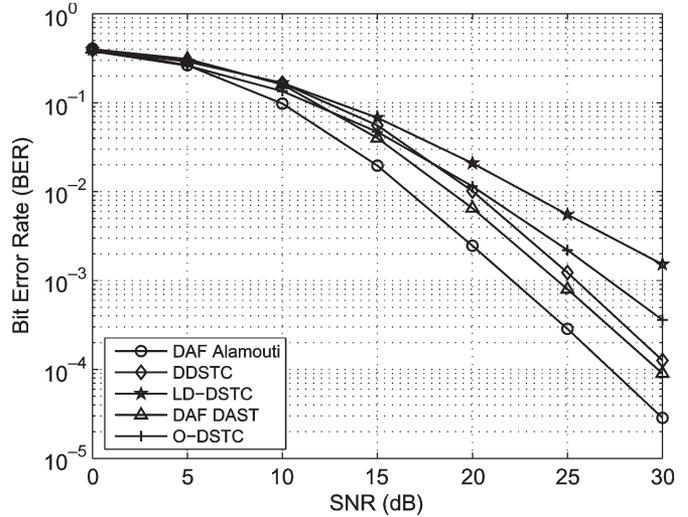
Fig. 4. BER for two relays with a data rate of 1 bit/sym.
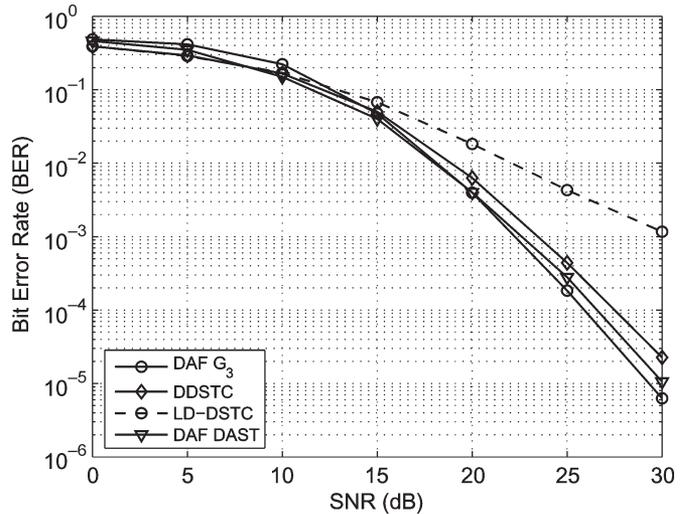
Fig. 5. BER for three relays with a data rate of 1 bit/sym.

transmitted bit per symbol (1 bit/sym) is achieved. For the DAF system, the power of the relay nodes that have erroneously decoded is not reallocated to other relay nodes. Clearly, DAF-based systems outperform AAF-based systems,[4] but this is under the assumption that each relay node can decide whether it has correctly decoded or not. Intuitively, the DAF protocol will deliver signals that are less noisy to the destination. The noise is suppressed at the relay nodes by transmitting a noise-free version of the signal. The AAF delivers more noise to the destination due to noise propagation from the relay nodes. However, the assumption of correct decision at the relay nodes imposes practical limitations on the DAF systems; otherwise, error propagation [3] may occur, because of the errors at the relay nodes. Error propagation would highly degrade system bit error rate (BER) performance.

Fig. 5 shows the simulation results for two DAF systems using the $\mathcal{G}_3$ ST block code of [26] and the diagonal STC (DAF DAST), LD-DSTC, and DDSTC. For fair comparison,

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[4]DDSTC is based on the AAF protocol.
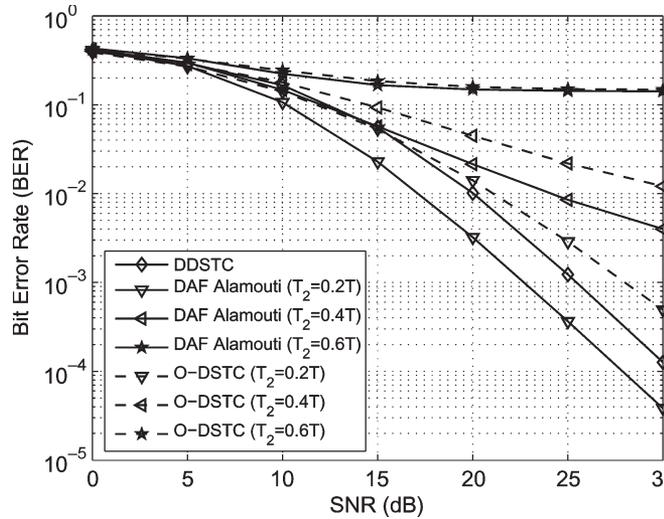
Fig. 6. BER performance with propagation delay mismatch. Two relay cases.

Fig. 7. BER performance with propagation delay mismatch. Three relay cases.

the number of transmitted bits per symbol is fixed at 1 bit/sym. The $\mathcal{G}_3$ ST block code has a data rate of 1/2 [26], which results in an overall system data rate of 1/3. Therefore, eight-phase phase-shift keying (8-PSK) modulation is employed for the system that uses the $\mathcal{G}_3$ ST block code. For the other three systems, QPSK modulation is used, as these systems have a data rate of 1/2. For the DAF system, the power of the relay nodes that erroneously decoded is not reallocated. Clearly, DAF-based systems outperform AAF-based systems under the same constraints previously stated. It is noteworthy that the performance of the LD-DSTC is not optimized since the LD matrices are randomly selected based on the isotropic distribution on the space of $n \times n$ unitary matrices, as in [14]. The performance of the LD-based codes can be improved by trying to optimize the selection of the LD matrices, which is out of the scope of this paper.

In the sequel, the effect of the synchronization errors on the system BER performance is investigated. Fig. 6 shows the case of having two relays helping the source and propagation delay mismatches of $T_2 = 0.2T,\ 0.4T$, and $0.6T$, where T is the time slot duration. Raised-cosine pulse-shaped waveforms were used with a rolloff factor of 0.2 and QPSK modulation. Clearly, the BER performance of the system greatly deteriorates as the propagation delay mismatch becomes larger. Fig. 7 shows the case of having three relays helping the source for different propagation delay mismatches. The DAF system using the $\mathcal{G}_3$ ST block code of [26] and the DDSTC were compared. For fair comparison, the number of transmitted bits per symbol is fixed at 1 bit/sym. Again, the $\mathcal{G}_3$ ST block code has a data rate of 1/2 [26], which results in an overall system data rate of 1/3. Therefore, 8-PSK modulation is employed for the system that uses the $\mathcal{G}_3$ ST block code. For the DDSTC, QPSK modulation is used, as the system has a data rate of 1/2. Raised-cosine pulse-shaped waveforms with a rolloff factor of 0.2 are used. Clearly, system performance is highly degraded as the propagation delay mismatch becomes larger. From Figs. 6 and 7, it is clear that the synchronization errors can greatly deteriorate system BER performance. The DDSTC bypasses this problem by allowing only one relay transmission at any time slot.
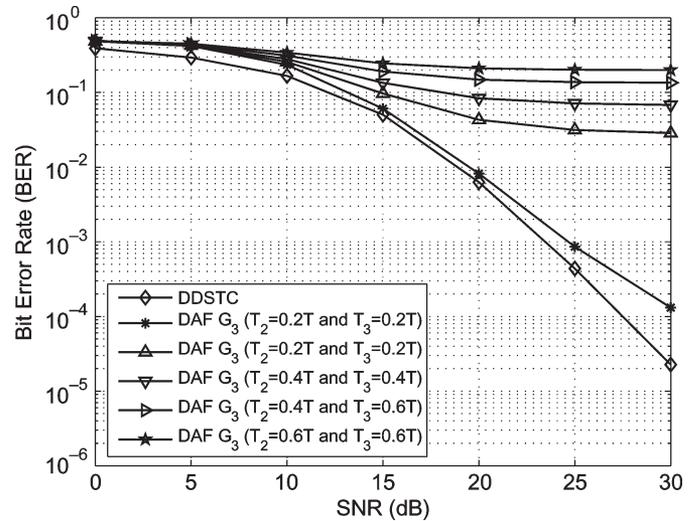
VI. CONCLUSION

The design of distributed ST codes in wireless relay networks is considered for different schemes, which vary in the processing performed at the relay nodes. For the DAF distributed ST codes, any ST code that is designed to achieve full diversity over MIMO channels can achieve full diversity under the assumption that the relay nodes can decide whether they have correctly decoded or not. A code that maximizes the coding gain over MIMO channels is not guaranteed to maximize the coding gain in the DAF DSTC. This is due to the fact that not all of the relays will always transmit their code columns in the second phase. Then, the code design criteria for the AAF distributed ST codes were considered. In this case, a code designed to achieve full diversity over MIMO channels will also achieve full diversity. Furthermore, a code that maximizes the coding gain over MIMO channels will also maximize the coding gain in the AAF distributed ST scheme.

The design of DDSTC for wireless relay networks was investigated. In DDSTC, the diagonal structure of the code was imposed to simplify the synchronization between randomly located relay nodes. Synchronization mismatches between the relay nodes cause intersymbol interference, which can greatly degrade system performance. DDSTC relaxes the stringent synchronization requirement by allowing only one relay to transmit at any time slot. The code design criterion for the DDSTC based on minimizing the PEP was derived, and the design criterion was found to maximize the minimum product distance. This is the same criterion used to design DAST codes and full-rate, full-diversity SF codes.

REFERENCES

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[4] W. Su, A. K. Sadek, and K. J. R. Liu, "Cooperative communication protocols in wireless networks: Performance analysis and optimum power allocation," *Wireless Pers. Commun.*, vol. 44, no. 2, pp. 181–217, Jan. 2008.

[5] A. K. Sadek, W. Su, and K. J. R. Liu, "Multinode cooperative communications in wireless networks," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 341–355, Jan. 2007.

[6] A. Robeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.

[7] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[8] R. Pabst, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sep. 2004.

[9] K. G. Seddik, A. K. Sadek, W. Su, and K. J. R. Liu, "Outage analysis and optimal power allocation for multinode relay networks," *IEEE Signal Process. Lett.*, vol. 14, no. 6, pp. 377–380, Jun. 2007.

[10] M. Dohler, B. A. Rassool, and A. H. Aghvami, "Performance evaluation of STTCs for virtual antenna arrays," in *Proc. IEEE VTC—Spring*, Apr. 2003, vol. 1, pp. 57–60.

[11] B. Sirkeci-Mergen and A. Scaglione, "Randomized distributed space-time coding for cooperative communication in self-organized networks," in *Proc. IEEE Workshop SPAWC*, Jun. 5–8, 2005, pp. 500–504.

[12] P. A. Anghel, G. Leus, and M. Kaveh, "Multi-user space-time coding in cooperative networks," in *Proc. ICASSP*, Apr. 6–10, 2003, pp. IV-73–IV-76.

[13] S. Barbarossa, L. Pescosolido, D. Ludovici, L. Barbetta, and G. Scutari, "Cooperative wireless networks based on distributed space-time coding," in *Proc. IEEE IWWAN*, May 31–Jun. 3, 2004, pp. 30–34.

[14] Y. Jing and B. Hassibi, "Distributed space-time codes in wireless relay networks," in *Proc. 3rd IEEE Sensor Array Multi-Channel Signal Process. Workshop*, Jul. 18–21, 2004, pp. 249–253.

[15] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1804–1824, Jul. 2002.

[16] K. G. Seddik, A. K. Sadek, and K. J. R. Liu, "Protocol-aware design criteria and performance analysis for distributed space-time coding," in *Proc. IEEE GLOBECOM*, Dec. 2006, pp. 1–5.

[17] M. O. Damen, K. Abed-Meraim, and J. C. Belfiore, "Diagonal algebraic space-time block codes," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 628–636, Mar. 2002.

[18] W. Su, Z. Safar, and K. J. R. Liu, "Full-rate full-diversity space-frequency codes with optimum coding advantage," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 229–249, Jan. 2005.

[19] W. Su, Z. Safar, and K. J. R. Liu, "Towards maximum achievable diversity in space, time, and frequency: Performance analysis and code design," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1847–1857, Jul. 2005.

[20] P. Merkey and E. C. Posner, "Optimum cyclic redundancy codes for noisy channels," *IEEE Trans. Inf. Theory*, vol. IT-30, no. 6, pp. 865–867, Nov. 1984.

[21] M. K. Simon and M. S. Alouini, *Digital Communication over Generalized Fading Channels: A Unified Approach to the Performance Analysis*. Hoboken, NJ: Wiley, 2000.

[22] S. Siwamogsatham, M. P. Fitz, and J. H. Grimm, "A new view of performance analysis of transmit diversity schemes in correlated Rayleigh fading," *IEEE Trans. Inf. Theory*, vol. 48, no. 4, pp. 950–956, Apr. 2002.

[23] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.

[24] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[25] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[26] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 3, pp. 451–460, Mar. 1999.

[27] I. S. Gradshteyn and I. M. Ryshik, *Table of Integrals, Series and Products*, 6th ed. New York: Academic, 2000.

[28] X. Li, "Space-time coded multi-transmission among distributed transmitters without perfect synchronization," *IEEE Signal Process. Lett.*, vol. 11, no. 12, pp. 948–951, Dec. 2004.

[29] Y. Mei, Y. Hua, A. Swami, and B. Daneshrad, "Combating synchronization errors in cooperative relays," in *Proc. IEEE ICASSP*, Mar. 19–23, 2005, pp. iii/369–iii/372.

[30] H. L. Van Trees, *Detection, Estimation, and Modulation Theory—Part I*. Hoboken, NJ: Wiley, 1968.

[31] C. Schlegel and D. J. Costello, Jr., "Bandwidth efficient coding for fading channels: Code construction and performance analysis," *IEEE J. Sel. Areas Commun.*, vol. 7, no. 9, pp. 1356–1368, Dec. 1989.

[32] K. Boull and J. C. Belfiore, "Modulation schemes designed for the Rayleigh channel," in *Proc. CISS*, Mar. 1992, pp. 288–293.

[33] X. Giraud, E. Boutillon, and J. C. Belfiore, "Algebraic tools to build modulation schemes for fading channels," *IEEE Trans. Inf. Theory*, vol. 43, no. 3, pp. 938–952, May 1997.

[34] J. Boutros and E. Viterbo, "Signal space diversity: A power- and bandwidth-efficient diversity technique for the Rayleigh fading channel," *IEEE Trans. Inf. Theory*, vol. 44, no. 4, pp. 1453–1467, Jul. 1998.

[35] Y. Jing and H. Jafarkhani, "Using orthogonal and quasi-orthogonal designs in wireless relay networks," in *Proc. IEEE GLOBECOM*, Dec. 2006, pp. 1–5.

[36] T. Kiran and B. S. Rajan, "Distributed space-time codes with reduced decoding complexity," in *Proc. IEEE ISIT*, Jul. 2006, pp. 542–546.

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