Adaptive Spectrum Hole Detection Using Sequential Compressive Sensing

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***Abstract*—Spectrum Sensing in wideband cognitive radio networks is considered one of the challenging issues facing opportunistic utilization of the frequency spectrum. Collaborative compressive sensing has been proposed as an effective technique to alleviate some of these challenges through efficient sampling that exploits the underlying sparse structure of the measured frequency spectrum. In this paper, we propose to model this problem as a compressive support recovery problem, and apply the adaptive Sequential Compressive Sensing (SCS) approach to recover spectrum holes. We propose several fusion techniques to apply the proposed approach in a collaborative manner. The experimental analysis through simulations shows that the proposed scheme can substantially increase the probability of spectrum hole detection as compared to traditional CS recovery approaches while using a very low sampling rate analog to information converter, and without requiring the knowledge of any statistical information about the environmental noise.**

***Index Terms*—Cognitive Radios, Collaborative Spectrum Sensing, Compressive Sensing, Sequential Compressive Sensing.**

I. Introduction

Cognitive radio (CR) technology is gaining more ground each day as a promising technology to mitigate the limited availability of radio spectrum resources. However, spectrum sensing in wideband CR networks remains as one of the most challenging issues facing the widespread of this technology. The Secondary User (SU) needs to sense the spectrum in order to detect the unoccupied channels available for opportunistic use.

Several issues cause such challenges to spectrum sensing. One of these issues is the need to sample the signal at a high sampling rate especially in wideband networks. This requires expensive, complicated Analog to Digital Converter (ADC). The current technology forms an obstacle to design such a high sampling rate with wide dynamic range ADC [1]. Another issue, known as the Hidden Primary User problem, arises due to severe multipath fading and shadowing in the sensed channel. In this case, the SU cannot detect the Primary User (PU) signal leading to interference at the PU receiver. Collaborative spectrum sensing has been proposed to solve this problem, but at the expense of increased overhead in the network.

Compressive Sensing (CS) acquisition techniques have been proposed as effective means to resolve similar issues in different applications. Under the assumption that the signal spectrum is sparse in some domain, CS algorithms can capture the signal at much lower sampling rate compared to the Nyquist rate. Recently, adaptive sensing techniques such as distilled sensing [2] were proven effective in signal acquisition in noisy environments using multi-acquisition processes with adaptive Gaussian measurement matrix focusing the sensing energy on the non-zero components. This approach was extended to incorporate compressive signal acquisition in [3]. The Sequential Compressive Sensing approach (SCS) proposed in [4] provides a complete adaptive algorithm with different acquisition and refinement steps where adaptive sparse measurement matrix is used to capture the sparse signal.

Several attempts to use CS for compressive spectrum sensing were presented in the literature. The sparsity assumption is valid in most of the scenarios of spectrum sensing due to the underutilized nature of the spectrum [5]. In [6], a wavelet spectrum detection method is used to find the edges of the piecewise constant Power Spectrum Density (PSD). A following approach was proposed in [7] and [8], where the wideband analog signal is directly captured by an Analog to Information Converter (AIC), solving the bottleneck in the sampling rate presented in the previous scheme, collaborative distributed spectrum sensing was also considered. A different collaborative approach using Kronecker Compressive Sensing was proposed in [9]. Another interesting approach, which is more relevant to this work, was proposed in [10]. Instead of recovering the edges of the spectrum, it was assumed that the spectrum has a slotted frequency segmentation structure. CS is utilized to recover the spectrum at a low sampling rate at each CR. A distributed fusion approach is proposed based on a consensus averaging technique for multi-hop large networks.

Most of these approaches require the knowledge of the noise statistics either for the recovery algorithm or for the process of setting the threshold needed to recover the support of the spectrum. Due to noise uncertainty, a virtual wall, called *SNR wall*, appeared in [11]. This SNR wall limits the sensitivity of the SUs by the amount of noise uncertainty.

In order to overcome these limitations, we propose modelling spectrum hole detection as a support recovery problem. We find this formulation more practical since in cognitive radio we are solely interested in the detection of the holes in the frequency spectrum. We deploy the adaptive SCS approach presented in [4] for spectrum hole detection. This enables us to recover the spectrum holes with the minimum number of measurements, and alleviates the need for statistical information about the noise, resulting in an increased robustness to the noise uncertainty problem. Moreover, it enables SUs to detect the Primary User's (PUs) signal at low SNR, which increases the sensitivity of the SU receiver.

We also propose a collaborative version of the proposed spectrum sensing scheme in which the sequential sensing is distributed among several SUs and the support is recovered at a central fusion center (FC). We experiment our proposed scheme with the decision fusion, the quantized data fusion, and the data fusion rules at the FC. We compare the performance of each of these algorithms with the corresponding non-adaptive compressive sensing based algorithm.

II. Signal Model and Problem Statement

We consider a network of J cognitive radio terminals, distributed randomly in a certain geographic area. Each CR locally monitors an OFDMA signal with M channels (subcarriers). We assume a predefined channel location and unknown power spectrum density level for the PU in each channel similar to [10]. The problem of spectrum holes detection is to determine whether each of these channels is occupied or available for opportunistic use. We will address the problem for the scenarios of a single SU detection and collaborative spectrum sensing through a centralized fusion center (FC). The channel between any PU and the SUs is considered to be a multipath-fading channel with additive white Gaussian noise.

Consider I active primary users, whose signals are represented by $\tilde{s}_i(t)$, where I is much smaller than the total number of channels M. This is justified by the low percentage of spectrum occupancy by active radios. The received signal at the j-th SU from all PUs can be modeled as follows

$$x_j(t) = \tilde{x}_j(t) + w_j(t), \tag{1}$$

where $\tilde{x}_j(t) = \sum_{i=1}^{I} \tilde{h}_{i,j}(t) * \tilde{s}_i(t)$ is the noise-free received signal from PUs, $\tilde{h}_{i,j}(t)$ is the impulse response of the channel from the i-th PU to the j-th SU, '$*$' denotes circular convolution[1], and $w_j(t)$ is the additive white Gaussian noise at the j-th SU. Equation (1) can be written in a discrete vector form as follows

$$\boldsymbol{x}_j = \tilde{\boldsymbol{x}}_j + \boldsymbol{w}_j = \sum_{i=1}^{I} \tilde{\boldsymbol{h}}_{i,j} * \tilde{\boldsymbol{s}}_i + \boldsymbol{w}_j, \tag{2}$$

where $\boldsymbol{x}_j$, $\tilde{\boldsymbol{s}}_i$ $\tilde{\boldsymbol{h}}_{i,j}$, $\tilde{\boldsymbol{x}}_j$, and $\boldsymbol{w}_j$ are $M \times 1$ vectors. The signal in frequency domain can be represented by taking the Discrete Fourier Transform (DFT) of equation (2), as follows

$$\boldsymbol{X}_j = \tilde{\boldsymbol{X}}_j + \boldsymbol{W}_j = \sum_{i=1}^{I} \tilde{\boldsymbol{H}}_{i,j} \tilde{\boldsymbol{S}}_i + \boldsymbol{W}_j, \tag{3}$$

[1]The received signal is assumed to be OFDMA signal with cyclic prefix

where $\tilde{\boldsymbol{H}}_{i,j}$ is an $M \times M$ diagonal matrix, whose main diagonal is the M point DFT of $\tilde{\boldsymbol{h}}_{i,j}$, and $\boldsymbol{X}_j$, $\tilde{\boldsymbol{S}}_i$, $\boldsymbol{W}_j$ are the frequency-domain versions of $\boldsymbol{x}_j$, $\tilde{\boldsymbol{s}}_i$, $\boldsymbol{w}_j$, respectively. Equation (3) can be stacked in a matrix form as

$$\boldsymbol{X}_j = \acute{\boldsymbol{H}}_j \acute{\boldsymbol{S}} + \boldsymbol{W}_j. \tag{4}$$

where $\acute{\boldsymbol{H}}_j = [\tilde{\boldsymbol{H}}_{1,j}, \tilde{\boldsymbol{H}}_{2,j}, \ldots, \tilde{\boldsymbol{H}}_{I,j}]$ is an $M \times MI$ matrix and $\acute{\boldsymbol{S}} = [\tilde{\boldsymbol{S}}_1^T, \tilde{\boldsymbol{S}}_2^T, \ldots, \tilde{\boldsymbol{S}}_I^T]^T$ is an $MI \times 1$ vector. However, this representation results in increasing the dimension of the spectrum signal by a factor of I. Alternative formulation can be obtained since we assume that in the primary network every PU is assigned a different channel, i.e., there is at most one active PU transmitter on each channel, and assuming flat fading gain on each of the narrow sub-bands. Thus, we can represent the PUs combined spectrum as an $M \times 1$ vector such that $\boldsymbol{S} = \sum_{i=1}^{I} \tilde{\boldsymbol{S}}_i$. We can also construct a combined Channel State Information (CSI) matrix $\boldsymbol{H}_j$ as a diagonal matrix, where each diagonal element corresponds to the channel gain between the j-th SU and the active PU occupying this channel. The sensed spectrum can be represented as

$$\boldsymbol{X}_j = \boldsymbol{H}_j \boldsymbol{S} + \boldsymbol{W}_j = \tilde{\boldsymbol{X}}_j + \boldsymbol{W}_j. \tag{5}$$

Instead of using analog to digital converter with sampling rate higher than the Nyquist rate, the SU receiver collects a compressed measurement of the analog signal $x_j(t)$. This is performed using an Analog to Information Converter (AIC) with much lower sub-Nyquist sampling rate [12]. The underlying assumption is that the signal is sparse in the frequency domain as the number of occupied channels I is much smaller than the total number of channels M. The $K \times 1$ measurement vector $\boldsymbol{y}_j$ collected at the j-th SU from $x_j(t)$ is represented as follows

$$\boldsymbol{y}_j = \boldsymbol{\Phi}_j \boldsymbol{x}_j, \tag{6}$$

where $I < K \ll M$, and $\boldsymbol{\Phi}_j$ is a $K \times M$ random measurement matrix whose entries are independent and identically distributed random variables drawn from some probability distribution. This scheme reduces the sampling rate by a factor of K/M. Substituting from equation (5), equation (6) can be rewritten as

$$\boldsymbol{y}_j = \boldsymbol{\Phi}_j \boldsymbol{F}^{-1} \boldsymbol{X}_j = \boldsymbol{\Phi}_j \boldsymbol{F}^{-1} \boldsymbol{H}_j \boldsymbol{S} + \check{\boldsymbol{W}}_j, \tag{7}$$

where $\boldsymbol{F}^{-1}$ is the $M \times M$ inverse DFT matrix and $\check{\boldsymbol{W}}_j = \boldsymbol{\Phi}_j \boldsymbol{F}^{-1} \boldsymbol{W}_j$. The spectrum sensing problem is to estimate the spectrum $\boldsymbol{S}$ from the low rate measurement vector y_j. However, as we are interested only in investigating the status of the channels (free or occupied), the problem is converted into a spectrum detection problem. Our aim is to detect the binary decision vector $\boldsymbol{d}$, which describes the status of the channels as follows

$$\boldsymbol{d} = (\|\boldsymbol{S}\| > 0). \tag{8}$$

In order to fully recover $\boldsymbol{S}$, it is required to know the channel state information (CSI). However, one advantage of our proposed formulation is that we can recover $\boldsymbol{d}$ efficiently in the absence of CSI by noting that both $\tilde{\boldsymbol{X}}_{\boldsymbol{j}}$ and $\boldsymbol{S}$ share the same support. This results from the diagonal structure of the CSI matrix with non-zero diagonal components.

III. Non-Cooperative Compressive Spectrum Sensing

In this section, we deal with spectrum sensing at a single SU. Each SU senses the compressed spectrum and makes its own decision based on the received signal. For the sake of comparison, we summarize the non-adaptive based compressive scheme similar to [10] in section III-A followed by our proposed Sequential Compressive Sensing algorithm in section III-B.

A. Non-adaptive compressive spectrum detection

We consider the case of a single SU which uses a non-adaptive compressive spectrum detection scheme. In this case, we recover the common spectrum $\boldsymbol{S}$ and then compare it to a threshold in order to detect the occupied channels.

The spectrum vector $\boldsymbol{S}$ can be recovered from the compressed measurements $\boldsymbol{y}_j$ in equation (7) by solving the following quadratic constrained linear program (QCLP) problem, known as Basis Pursuit Denoising (BPDN),

$$\arg\min \|\boldsymbol{S}\|_1 \text{ s.t. } \left\|\boldsymbol{\Phi}_j \boldsymbol{F}^{-1} \boldsymbol{H}_j \boldsymbol{S} - \boldsymbol{y}_j\right\|_2 \leq \epsilon_j. \quad (9)$$

This problem can be efficiently solved provided that the noise level is bounded by a parameter ϵ_j [13]. For illustration simplicity, equation (9) can be written in a compact form as

$$\hat{\boldsymbol{S}}_j = \text{BPDN}\left(\boldsymbol{y}_j, \boldsymbol{\Phi}_j \boldsymbol{F}^{-1} \boldsymbol{H}_j, \epsilon_j\right), \quad (10)$$

where $\hat{\boldsymbol{S}}_j$ is the recovered spectrum at the j-th SU. The next step is to find the decision vector $\hat{\boldsymbol{d}}_j$ at the j-th SU as

$$\hat{\boldsymbol{d}}_j = (|\hat{\boldsymbol{S}}_j| > \eta_j), \quad (11)$$

where η_j is a threshold calculated under Neyman-Pearson detection settings for a given probability of false alarm. This computation requires the knowledge of the noise statistics, which is not always available at the SU receiver. In addition, the recovery algorithm introduces additional internal error, as ϵ_j does not completely bound the noise such that $\|\boldsymbol{S} - \hat{\boldsymbol{S}}_j\|_2 \geqslant \epsilon_j$. All these factors lead to noise uncertainty, which subsequently lead to what is known as the SNR wall problem that decreases the sensitivity of the SU receiver [11].

B. Sequential compressive spectrum detection

Instead of recovering the complete spectrum and then use the noise statistics to find a suitable threshold, we propose to directly use a compressive support recovery algorithm to recover the spectrum support.

We use the sequential compressive sensing [4], which is a compressive support recovery algorithm that was shown to achieve high recovery performance under low SNR. We assume that the input signal $\tilde{\boldsymbol{x}}_j$ maintains a fixed support in the frequency domain during the acquisition processes but the additive noise terms at each acquisition stage are independent.

SCS is an iterative algorithm in which elementary estimation of the signal support is obtained through t_1 acquisition processes (steps), then the search space is refined in an adaptive manner through s iteration, where t_1 and s are calculated as shown in Algorithm 1. At each step, the signal $\boldsymbol{x}_j$ is captured at sampling rate much lower than the Nyquist rate by a factor of K/M through the measurement matrix $\boldsymbol{A_t F}$ as

$$\boldsymbol{y}_t = \boldsymbol{A}_t \boldsymbol{F} \boldsymbol{x}_j = \boldsymbol{A}_t \boldsymbol{F} \tilde{\boldsymbol{x}}_j + \tilde{\boldsymbol{w}}_{t,j}, \quad (12)$$

where the subscript t denotes the t-th acquisition step, $\boldsymbol{F}$ is the $M \times M$ DFT matrix, and $\boldsymbol{A}_t$ is a $K \times M$ sparse matrix. Each column in the matrix $\boldsymbol{A}_t$ will have only one non-zero element which has a value α or $-\alpha$. The amplitude and the locations of α are independent and their locations are drawn from i.i.d. random variables. The structure of the matrix $\boldsymbol{A}_t$ is preferred due to the low complexity inherent in the multiplication by a sparse matrix as compared to the dense Gaussian measurement matrices [14]. After the acquisition of the signal, a back-projected initial estimation of the signal is calculated for each acquisition step as

$$\bar{\boldsymbol{x}}_t = \boldsymbol{A}_t^T \boldsymbol{y}_t. \quad (13)$$

The signs of these different vectors are summed as

$$\breve{\boldsymbol{x}}_i = \sum_{t=1}^{t_1} \text{sgn}(\bar{x}_{i,t}), \quad (14)$$

where the subscript i denotes the i-th iteration. This summation of signs works like a majority voting for the different acquisition processes. Assuming a zero mean AWGN process, the signs of the non-active channels (corresponding to the noise at different acquisition steps) will cancel each other with high probability. Meanwhile, the signs corresponding to active channels are accumulated. This assures us that the noise will not alter the sign of an active channel with high probability.

In the first stage, the SCS is concerned with the detection of the non-zero positive spectrum elements, so if the sign of the m-th channel ($m \in 1, 2, \dots, M$) in the signs' vector $\breve{\boldsymbol{x}}_i$ retains negative after t_1 acquisition processes, it is inferred that the noise alters it, and this channel is declared as a free channel. The new refined search space vector could be obtained by finding the indices of the positive numbers in the vector $\breve{\boldsymbol{x}}_i$.

In the next iteration, the columns of the declared free channels are set to zero in the matrix $\boldsymbol{A}_t$, consequently, we can concentrate the sensing energy in the channels expected to be occupied and this process is repeated s times. At each iteration about 50% of the remaining free channels are announced.

As suggested in [4], the same process is repeated in the second stage for detection of the non-zero negative spectrum elements. The new refined search space vector could be obtained by finding the indices of the negative numbers in the vector $\breve{\boldsymbol{x}}_i$. The overall support of the signal is the union of both the positive and negative detected spectrum elements in the two stages.

IV. Cooperative Spectrum Sensing

Cooperation between multiple SUs in the sensing process was introduced to overcome some of the problems that face spectrum sensing because of noise uncertainty, fading, and shadowing. Cooperative sensing alleviates the problem of hidden PU [1], increases the probability of detection and decreases the probability of false alarm. In addition, it can further reduce the compression ratio and the sampling rate in the compressive sensing based techniques. On the other hand, cooperation can lead to data overload on the communication network. The cooperation in this setting is performed using two well known fusion techniques, namely, Decision Fusion and Data Fusion.

In Decision Fusion approach, each SU senses the spectrum and makes its own decision using any of the approaches presented in section III. Then each SU sends its M bit decision to the FC. The FC applies OR decision rule. In this rule, if only one SU receiver claims that the channel is occupied, the FC marks this channel as occupied and this channel is blocked from opportunistic use by the Sus. This subsequently leads to improvement in the probability of detection and ensures the best protection to the PU as compared to other fusion rules such as majority-voting or AND rules. However, this comes at the expense of increasing the probability of false alarm. The decision at FC could be formulated as

$$\hat{\boldsymbol{d}} = \left(\sum_{j=1}^{J} \hat{\boldsymbol{d}}_j \geqslant 1 \right) \tag{15}$$

In Data Fusion, compressive sensing can lead to a more efficient cooperation scheme between the multiple SUs. We present a novel collaborative data fusion technique based on the sequential compressive sensing presented in Section IV-B. For the sake of comparison, we will first present a non-adaptive data fusion technique based on BPDN in Section IV-A [10].

A. Optimal global recovery using BPDN

In this method, each cognitive radio sends its compressed measurement data to a Fusion Center (FC). The FC uses the algorithm given in Section III-A to recover the common spectrum and this is done by stacking all the measurements in one vector then the common spectrum can be recovered by solving the following BPDN with extended measurements as

$$BPDN \left(\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \vdots \\ \boldsymbol{y}_J \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Phi}_1 \boldsymbol{F}^{-1} \boldsymbol{H}_1 \\ \boldsymbol{\Phi}_2 \boldsymbol{F}^{-1} \boldsymbol{H}_2 \\ \vdots \\ \boldsymbol{\Phi}_J \boldsymbol{F}^{-1} \boldsymbol{H}_J \end{bmatrix}, \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_J \end{bmatrix} \right). \tag{16}$$

Using such algorithm requires each cognitive radio to send to the FC its measurements, channel state information, and its noise bounding parameter ϵ_j. This has a drawback of introducing high traffic overhead over the network.

B. Data Fusion algorithm using SCS

We utilize the adaptive nature of the SCS algorithm to design a novel algorithm for data fusion in cooperative spectrum sensing. The complete algorithm is shown in Algorithm 1. Instead of sending the raw data to the FC, each SU process the data and iterates to generate $\breve{\boldsymbol{x}}_i$ using equation (14) in a way similar to the non-cooperative case in [4]. These signs vectors are sent to the FC from all the Sus. The FC sums the signs from all SUs, comparing it to zero, then send back the decision for the first iteration to the SU. These steps are repeated for s times as shown in Algorithm 1.

This algorithm shares some similarity with the decision fusion algorithm mentioned at the beginning of this section as most of the processing is performed at each local SU with minimum load over the communication network. However, our proposed algorithm achieves higher performance gains because each SU sends the signs vector of the signal and this makes the algorithm similar in a sense to a quantized soft combining algorithm. The proposed algorithm achieves a probability of detection higher than that of the decision fusion, which can be considered as a hard combining algorithm.

Meanwhile, distributing the acquisition process among several nodes reduces the number of acquisition repetitions, which in return decreases the complexity of the algorithm. All these merits come at the expense of increased traffic between the SUs and FC. As each SU sends $sM \log_2 t_1$ bits compared to M bits in the decision fusion algorithm where $\log_2 t_1$ is the number of bits needed to code one element in the summation vector. This is still much lower than the traffic overhead in the data fusion algorithm using BPDN.

V. Simulation Results

In this section, we perform numerical simulations to illustrate the performance of the proposed sequential approaches. In all of our experiments, we fixed the sparsity order by considering a spectrum of interest with $M = 30$ subchannels and $I = 3$ active primary users. As the spectrum utilization increases, the number of acquisition steps t_1 increases. Consequently, both the computational complexity and measurement budget are increased. The compression ratio is defined as the ratio between the number of measurements K and the dimension of the signal M.

The channel is modeled as a multipath fading channel with a number of taps $Np = 4$. The gains of these taps are drawn from a Rayleigh distribution. The received signal is corrupted by an additive white Gaussian noise and the signal to noise ratio is considered as the ratio between the average of the received signal power and the noise power.

Since we model the problem as a detection problem, we evaluate the performance of the different approaches using the probability of detection (P_d) for a fixed probability of false alarm (P_{fa}). Note that the probability of detection in our problem refers to the probability of detecting the active primary users while false alarm errors refer to announcing a channel as occupied while being empty. In the first set of experiments, a single SU locally senses the compressed

Algorithm 1 SCS for collaborative SUs using quantized data

Input:
$\boldsymbol{x}_j(M \times 1)$, $\boldsymbol{F}(M \times M)$, K, and parameter $\delta > 0$
Internal Algorithm Parameters:
$s \underset{\sim}{\propto} \lceil \log_2 \log M \rceil$ (number of iterations)
$t_1 \underset{\sim}{\propto} \lceil \log \frac{2}{\delta} + \log k + \log \log_2 \log M \rceil$
(The number of acquisition processes per iteration)
$\alpha = \frac{K}{6}$ (magnitude of non zero entities)
$I_1 = \{1, 2, \ldots, M\}$ (initial index set)
for $i = 1$ **to** s **do**
  for $j = 1$ **to** J **do**
    for $t = 1$ **to** t_1 **do**
      Generate measurement matrix : $\boldsymbol{A}_t(K \times M)$
      Mask : Set ℓ column of $\boldsymbol{A_t}$ to zero $\forall\, \ell \notin I_i^p$
      Signal acquisition : $\boldsymbol{y}_t = \boldsymbol{A}_t \boldsymbol{F} \boldsymbol{x}_j = \boldsymbol{A}_t \boldsymbol{F} \tilde{\boldsymbol{x}}_j + \tilde{\boldsymbol{w}}_{t,j}$
      Backproject : $\bar{\boldsymbol{x}}_t = \boldsymbol{A}_t^T \boldsymbol{y}_t$
    end for
    local Sign summation at SUs : $\breve{\boldsymbol{x}}_{i,j} = \sum_{t=1}^{t_1} \mathrm{sgn}(\bar{x}_{i,t,j})$
  end forOperations at FC
  Centralized sign summation at FC: $\acute{\boldsymbol{x}}_i = \sum_{j=1}^{J} \breve{\boldsymbol{x}}_{i,j}$
  Refine search space : $I_{i+1} = \{i \in I_i : \acute{x}_i > 0\}$
end for
Output:
$I_s = I_s$ (total support recovery)

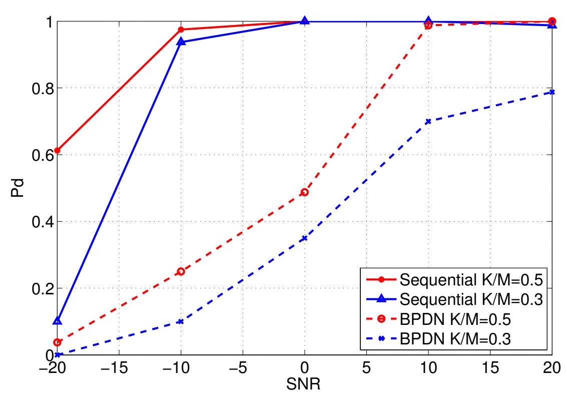

Fig. 1. Pd via SNR at different compression ratios calculated locally at a single SU receiver with probability of false alarm = 0.01

spectrum and then it makes its own decision. We compare the performance of the sequential recovery algorithm to that of the BPDN program under different SNRs and compression ratios, measured at the same 1% probability of false alarm.

Figure 1 elucidates a noticeable performance improvement under 0 dB SNR as it perfectly recovers the active channels at a low compression ratio using only 30% of the Nyquist measurements, which elevates the need for highly complex ADC to capture the wideband signal.

We also note that the sequential technique provides high recovery sensitivity as it can discover 60% of the active sub

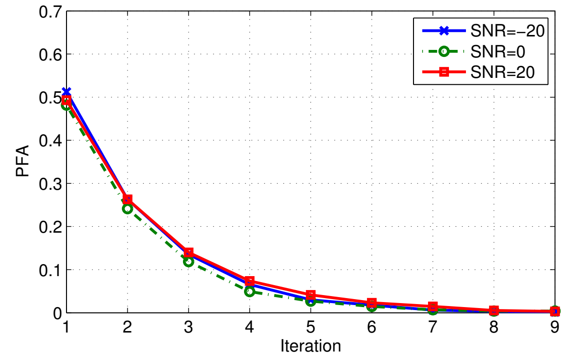

Fig. 2. The probability of false alarm at single SU via the number of iterations in the sequential algorithm at compression ratio=0.5.

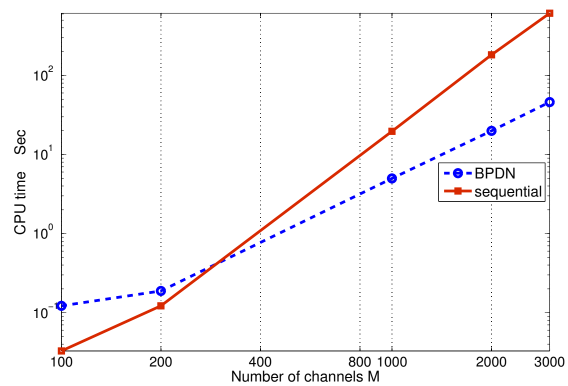

Fig. 3. The CPU time via the number of channels M and $I = 0.1M$ at compression ratio=0.5, SNR= 0.

channels at SNR=-20 dB with 50% of the Nyquist measurements. The recovery at such low SNR as compared to the BPDN method is attributed to its blind detection method (in the absence of noise statistics). Therefore, it could overcome the SNR wall problem. Although the sequential CS is an iterative algorithm, it is evident from Fig. 2 that from the first iteration it can reach a probability of detection as high as 100% at SNR= 0 dB (as seen in Fig. 1), which fully protects the PUs while allowing 50% of free channels for opportunistic use at 0.5 compression ratio. The P_{fa} decreases as the number of iterations increases allowing for the detection of more free channels for opportunistic use. We can reach P_{fa} as low as 0.01 after only 6 iterations, which follows the behavior of SCS proved in [4], as the SCS probability of false alarm reduces to half of its value with each iteration independent of the value of the SNR while maintaining the non-zero components with high probability. The CPU time in Fig. 3 shows that the SCS has lower computational complexity, measured by CPU processing time, as compared to BPDN for low number of channels. However, as the number of channels increase, BPDN requires fewer computations for the same compression ratio. We note that this computation difference is compensated by the possible reduction in the number of measurements k

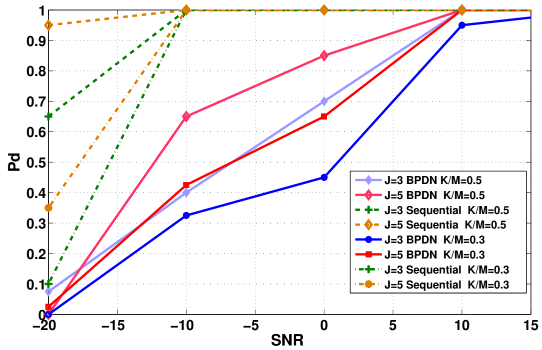

Fig. 4. Probability of detection for centralized collaborative SU, using OR decision fusion rule while maintaining a P_{fa} of 0.01.

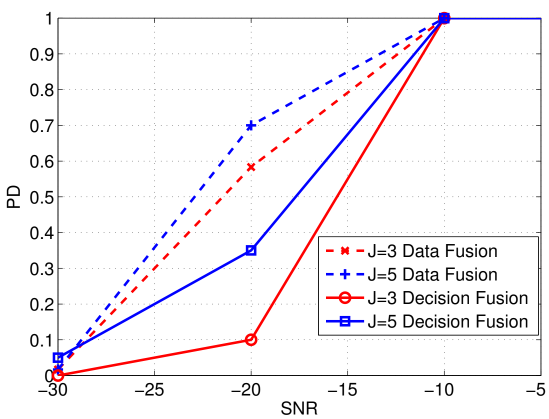

Fig. 5. Probability of detection for centralized collaborative SU using Data Fusion and Decision Fusion for different SNR values at a compression ratio $= 0.3$ while maintaining a P_{fa} of 0.01.

required to maintain the same high probability of detection in the two algorithms.

In the second set of experiments, we examine the effect of cooperation among SUs. In the Decision Fusion scenario, each SU sends its decision to the FC. The FC applies OR Fusion Rule to maximize the P_d because we are primarily concerned by protecting the PUs signals.

Figure 4 shows the effectiveness of cooperative sequential approach even when the signal is subject to harsh conditions in terms of very low SNR$= -15dB$ and using a very low compression ratio $= 0.3$. A system consisting of 5 SUs can achieve $P_d = 0.7$. The collaborative techniques reduce the effects of fading and hidden primary user on the P_d so as the number of SUs J increases the P_d increases.

In the last experiment, we evaluate the performance of the proposed Data Fusion Sequential approach versus the Decision Fusion Sequential approach as shown in Fig. 5. It is evident from this figure that the Data Fusion algorithm achieves a significant improvement in the Pd as compared to the Decision Fusion scenario, especially at very low SNR (about -20 dB). In addition to that, data fusion algorithms distribute the computational complexity among SUs and FC at the expense of higher traffic overload on the network by the factor $s \log_2 t_1$.

VI. Conclusion

In this paper, we have presented an adaptive approach for collaborative spectrum sensing for cognitive radio networks. The proposed approach exploits the sparsity of the spectrum in the frequency domain and the nature of the problem, which requires basically the detection of the spectrum support. The Sequential Compression Sensing (SCS) algorithm has been applied as a support recovery algorithm for both the single SU and the multiple collaborative SUs network. The proposed SCS based approach shows improvement over the non-sequential based approach in terms of the probability of PU signal detection while maintaining a fixed probability of false alarm. The proposed approach also works under low compression ratio, which allows for a significant reduction in sampling rate, and relaxes the constraints on the ADCs. A data fusion algorithm has been developed and compared to the decision fusion scenario. Data fusion rule outperforms decision fusion at the expense of traffic overload on the network between the SUs and the FC.

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