Optimal Power Allocation for Layered Broadcast Over Amplify-and-Forward Relay Channels

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***Abstract*—In this paper, we consider a fading relay channel where the source uses multilayer source coding with successive refinement. The source layers are transmitted using superposition coding at the source with optimal power allocation. The relay uses the simple half-duplex amplify-and-forward strategy. The destination applies successive interference cancellation after optimally combining the direct and relayed signals. The power allocation for the source layers at the source is subject to optimization in order to maximize the expected user satisfaction that is defined by a utility function of the total decoded rates at the destination. We propose an approximation for the distribution of the end-to-end channel quality. We assume that only the channel statistics are known. We characterize the expected utility function in terms of the channel statistics of the fading channels, and we solve the optimization problem for any number of source layers. We provide numerical examples to show the prospected gains of using the relay on the expected utility for different channel conditions. Furthermore, we obtain that for some conditions, it is optimal to send only one layer.**

I. Introduction

As well known, wireless networks are characterized by varying channel qualities due to location based long-term variations, and signal propagation based short-term variations. Thus, maintaining a constant Quality-of-Service (QoS) for all users in every instantaneous moment is infeasible. For this reason, the use of "multilayer" transmission schemes, which combine successive refinement layered source coding [1] with ordered protection levels at the physical layer, is a favorable choice. In addition to its practical merits in multimedia broadcasting and multicasting applications, the multilayer transmission schemes have gained a lot of interest in the information theory and the communication theory literature, where most researchers are interested in the "broadcast approach" since it is the optimal transmission strategy. In this case, the source layers are protected using different channel codewords and transmitted jointly using superposition coding at the physical layer [2], [3]. The receiver decodes the layers in order, up to the supported layer by its channel condition, using successive interference cancellation (SIC). For brevity and due to the scope of this paper, we do not provide a survey of the papers that examined this topic. The interested readers can refer to [4], [5] and the references therein.

The design parameters that are subject to optimization in the broadcast approach are the allocated rates and power ratios of the different source layers. This is a rigorous problem that has been solved in [4], [5], where the optimization objective was defined to be a utility maximization problem. This problem formulation can fit different applications like maximizing the expected rate or minimizing the expected distortion. The interesting contribution of [4], [5] is that they provided generic algorithms to solve the optimization problem for any number of source layers and for any concave increasing utility function and for any channel statistical model that fits some conditions. Furthermore, their algorithms have linear computation complexity with respect to the number of layers.

Our main interest in this work is in the application of multilayer transmission using the broadcast approach in the context of relay-assisted networks. Our initial (and recent) contribution in this topic was by considering decode-and-forward (DF) relays [6]. While in this paper, we are interested in the amplify-and-forward (AF) relay scenario. In particular, we examine the extension of the optimization framework presented in [4], [7] to the relay channel case. For brevity, we refer here to only few examples of important contributions in this interesting topic [8]–[13].

We show in this paper that, unlike the DF relay case [6], the application of the algorithm presented in [4] to solve the optimization problem assuming AF relaying is feasible. This means that we can solve the power allocation problem for any number of source layers while maintaining a linear computation complexity with respect to the number of source layers. Notice that the expected utility function in our problem is a function of the channel statistics of the three links in the channel (i.e. source-destination, source-relay and relay-destination). So, we need to characterize analytically the end-to-end channel statistics in terms of the statistics of the three links of the channel model in order to be able to apply the algorithm presented in [4]. This is the main bottleneck in our problem. However, we propose a simple and useful approximation of the end-to-end channel quality given that all three links in our channel model are Rayleigh faded. Furthermore, we provide numerical examples to show the optimal power allocation for two different utility functions and to demonstrate the gains of relaying over the case when the relay is not utilized.

II. System Model and Problem Formulation

A. System Model and Transmission Scheme

We consider a system that consists of three nodes; source, destination and relay. We assume that the source is Gaussian and it is encoded into independent M layers, $L = [L_1, L_2, \cdots, L_M]$, with fixed rates $R = [R_1, R_2, \cdots, R_M]$, with power ratios $\alpha = [\alpha_1, \alpha_2, \cdots \alpha_M]$ of the total source

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power P_s, and with each layer successively refining the information from the lower layers. Therefore, the source transmits layer L_i with a power $P_i = \alpha_i P_s$. The relay is half-duplex and applies amplify-and-forward strategy (AF) [8]. Therefore, the transmission is carried over two consecutive time slots of equal duration and bandwidth. The source broadcasts the layers to the relay and the destination using superposition coding in the first time slot.

In the second time slot, the relay forwards the signal that was received from the source after amplifying it. The power of the relay is denoted by P_r. Notice that the power ratios of the source layers at the relay preserve the same ratios like the source node since the relay just amplifies the layers without decoding and regenerating them.

Two copies of the layers are received at the destination in the two time slots. The destination utilizes both copies in order to decode the source information up to the number of layers that can be decoded reliably based on the end-to-end instantaneous channel quality. The layers are decoded with successive interference cancellation (SIC). Thus, to decode layer L_i, the destination must be able to decode all "higher priority" layers first (i.e. all L_j where $j < i$).

We assume that the three nodes are equipped with a single antenna. We denote the instantaneous Signal-to-Noise-Ratio (SNR) over the three links of the relay channel using γ_{sr}, γ_{sd} and γ_{rd} for the source-relay, source-destination, and relay-destination links, respectively. We assume that the source and the relay transmit using constant power. Furthermore, we assume that the channel gain, and consequently the SNR, stay constant for the duration of one transmission block, which consists of two consecutive time slots. However, γ_{sr}, γ_{sd} and γ_{rd} vary from one channel block to another randomly. Furthermore, we assume that the source and the relay do not know the instantaneous values of the SNRs.

In this work, we assume that the variation (i.e. fading) of the channels' gain is Rayleigh distributed. Hence, the probability density function (PDF) of the channels follow an exponential distribution, and they are given as

$$f_{\mathsf{sd}}\left(\gamma_{\mathsf{sd}}\right) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{sd}}}{\bar{\gamma}}\right), \quad f_{\mathsf{sr}}\left(\gamma_{\mathsf{sr}}\right) = \frac{1}{m_1\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{sr}}}{m_1\bar{\gamma}}\right),$$
$$f_{\mathsf{rd}}\left(\gamma_{\mathsf{rd}}\right) = \frac{1}{m_2\bar{\gamma}} \exp\left(\frac{-\gamma_{\mathsf{rd}}}{m_2\bar{\gamma}}\right), \tag{1}$$

for the source-destination, source-relay and relay-destination channels, respectively. In (1), $\bar{\gamma}$ is the average SNR for the direct source-destination link and m_1 and m_2 are the ratios between the average SNR of the source-relay and the relay-destination links to the source-destination link, respectively. We assume that $\bar{\gamma}$, m_1 and m_2 are known at the source node which utilizes its knowledge of the average channel qualities of the three links in the optimization of the power allocation α_i's over the layers in order to maximize the expected utility function, denoted U, of the total decoded rate, denoted $\bar{R}$, at the destination.

B. End-to-End Channel Condition

The two copies of the layers y_{sd} and y_{rd} received from the source and the relay in the two time slots, respectively, are combined at the destination using maximum ratio combining (MRC). Therefore, the "combined" signal can be given as

$$y_c = a y_{sd} + b y_{rd}, \tag{2}$$

where a and b are the combining ratios, and

$$y_{sd} = h_{sd}\Sigma_{i=1}^{M} L_i + n_{sd}, \; y_{sr} = h_{sr}\Sigma_{i=1}^{M} L_i + n_{sr}, \tag{3a}$$
$$y_{rd} = h_{rd} A_r y_{sr} + n_{rd}, \tag{3b}$$

where h_{sd}, h_{sr} and h_{rd} are the independent channel gains, n_{sd}, n_{sr} and n_{rd} are the independent noise signals with variance N_o for the three links of the relay channel, and A_r is the amplifying gain at the relay node that is a function of the power constraint at the relay P_r. Hence, $A_r = \sqrt{P_r/(|h_{sr}|^2 P_s + N_o)}$. It can be shown that the signal to noise ratio of the combined signal with SIC for layer L_i can be easily written as

$$\mathrm{SNR}_c^{(L_i)} = \frac{|a h_{sd} + b h_{rd} h_{sr} A_r|^2 \alpha_i P_s}{N_o\left(|a|^2 + |b|^2 + |b h_{rd} A_r|^2\right) + |a h_{sd} + b h_{rd} h_{sr} A_r|^2 \sum_{m>i}^{M} \alpha_m P_s}. \tag{4}$$

In order to get the MRC, we need to find the combining ratios a and b that will maximize $\mathrm{SNR}_c^{(L_i)}$. Therefore, we differentiate $\mathrm{SNR}_c^{(L_i)}$ with respect to a^* and find the nulls of the derivative $\frac{\delta \mathrm{SNR}_c^{(L_i)}}{\delta a^*} = 0$, which can be found after some mathematical derivations, that are omitted here for brevity, as

$$a = C h_{sd}^* \left(|h_{sr}|^2 P_s + |h_{rd}|^2 P_r + N_o\right), \tag{5a}$$
$$b = C h_{sr}^* h_{rd}^* \sqrt{P_r \left(|h_{sr}|^2 P_s + N_o\right)}, \tag{5b}$$

where C is an arbitrary constant. By substituting with (5) in (4), we can find the maximum SNR value for the layer L_i denoted by $\mathrm{SNR}_{\mathrm{MRC}}^{(L_i)}$, which yields

$$\mathrm{SNR}_{\mathrm{MRC}}^{(L_i)} = \frac{\alpha_i}{\frac{1}{\gamma} + \sum_{m>i}^{M} \alpha_m}, \; \gamma = \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1}, \tag{6a}$$

where γ denotes the end-to-end SNR (i.e. the SNR at the destination after combining the direct and relayed signals optimally).

In order for the destination to decode and make use of layer L_i, it must be able to decode this layer as well as all the previous layers. Therefore, the value of γ must satisfy the relation

$$R_j \le \frac{1}{2}\log\left(1 + \frac{\alpha_j}{\frac{1}{\gamma} + \sum_{m>j}^{M} \alpha_m}\right) \quad \forall j \le i. \tag{7}$$

This can be written as

$$\begin{aligned} \gamma \ge \bar{\gamma}_i &= \max\{\gamma_1, \gamma_2, \dots, \gamma_i\} \\ &= \max\left\{\bar{\gamma}_{i-1}, \frac{1}{\frac{\alpha_i}{2^{2R_i}-1} - \sum_{m>i}^{M} \alpha_m}\right\}, \end{aligned} \tag{8}$$

where $\bar{\gamma}_i$, named as γ threshold, is the constraint on γ for the destination to be able to decode all the layers up to layer L_i, and γ_j is the minimum value for γ required to decode the layer L_j after correctly canceling all the previous layers, and can be written as

$$\gamma_j = \frac{1}{\frac{\alpha_j}{2^{2R_j}-1} - \sum_{m>j}^{M} \alpha_m}. \tag{9}$$

It can be seen that the γ threshold values depends on the power allocated to each layer. The destination only decodes the layers whose thresholds are below the instantaneous end-to-end channel condition γ.

C. Problem Formulation

Similar to [4], we formulate the optimization problem as maximizing the expected user satisfaction that is defined by a utility function $U(\bar{R})$ of the total decoded rate $\bar{R}$ at the destination. The utility function can be flexibly defined to employ many special cases such as minimizing the expected distortion of a Gaussian source, e.g. [14], [15], or maximizing the expected rate, e.g. [16]. The optimization problem is to optimally allocate the power among the layers, i.e., $\alpha_i' s$, such that the expectation of the utility function $E[U(\bar{R})]$ is maximized. Hence, we can formulate the optimization problem as follows

$$\max_{\alpha} \quad \int_0^{\infty} f_\gamma(\gamma) \quad U\left(\bar{R}(\gamma)\right) d\gamma \tag{10a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \qquad \alpha_i \geq 0 \qquad \forall i, \tag{10b}$$

where $f_\gamma(\gamma)$ is the probability density function (PDF) of the end-to-end channel quality γ.

As described in details in [4], the problem in (10) can be equivalently written as

$$\min_{\alpha, \bar{\gamma}} \qquad \sum_{i=1}^{M} c_i \, F_\gamma(\bar{\gamma}_i(\alpha)) \tag{11a}$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \qquad \alpha_i \geq 0 \qquad \forall i, \tag{11b}$$

$$\bar{\gamma}_i = \max\left\{\bar{\gamma}_{i-1}, \frac{1}{\frac{\alpha_i}{2^{2R_i}-1} - \sum_{m>i}^{M} \alpha_m}\right\} \qquad \forall i, \tag{11c}$$

where $U_i = U\left(\sum_{m=1}^{i} R_m\right)$, $U_0 = 0$, $c_i = U_i - U_{i-1}$, and $F_\gamma(\gamma)$ is the cumulative distribution function (CDF) of the end-to-end channel quality γ.

An efficient and optimal solution of this problem was described in [4, Table I]. The solution is based on a change of optimization variables step which enabled the application of bisection search methods with linear computation complexity with respect to the total number of layers M. We can apply that algorithm to our problem as well. However, the missing step will be to obtain the PDF of the end-to-end channel quality γ. This is discussed in Section III.

Before we end this section, we like to highlight the main difference between the AF relay case that is discussed in this paper, and the DF relay case that we considered in [6]. In the case of DF, the problem becomes more difficult to solve since the optimal power allocation at the relay may not necessarily follow the same power allocation at the source. Hence, the optimal power allocation at the relay should be considered as well. As a result, the number of optimization variables increases considerably (since the power allocation at the relay will be conditional on the number of layers decoded at the relay). Hence, the number of optimization variables becomes $\frac{M(M+3)}{2} - 1$, where M is the total number of layers. Furthermore, the solution presented in [4] cannot be applied. So, numerical random search methods should be applied which becomes inefficient and expensive in terms of the computation load as the number of optimization variables increases.

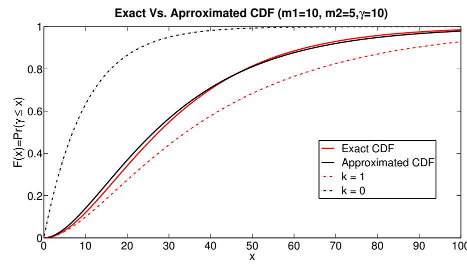


Fig. 1: The approximated CDF Vs. the true CDF for γ with $\bar{\gamma} = 10$, $m_1 = 10$, and $m_2 = 5$.

III. End-To-End Channel Approximation

We aim in this Section to find the PDF (or equivalently CDF) for γ, given (6a), in terms of the PDFs of γ_{sr}, γ_{sd} and γ_{rd}. The exact characterization of the CDF of γ is not straightforward. So, alternatively, we propose to use an approximation for it as follows. It can be easily shown that the value of γ can be bounded as

$$\gamma_{sd} < \gamma \leq \gamma_{sd} + \min\left(\gamma_{sr}, \gamma_{rd}\right). \tag{12}$$

So, intuitively, we can in general rewrite the definition of γ approximately as

$$\gamma \approx \gamma_{sd} + k \, \min\left(\gamma_{sr}, \gamma_{rd}\right), \tag{13}$$

where the appropriate value for k should be used $(0 < k \leq 1)$ such that the CDF of γ as defined in (13) becomes as close as possible to the exact CDF of γ as defined in (6a). We have done this task for different values of m_1 and m_2, and for different values of $m_1\bar{\gamma}$, and $m_2\bar{\gamma}$ to get a close approximation for the CDF of γ (results of the best values of k are omitted due to space limitations).

Based on the proposed approximation formula, we can easily write the CDF of γ using the definition in (13) as

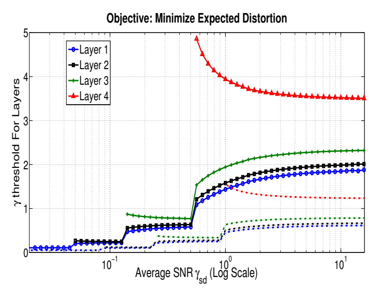
$$F_\gamma(\gamma) = 1 - \frac{\beta_3}{\beta_3 - \beta'} e^{-\gamma(\beta')} + \frac{\beta'}{\beta_3 - \beta'} e^{-\gamma\beta_3}. \tag{14}$$

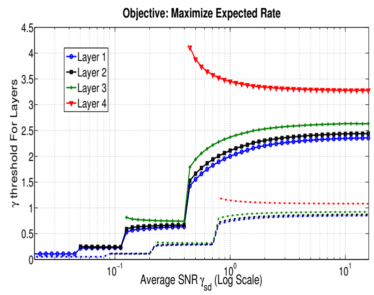
where $\beta' = \frac{\beta_1+\beta_2}{k}$, $\beta_1 = \frac{1}{\bar{\gamma}}$, $\beta_2 = \frac{1}{m_1\bar{\gamma}}$, and $\beta_3 = \frac{1}{m_2\bar{\gamma}}$.

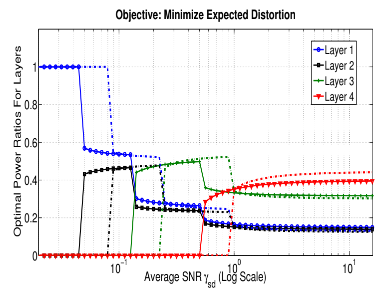
Fig. 1 shows the CDF for the approximated γ in (14) compared to the CDF of the exact γ as defined in (6a), which is obtained numerically, for some values of $\bar{\gamma}$, m_1, and m_2. This figure demonstrates that the approximation given by (13) is appropriate.

IV. Numerical Results

In this section we present some numerical results for the case of Rayleigh fading channels described in (1). The proposed algorithm in [4, Table I] is applied for a source example consisting of four layers with a sum rate of 1 Mbps and transmitted over 1 MHz bandwidth. The rates of the source layers are 75.5, 80.4, 240 and 642 Kbps respectively. We consider two different utility functions; namely, $U(\bar{R}) = 1 - 2^{-2\bar{R}}$, which corresponds to minimizing the expected distortion of



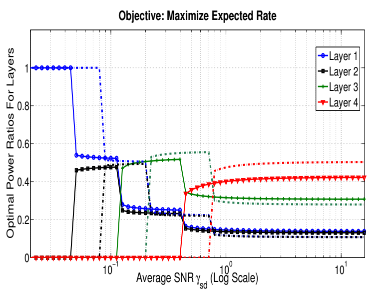


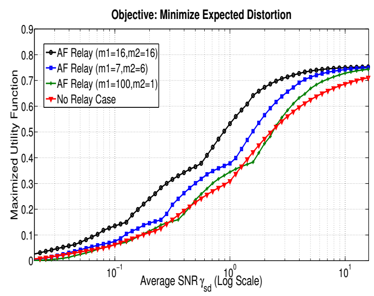


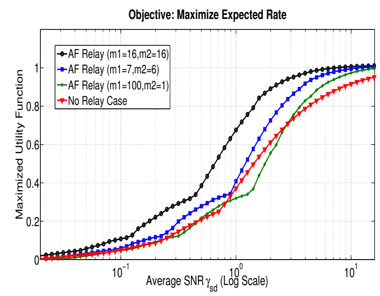
(a) The threshold γ values of the layers versus the average SNR value with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.

(b) The threshold γ values of the layers versus the average SNR value with $U(\bar{R}) = \bar{R}$.

(c) The relative power ratios of the layers versus the average SNR value with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.







(d) The relative power ratios of the layers versus the average SNR value with $U(\bar{R}) = \bar{R}$.

(e) The maximized average utility function versus the average SNR value with $U(\bar{R}) = 1 - 2^{-2\bar{R}}$.

(f) The maximized average utility function versus the average SNR value with $U(\bar{R}) = \bar{R}$.

Fig. 2: Simulation Results

a Gaussian source and $U(\bar{R}) = \bar{R}$, which corresponds to maximizing the expected total rate at the destination.

Figs. 2(a) and 2(b) show the optimal γ thresholds for the layers with the target of minimizing expected distortion and maximizing expected rate, respectively, while Figs. 2(c) and 2(d) show the optimal power ratios for these cases. The power ratios and γ thresholds are plotted against average SNR of the source-destination channel. The solid curves corresponds the case when the relay is used with $(m_1, m_2) = (16, 16)$ which is the best case for the relay position (i.e., relay in the mid point of the LOS between source and destination), with the assumption that the power of the signal $P \propto \frac{1}{d^4}$, where d is the distance. The dashed curves corresponds the case when no relay is used for comparison.

It can be seen that for some average SNR values it might be optimal to send only one layer, and as the average SNR value exceeds certain thresholds the number of layers increases. That is because as the average SNR increases, the channel condition becomes better, and the upper layers will be decoded reliably. It is obvious that the solid curves are shifted versions to the left of the dashed curves. This means that it is optimal to send higher number of layers for lower values of average SNR when a relay is used even in the worst case. Therefore, the destination becomes more capable of decoding more layers refining the information even when its direct channel with the source has low SNR. However, we can notice that the γ threshold values for the layers when no relay is used are lower compared to the case for the relay-assisted. The reason is the multiplexing loss due to transmitting over two time slots.

Since the utility function for maximizing the rate is linear, then the solution gives more importance for the higher layers as expected. This can be achieved by allocating more power for the higher layers as seen in Figs. 2(b) and 2(d). On the other hand, the solution for minimizing the average distortion gives more importance to the lower layers as in Figs. 2(a) and 2(c), and hence the lower layers are allocated more power, and it becomes optimal to send the higher layers for higher values of the average SNR compared to the case of maximizing the expected rate.

In Figs. 2(e) and 2(f) we plot the maximized utility function with the target of minimizing expected distortion and maximizing expected rate, respectively. It can be shown for the worst relay position case with $(m_1, m_2) = (100, 1)$ (i.e., relay near source or near destination), that the maximum expected utility is close (and maybe less than) the no-relay case. This is because the channel gains of the relay channel are not high in this case. Therefore, the prospected gain due to channel diversity of the relay channel will be opposed by the multiplexing loss due to the transmission over two time slots. Furthermore, the gain with respect to the no-relay case increases for the relay-assisted case with $(m_1, m_2) = (7, 6)$, and with $(m_1, m_2) = (16, 16)$ which is the best case (Relay in the mid point of the LOS between source and destination).

V. Conclusion

In this paper, we have considered layered source coding using superposition coding at the transmitter with successive interference cancellation at the receiver. The transmission is relay-aided, and the relay applies amplify and forward strategy. The objective is to maximize the expected user satisfaction that is defined by a utility function of the total decoded rate at the destination. However, we needed to obtain the end-to-end channel statistics analytically. So, we have proposed a simple and appropriate approximation for the AF relay scenario. Several numerical examples were obtained for two different utility functions, which were maximizing the expected rate and minimizing the expected distortion of a Gaussian source. The numerical results demonstrated the gains of relaying.

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