A Feedback-Based Access Scheme for Cognitive-Relaying Networks

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***Abstract*—In this paper, we consider a cognitive relaying network in which the secondary user accesses the channel with a certain access probability that depends on the feedback information sent by the primary destination. In addition, the secondary user is granted relaying capabilities by which it can relay primary traffic that was unsuccessfully transmitted by the primary user. We show that this proposed scheme enhances the performance of the secondary user as well as the primary user, while the QoS requirements of the primary user is unviolated. The secondary user can avoid sure collisions with the primary transmissions exploiting the feedback information from the primary user. Also, due to the fact that relaying the unsuccessfully transmitted primary traffic increases the availability of the channel for its own packets, the secondary throughput is increased and the primary delay is decreased.**

I. INTRODUCTION

Wireless communication is becoming more and more challenging as more users compete for limited bandwidth. Surprisingly, in some spectrum locations and at some times, 70% of the spectrum is idle [1]. Cognitive Radio was recently introduced in this context in which the unlicensed user (or secondary user (SU) or cognitive user) can reuse the unused spectrum by the licensed user (or primary user (PU)) in order to increase the total spectral efficiency [2], [3]. The PU can access the channel any time as long as it has a packet to transmit, while the SU seeks idle instants to transmit. The most important condition is that the activity of the SU doesn't violate the QoS requirements of the PU.

Cooperative scenarios have been introduced lately in which a cooperating terminal relay packets for other terminals over the so called relay channel in order to increase the channel availability for its own packets [4]. Similar scenarios are proposed in which the PU has the authority to access the channel whenever it has a packet to transmit. A primary packet unsuccessfully transmitted by the PU and successfully transmitted to the SU is stored in a relay queue at the SU. On the other hand, the SU waits for the opportunity of an idle instant to transmit either the relayed packets or its own packets and in most studies priority is given to the relayed traffic in a way that guarantees QoS requirements of the PU. These scenarios have proved to enhance cognitive node performance.

Throughput analysis of a basic cognitive network, consisting of one primary source-destination pair and one secondary source destination pair, is studied in [5]. The maximum sustainable average throughput of SU is studied given the average throughput of the PU, pre-determined independently, and a comparison is given in two cases, a SU with and without relaying capabilities. In an energy-constrained environment, a tradeoff arises between the average packet delay and the user's lifetime. The relative location of the SU with respect to the PU is an important factor in this problem. In [6], the effect of this relative location on the average delay of both the primary and cognitive users is studied provided that the cognitive user has limited lifetime and it offers prioritized relaying capabilities to primary traffic that the PU failed to transmit. If the SU transmitter has a limited energy budget, forwarding the PU packets comes at the expense of the SU throughput since the SU wastes energy sending primary packets, [7] suggested an admission control algorithm; the cognitive user controls the amount of relayed traffic through what it calls an admission control parameter. This admission control parameter is obtained such that the delay and power budget constraints are satisfied. A multi-packet reception (MPR) model is studied in [8]. The SU senses the channel and it transmits its packets with probability one when the PU is idle, and it transmits with a certain probability p when the PU is active. The trick was to find the optimal p^* that maximizes the throughput taking into account the tradeoff between simultaneous transmissions with reduced reception probability and a single transmission with high reception probability. This problem was introduced for the two cases, a SU with and without relaying capabilities.

Our contribution in this work is to introduce a relaying scheme based on feedback information from the primary destination. This paper is an extension of [9] adding relaying capabilities to the cognitive user and taking channel impairments into account. The SU is allowed to keep the successfully received primary packet which were not received by the primary destination. A packet which could not be transmitted neither to the primary destination nor to the SU is retransmitted by the PU and, in this case, the SU is obliged to stay silent in the next time slot because a sure primary retransmission will take place. This scheme outperforms [9], in which the SU also utilizes the feedback information in the system but does not have relaying capabilities, when the primary link suffers a high outage probability. Instead

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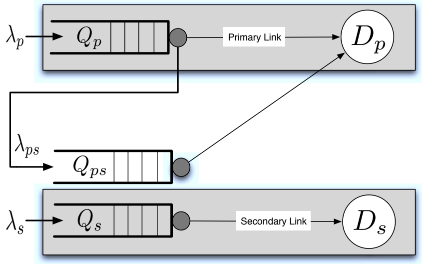

Figure 1: MAC layer of a cognitive-relaying scenario with one primary link and one secondary link

of several retransmissions by the PU suffering high outage probability, the SU simply relays those overhead packets, yet lost at the primary destination, increasing spectrum availability for its own packets. Accordingly, the secondary throughput is increased and the PU average packet delay is decreased, given that the SU perceives a better channel between itself and the primary destination.

The rest of the paper is organized as follows. The system model is introduced in Section II. The system analysis is given in Section III and numerical results are presented in Section IV. The paper is concluded in Section V.

II. SYSTEM MODEL

We consider a system of one PU and one SU as depicted in Figure 1. The PU has one buffer, Q_p, for storing the incoming packets. The SU has two buffers, Q_s for storing its own packets and Q_r for storing the packets to be relayed in case of unsuccessful reception at the primary destination. All buffers are assumed to be of infinite lengths. A time-slotted transmission scheme is considered where the slot duration is equal to the transmission time of one packet and all packets are of equal length. The primary and secondary traffic are assumed to be independent. The packet arrival process at the PU queue Q_p is Bernoulli with mean λ_p. The primary packets are assumed to arrive any time during the time slot, but it waits anyway for the next time slot to be processed. The PU has an unconditional access to the channel whenever it has a packet.

The SU doesn't apply any spectrum sensing technique before accessing the channel. It transmits into the channel according to a random access policy giving priority to the relayed packets in Q_r, and it is assumed to always have a packet to send in its buffer Q_s. This access probability is chosen so as to maximize the SU throughput without violating the stability of the PU queue. A transmission is unsuccessful in two cases, channel outage and collision; in both cases the packet is considered lost. A collision occurs when both the PU and the SU access the channel at the same time.

In any time slot, the PU transmits a packet to the primary destination, given that the PU has unconditional priority to access the channel whenever it has a packet to transmit. This packet is received by both the primary destination and the SU. Thus, we have the following four cases:

- Case-I: the packet is successfully received by the primary destination and the SU. In this case an ACK is sent by both, the primary destination and the SU which results in the packet being dropped from Q_p and also dropped from Q_r (upon the reception of an ACK from the primary destination).
- Case-II: the packet is successfully received by the primary destination but not successfully received by the SU. In this case an ACK is sent by the primary destination and a NACK is sent by the SU which results in the packet being dropped from Q_p.
- Case-III: the packet is not successfully received by the primary destination but successfully received by the SU. In this case an ACK is sent by only the SU and a NACK is sent by the primary destination which results in the packet being dropped from Q_p and stored in Q_r (when receiving a NACK from primary destination).
- Case-IV: the packet is not successfully received by neither the primary destination nor the SU, a NACK is sent by both nodes. In this case the primary user retransmits the packet.

Note that this is a small deviation from the cognitive radio principle of transparency as the PU receives two acknowledgements for the same packet [5]. In our work, we assume to have two independent feedback channels from the primary destination and the SU to the primary transmitter[1]

The SU exploits the feedback in these scenarios to enhance the access probability to the channel. In cases I, II and III, the SU can access the channel with a random probability a_s in the next time slot. In case-IV, the SU enters a BACK-OFF mode, i.e., it remains silent in the next time slot avoiding a sure collision as the PU will be transmitting with probability one. A packet is successfully transmitted by the SU if, in a given time slot, the PU is idle and there is no outage in the channel on which the packet is being transmitted (S-P or S-S), where the S-P channel is the channel between SU and primary destination, and the S-S channel is the channel between the secondary transmitter and receiver. Priority is given to packets in Q_r, i.e., the primary user's relayed packets.

III. SYSTEM ANALYSIS

In this section, we present the analysis of the proposed scheme and compare it to the one presented in [9]. Our objective is to maximize the secondary throughput without violating the primary QoS represented by the queue stability.

We compare two schemes, the feedback access scheme with no relaying capability named the no-relaying scheme and the feedback access scheme with relaying capability named the relaying scheme.

A. Relaying Scheme

1) Secondary throughput analysis: The Markov chain modeling of the primary queue is shown in Figure 2. $\overline{\lambda}_p$ denotes 1-λ_p, this notation is used throughout the paper[2]. η is the probability of successful reception of a primary packet either

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[1] Collisions in the feedback channels can be avoided by separating the two feedback channels in the time- or frequency-domain.

[2] Throughout the paper $\bar{x}$ is used to denote 1-x, i.e, $\bar{x} = 1 - x$.

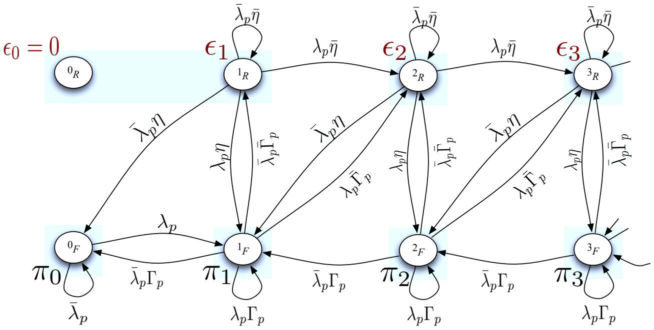

Figure 2: Markov chain of PU

at the primary destination or at the SU when the SU is in the BACK-OFF mode, i.e., when the PU is doing a retransmission. Γ_p is the probability of successful transmission of a primary packet either to primary destination or the SU when the SU is not in BACK-OFF mode, but decides not to access the channel. Each state is represented by N_F or N_R where N represents the number of packets in the queue at a certain instant, F means that the PU is transmitting a packet for the first time and R means that the PU is undergoing a retransmission. A retransmission takes place when a collision occurs or when there is outage in, both, the P-P and the P-S links. Recall that the P-P link is the link between the PU and the primary destination whereas the P-S link is the link between the PU and the SU.

Define $P_{\text{out},pp}$, $P_{\text{out},ps}$, $P_{\text{out},sp}$ and $P_{\text{out},ss}$ as the outage probability in the P-P, P-S, S-P and P-P links, respectively. π_k and ϵ_k are the probabilities of the primary queue having k packets at a certain time instant in case of first transmission and retransmission, respectively. Following the proposed system model, it is straightforward to show that

$$\begin{aligned} \eta &= 1 - P_{\text{out},pp} \cdot P_{\text{out},ps}, \\ \Gamma_p &= \eta \bar{a}_s. \end{aligned}$$

Towards characterizing the secondary throughput, we write, next, the balance equations. First, the balance equation around state 0_F is given by

$$\pi_0 \lambda_p = \epsilon_1 \overline{\lambda}_p \eta + \pi_1 \overline{\lambda}_p \Gamma_p. \tag{1}$$

The balance equation around state 1_R is given by

$$\epsilon_1(\overline{\lambda}_p \eta + \lambda_p \eta + \lambda_p \overline{\eta}) = \pi_1 \overline{\lambda}_p \overline{\Gamma}_p,$$

from which we get

$$\epsilon_1 = \frac{\overline{\lambda}_p \overline{\Gamma}_p}{\lambda_p + \overline{\lambda}_p \eta} \pi_1, \tag{2}$$

$$\pi_1 = \frac{\lambda_p + \overline{\lambda}_p \eta}{\overline{\lambda}_p \overline{\Gamma}_p} \epsilon_1. \tag{3}$$

Substituting from (3) into (1), we get

$$\pi_0 \lambda_p = \epsilon_1 \left[\overline{\lambda}_p \eta + \overline{\lambda}_p \Gamma_p \frac{\lambda_p + \overline{\lambda}_p \eta}{\overline{\lambda}_p \overline{\Gamma}_p} \right].$$

After some straightforward manipulation, we get

$$\epsilon_1 = \frac{\lambda_p \overline{\Gamma}_p}{\Psi} \pi_0, \tag{4}$$

where $\Psi = \overline{\lambda}_p \eta + \lambda_p \Gamma_p$. Substituting from (4) into (3) yields

$$\pi_1 = \frac{\lambda_p (\lambda_p + \overline{\lambda}_p \eta)}{\overline{\lambda}_p \Psi} \pi_0.$$

The balance equation around state 1_F is given by

$$\begin{aligned} &\pi_1(\overline{\lambda}_p \Gamma_p + \overline{\lambda}_p \overline{\Gamma}_p + \lambda_p \overline{\Gamma}_p) = \\ &\qquad \pi_0 \lambda_p + \epsilon_1 \lambda_p \eta \epsilon_2 \overline{\lambda}_p \eta + \pi_2 \overline{\lambda}_p \Gamma_p, \end{aligned}$$

which yields

$$\epsilon_2 \overline{\lambda}_p \eta + \pi_2 \overline{\lambda}_p \Gamma_p = \frac{\lambda_p^2 \overline{\Gamma}_p}{\overline{\lambda}_p \Psi} \pi_0. \tag{5}$$

The balance equation around state 2_R is given by

$$\begin{aligned} &\epsilon_2(\overline{\lambda}_p \eta + \lambda_p \eta + \lambda_p \overline{\eta}) = \\ &\qquad \epsilon_1 \lambda_p \overline{\eta} + \pi_1 \lambda_p \overline{\Gamma}_p + \pi_2 \overline{\lambda}_p \overline{\Gamma}_p, \end{aligned}$$

which yields

$$\epsilon_2(\lambda_p + \overline{\lambda}_p \eta) - \pi_2 \overline{\lambda}_p \overline{\Gamma}_p = \frac{\lambda_p^2 \overline{\Gamma}_p}{\overline{\lambda}_p \Psi} \pi_0. \tag{6}$$

From (5) and (6), we get

$$\begin{aligned} \epsilon_2(\lambda_p + \overline{\lambda}_p \eta) - \pi_2 \overline{\lambda}_p \overline{\Gamma}_p &= \epsilon_2 \overline{\lambda}_p \eta + \pi_2 \overline{\lambda}_p \Gamma_p, \\ \epsilon_2 &= \frac{\overline{\lambda}_p}{\lambda_p} \pi_2. \end{aligned} \tag{7}$$

Substituting from (7) into (5), it can be shown that

$$\pi_2 = \frac{\lambda_p \overline{\Gamma}_p}{\overline{\Psi}^2} \left[\frac{\lambda_p \overline{\Psi}}{\overline{\lambda}_p \Psi} \right]^2 \pi_0. \tag{8}$$

Substituting from (8) into (7), we get

$$\epsilon_2 = \frac{\lambda_p^2 \overline{\Gamma}_p}{\overline{\lambda}_p \Psi^2} \pi_0.$$

The balance equation around state 2_F is given by

$$\pi_2(\overline{\lambda}_p \Gamma_p + \overline{\lambda}_p \overline{\Gamma}_p + \lambda_p \overline{\Gamma}_p) = \epsilon_2 \lambda_p \eta + \epsilon_3 \overline{\lambda}_p \eta + \pi_3 \overline{\lambda}_p \Gamma_p,$$

which yields

$$\epsilon_3 \overline{\lambda}_p \eta + \pi_3 \overline{\lambda}_p \Gamma_p = \pi_2 \overline{\Psi}. \tag{9}$$

The balance equation around state 3_R is given by

$$\epsilon_3(\overline{\lambda}_p \eta + \lambda_p \eta + \lambda_p \overline{\eta}) = \epsilon_2 \lambda_p \overline{\eta} + \pi_2 \lambda_p \overline{\Gamma}_p + \pi_3 \overline{\lambda}_p \overline{\Gamma}_p,$$

which yields

$$\epsilon_3(\lambda_p + \overline{\lambda}_p \eta) - \pi_3 \overline{\lambda}_p \overline{\Gamma}_p = \pi_2 \overline{\Psi}. \tag{10}$$

From (9) and (10), we get

$$\begin{aligned} \epsilon_3 \overline{\lambda}_p \eta + \pi_3 \overline{\lambda}_p \Gamma_p &= \epsilon_3(\lambda_p + \overline{\lambda}_p \eta) - \pi_3 \overline{\lambda}_p \overline{\Gamma}_p \\ \epsilon_3 &= \frac{\overline{\lambda}_p}{\lambda_p} \pi_3. \end{aligned} \tag{11}$$

Substituting from (11) into (9), it can be shown that

$$\pi_3 = \frac{\lambda_p \overline{\Gamma}_p}{\overline{\Psi}^2} \left[\frac{\lambda_p \overline{\Psi}}{\overline{\lambda}_p \Psi} \right]^3 \pi_0. \tag{12}$$

Substituting from (12) into (11), it can be shown that

$$\epsilon_3 \quad = \quad \frac{\overline{\lambda}_p\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^3\pi_0.$$

Solving the balance equation around state 3_F, we get

$$\epsilon_4\overline{\lambda}_p\eta+\pi_4\overline{\lambda}_p\Gamma_p=\pi_3\overline{\Psi}. \tag{13}$$

Solving the balance equation around 4_R, we get

$$\epsilon_4(\lambda_p+\overline{\lambda}_p\eta)-\pi_4\overline{\lambda}_p\overline{\Gamma}_p=\pi_3\overline{\Psi}. \tag{14}$$

From (13) and (14), we get

$$\begin{aligned}\epsilon_4\overline{\lambda}_p\eta+\pi_4\overline{\lambda}_p\Gamma_p &= \epsilon_4(\lambda_p+\overline{\lambda}_p\eta)-\pi_4\overline{\lambda}_p\overline{\Gamma}_p,\\ \epsilon_4 &= \frac{\overline{\lambda}_p}{\lambda_p}\pi_4. \qquad (15)\end{aligned}$$

Substituting from (15) into (13), it can be shown that

$$\pi_4 \quad = \quad \frac{\lambda_p\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^4\pi_0. \tag{16}$$

Substituting from (16) into (15), we get

$$\epsilon_4 \quad = \quad \frac{\overline{\lambda}_p\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^4\pi_0.$$

Thus, the above analysis yields the steady-state probabilities in terms of π_0 as follows

$$\begin{aligned}\epsilon_0 &= 0\\ \epsilon_1 &= \frac{\lambda_p\overline{\Gamma}_p}{\Psi}\pi_0\\ \pi_1 &= \frac{\lambda_p(\lambda_p+\overline{\lambda}_p\eta)}{\overline{\lambda}_p\Psi}\pi_0,\end{aligned}$$

and for $k \geq 2$, it can be shown that

$$\begin{aligned}\pi_k &= \frac{\lambda_p\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^k\pi_0,\\ \epsilon_k &= \frac{\overline{\lambda}_p\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^k\pi_0.\end{aligned}$$

The normalization condition is given by $\sum\limits_{k=0}^{\infty}(\pi_k+\epsilon_k)=1$. Hence, we have

$$\pi_0+\pi_1+\epsilon_1+\sum_{k=2}^{\infty}(\pi_k+\epsilon_k)=1. \tag{17}$$

Substituting, we get

$$\pi_0\left\{1+\frac{\lambda_p(\lambda_p+\overline{\lambda}_p\eta)}{\overline{\lambda}_p\Psi}+\frac{\lambda_p\overline{\Gamma}_p}{\Psi}+\sum_{k=2}^{\infty}\frac{\overline{\Gamma}_p}{\overline{\Psi}^2}\left[\frac{\lambda_p\overline{\Psi}}{\overline{\lambda}_p\Psi}\right]^k\right\}=1. \tag{18}$$

Assuming that $\lambda_p < \Psi$, which will be shown to be the stability condition of this queue, we can show that

$$\pi_0\left\{1+\frac{\lambda_p^2+\lambda_p\overline{\lambda}_p\eta+\lambda_p\overline{\lambda}_p\overline{\Gamma}_p}{\overline{\lambda}_p\Psi}+\frac{\lambda_p^2\overline{\Gamma}_p}{\overline{\lambda}_p\Psi(\Psi-\lambda_p)}\right\}=1. \tag{19}$$

From which we can be shown that

$$\pi_0=\frac{\Psi-\lambda_p}{\eta}. \tag{20}$$

Now since the stability of the PU queue is attained when the probability of the queue being empty isn't equal to zero i.e., $\pi_0 \neq 0$; this is equivalent to having $\lambda_p < \Psi$.

In the remaining of this section, we shift our attention to the relay queue, Q_r, analysis. It holds the primary packets which have not been correctly received at the primary destination but received correctly at the SU. The Markov chain modeling this queue is shown in Figure 3. Each state represents the number of packets in the relay queue at a certain instant. β is the probability of successful transmission of a primary packet from Q_r and is given by

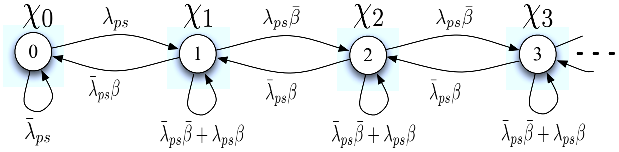

Figure 3: Markov chain of the relay queue

$$\beta=\pi_0\overline{P}_{\text{out},sp}a_s.$$

λ_{ps} is the arrival rate of primary packets to Q_r. χ_k is the probability of the relay queue having k packets at a certain time slot.

The balance equation around state 0 is given by

$$\begin{aligned}\chi_0\lambda_{ps} &= \chi_1\overline{\lambda}_{ps}\beta,\\ \chi_1 &= \frac{\lambda_{ps}}{\overline{\lambda}_{ps}\beta}\chi_0.\end{aligned}$$

The balance equation around state 1 is given by

$$\begin{aligned}\chi_1(\lambda_{ps}\overline{\beta}+\overline{\lambda}_{ps}\beta) &= \chi_0\lambda_{ps}+\chi_2\overline{\lambda}_{ps}\beta,\\ \chi_2\overline{\lambda}_{ps}\beta &= \left[\frac{\lambda_{ps}^2\overline{\beta}}{\overline{\lambda}_{ps}\beta}+\lambda_{ps}-\lambda_{ps}\right]\chi_0,\\ \chi_2 &= \frac{1}{\overline{\beta}}\left[\frac{\lambda_{ps}\overline{\beta}}{\overline{\lambda}_{ps}\beta}\right]^2\chi_0.\end{aligned}$$

The balance equation around state 2 is given by

$$\begin{aligned}\chi_2(\lambda_{ps}\overline{\beta}+\overline{\lambda}_{ps}\beta)=\chi_1\lambda_{ps}\overline{\beta}+\chi_3\overline{\lambda}_{ps}\beta,&\\ \chi_3\overline{\lambda}_{ps}\beta \quad = \quad \left[\frac{\lambda_{ps}^3\overline{\beta}_{ps}^2}{\overline{\lambda}_{ps}^2\beta^2}+\frac{\lambda_{ps}^2\overline{\beta}}{\overline{\lambda}_{ps}\beta}-\frac{\lambda_{ps}^2\overline{\beta}}{\overline{\lambda}_{ps}\beta}\right]\chi_0,&\\ \chi_3 \quad = \quad \frac{1}{\overline{\beta}}\left[\frac{\lambda_{ps}\overline{\beta}}{\overline{\lambda}_{ps}\beta}\right]^3\chi_0.&\end{aligned} \tag{21}$$

Therefore, for $k \geq 1$, we have

$$\chi_k=\frac{1}{\overline{\beta}}\left[\frac{\lambda_{ps}\overline{\beta}}{\overline{\lambda}_{ps}\beta}\right]^k\chi_0.$$

The normalization condition is given by $\sum_{k=0}^{\infty} \chi_k = 1$. Hence, we have $\chi_0 + \sum_{k=1}^{\infty} \chi_k = 1$; substituting for χ_k, we get

$$\chi_0 \left\{ 1 + \frac{1}{\overline{\beta}} \sum_{k=1}^{\infty} \left[\frac{\lambda_{ps}\overline{\beta}}{\overline{\lambda}_{ps}\beta} \right]^k \right\} = 1. \tag{22}$$

If $\lambda_{ps} < \beta$, we can show that

$$\chi_0 = \frac{\beta - \lambda_{ps}}{\beta}. \tag{23}$$

As argued before $\lambda_{ps} < \beta$ is the stability condition for the Q_r queue.

A primary packet is stored in Q_r if it is unsuccessfully transmitted by the primary user. Thus, we have two scenarios. First, when the SU is in the BACK-OFF mode, it does not access the channel and so a primary packet is stored in Q_r when there is an outage in the P-P link and no outage in the P-S link. Second, when the SU is not in the BACK-OFF mode and accesses the channel according to some access probability. In this case, a primary packet is stored in Q_r when there is an outage in the P-P link and no outage in the P-S link and the SU decides not to access the channel at this instant. Therefore, the arrival rate of the primary packets at the SU is given by

$$\lambda_{ps} = \sum_{i=1}^{\infty} \epsilon_i P_{out,pp} \overline{P}_{out,ps} + \sum_{j=1}^{\infty} \pi_j P_{out,pp} \overline{P}_{out,ps} \overline{a}_s, \tag{24}$$

which can be simplified to

$$\lambda_{ps} = P_{out,pp} \overline{P}_{out,ps} \left\{ 1 - \pi_0 - a_s \left[\pi_1 + \sum_{j=2}^{\infty} \pi_j \right] \right\}. \tag{25}$$

After manipulation, we can show that

$$\begin{aligned} \lambda_{ps} = {} & \frac{P_{out,pp} \overline{P}_{out,ps}}{\overline{\lambda}_p \eta^2} \times \\ & \left\{ \frac{\lambda_p^2 \eta (2\lambda_p - 1) a_s^2 + \lambda_p^2 (\lambda_p \overline{\eta} - \overline{\lambda}_p) a_s + \lambda_p \overline{\lambda}_p \eta}{(1 - \lambda_p a_s)} \right\}. \end{aligned} \tag{26}$$

Our final step in this analysis is to characterize the SU throughput defined as the rate of reception success at the secondary receiver. The SU makes a successful transmission when both Q_p and Q_r are empty, no outage in the S-S link and it makes a decision to access the channel at this instant. So the secondary throughput denoted by μ_s is given by

$$\mu_s = \pi_0 \chi_0 \overline{p}_{out,ss} a_s. \tag{27}$$

Finally, we need to find the access probability a_s that maximizes the secondary throughput μ_s. This is done using a simple one-dimensional exhaustive search; results are shown in the next section.

2) PU packet delay analysis: Next, we characterize the PU packet delay. As discussed before, the relaying capabilities granted to the SU increases the channel availability for its own packets, which, in turn, reduces the PU packet delay. A PU packet directly transmitted to the primary receiver experiences delay that is attributed to the time it spends in the primary queue Q_p; we refer to this delay as D_{Q_p}. On the other hand, a relayed packet spends some extra time in Q_r denoted as D_{Q_r}. Since the primary and secondary queues are not interacting[3], we can separate the delay analysis of both queues. First we quantify the average waiting time of packets in Q_p.

Applying Little's law [10],

$$E[D_{Q_p}] = \frac{\text{Expected number of packets in } Q_p}{\text{Arrival rate of packets to } Q_p}. \tag{28}$$

Therefore, we have

$$E[D_{Q_p}] = \frac{\epsilon_1 + \pi_1 + \sum_{k=2}^{\infty} k(\pi_k + \epsilon_k)}{\lambda_p}. \tag{29}$$

It can be shown that

$$E[D_{Q_p}] = \frac{(1 + \overline{\lambda}_p \eta a_s)(\Psi - \lambda_p)^2 + \lambda_P \overline{\Gamma}_p (2\overline{\lambda}_p \Psi - \lambda_p \overline{\Psi})}{\overline{\lambda}_p \Psi \eta (\Psi - \lambda_p)}. \tag{30}$$

Similarly, we can write the average waiting time of the relayed packets in Q_r as

$$E[D_{Q_r}] = \frac{\sum_{k=1}^{\infty} k(\chi_k)}{\lambda_{ps}}. \tag{31}$$

Substituting for χ_k yields

$$E[D_{Q_r}] = \frac{\overline{\lambda}_{ps}}{\beta - \lambda_{ps}}. \tag{32}$$

Now, we can write the average delay experienced by the primary packets as

$$E[D_p] = \frac{\lambda_p E[D_{Q_p}] + \lambda_{ps} E[D_{Q_r}]}{\lambda_p}. \tag{33}$$

B. No-Relaying Scheme

The no relaying scheme presented in [9] undergoes the same analysis given the fact that the SU has no relaying capabilities so a primary packet can reach its destination through the primary link only. η here is the probability of successful transmission of a primary packet to the primary destination when the SU is in the BACK-OFF mode and it is equal to 1-$P_{\text{out},pp}$. The SU successfully transmits its own packet when it accesses the channel during a primary-idle instant. The secondary throughput now becomes $\mu_s = \pi_0 \overline{P}_{\text{out},ss} a_s$. The PU packet delay in this case is the amount of time a packet spends in the PU queue before it is transmitted, so $E[D_p] = E[D_{Q_p}]$.

IV. NUMERICAL RESULTS

The cognitive relaying scheme with feedback shows a better performance than the one with no relaying capabilities in case of high outage in the P-P channel. For a small outage probability in the P-P channel, the two systems have very close performance.

In Figure 4, the secondary throughput μ_s is plotted against the arrival rate of packets to the primary queue, λ_P, for the

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[3] This is due to the assumption of always having packets to transmit at the secondary user.

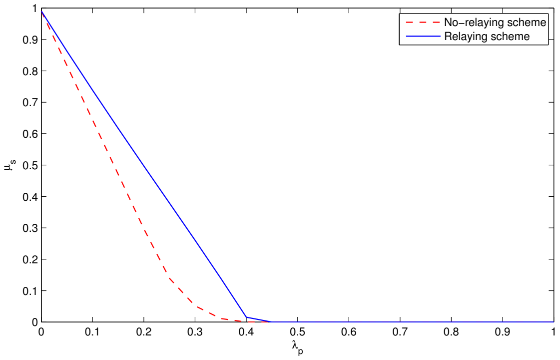

Figure 4: SU throughput vs. arrival rate of packets at PU

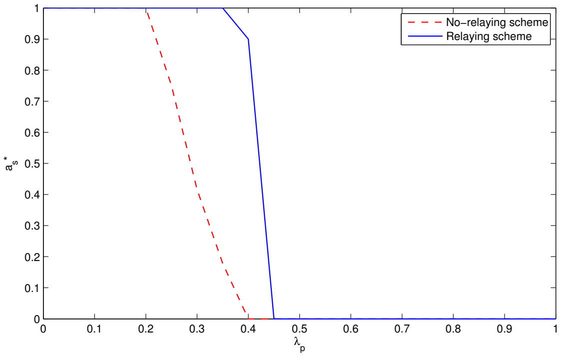

Figure 5: Optimal access probability for SU vs. arrival rate of packets at PU

two schemes. This is shown for the parameters $P_{\text{out},pp} = 0.6$, $P_{\text{out},ps} = 0.01$, $P_{\text{out},sp} = 0.05$, and $P_{\text{out},ss} = 0.01$.

In Figure 5, the optimal access probability a_s^* for the two schemes is plotted against the the arrival rate of packets to the primary queue, λ_P. The relaying scheme is shown to achieve higher access probability to the channel. This is due to the fact that relaying packets which couldn't be transmitted by PU it increases the channel availability for itself.

In Figure 6, the PU packet delay is plotted against the arrival rate of packets to the primary queue, λ_P. We can see that the PU gains from the relaying capabilities of the SU. The SU relays unsuccessfully transmitted primary packets instead of being retransmitted by the PU. This way the delay of these packets is decreased.

It is worth mentioning that the access probability for the SU includes the access probability acquired for both Q_r and Q_s, while the secondary throughput is only for the secondary packets in Q_s only.

V. CONCLUSION

An access scheme for cognitive relaying networks based on feedback information sent by the PU is introduced in this paper. The SU does not apply any sensing scheme, on the other hand it applies a random access policy. A packet unsuccessfully transmitted by the PU to the primary destination but successfully transmitted to the SU is stored in a relay queue at the SU in case it is successfully received by the SU. On the other hand, a packet that is not successfully received by neither the primary destination nor the SU is retransmitted by the PU. During primary retransmission, the SU backs-off avoiding a sure collision. This is proved to enhance the cognitive radio system performance from two aspects. First, avoiding sure collisions and second, relaying packets that could not be transmitted by the PU decreasing the number of retransmissions. The presented relaying scheme increases the channel availability for the SU and so increasing its throughput guaranteeing QoS requirements of the PU.

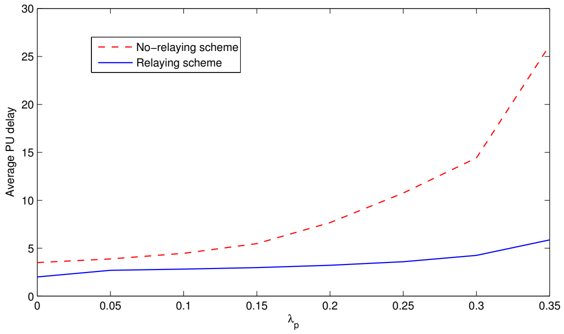

Figure 6: Average PU delay vs. arrival rate of packets at PU

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